Providing Future Secondary Teachers with a Mathematics Education

Lavanya Kashyap

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Edited by

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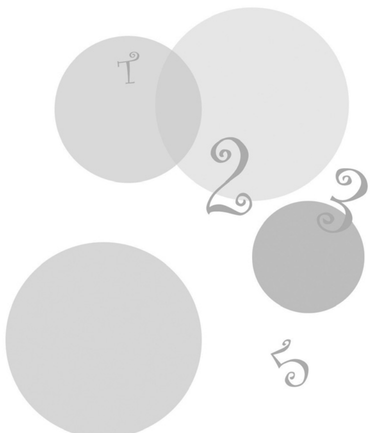
Table of Contents

Chapter 1	Fostering the development of mathematical competences in children of pre-school age and during compulsory education	**1**
Chapter 2	The role of information and communication technologies in other approaches to teaching high school mathematics	**94**
Chapter 3	Encouraging the development of students' cognitive domain within mathematics instruction	**139**
Chapter 4	The influence of convictions, views, norms, emotions and attitudes of mathematics teachers on the efficiency of mathematics instruction	**221**
Chapter 5	Examples of good teaching practice	**283**

1

Fostering the development of mathematical competences in children of pre-school age and during compulsory education



In this chapter the authors analyse segments of international research pertaining to pupils' achievements in mathematics. Presented are the comparative analyses of learning outcomes in mathematics from the neighbouring countries for the duration of compulsory education. The authors point to the continuous necessity for popularization of mathematics from the lower primary school age. Additionally, they highlight the fact that praise-worthy laws on inclusion, in other words laws that govern the inclusion of children with disabilities into formal instruction, should in our sorroundings move beyond words towards deeds, which is primarily the task of governments, and not necessarily of mathematics teachers. What is particularly emphasized is the need for improving the learning outcomes in mathematics among economically disadvantaged groups, such as the Roma people, with this section of the book providing certain guidelines for teachers of mathematics in these particular cases. Research on the pre-existing levels of mathematical knowledge of pupils points to the need for improving the level of written and spoken communication skills in pupils and teachers alike. Some of the scientists persist in encouraging new approaches which enable pupils to gain understanding of and acquire fundamental mathematical concepts (multiplicative concepts, plane mapping).

Regional comparison of the PISA 2009 results in the field of mathematical literacy

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Abstract. This work presents the analysis results on the requirements in PISA 2009 mathematical items. The analysis is based on all 35 mathematical literacy items, which were involved in PISA 2009 assessment. The research objectives were to determine what mathematical contents and activities are required in mathematical PISA 2009 items, to compare Croatian PISA results with the results of other countries in the region (Slovenia, Serbia, Montenegro), and to compare PISA and curricular requirements in Croatia. The results show that PISA 2009 requirements differ from the textbook and curricular requirements in Croatia to a great extent. The paper presents the detailed analysis of requirements in the items, in which the Croatian students showed poor results, compared to the OECD average and other countries from the region.

Keywords: Mathematics education, PISA assessment 2009, mathematical literacy, requirements

Introduction

Since the first Programme for International Student Assessment (PISA) in 2000, educational and public circles worldwide have paid a lot of attention to it in the context of national ranking as well as in commenting on the PISA theoretical background.

PISA examines reading literacy, mathematical literacy and scientific literacy among 15-year-old students (OECD, 2003).[1] In order to properly measure mathematical literacy, three components are distinguished: mathematical contents required for solving problems successfully, mathematical competencies, and the

[1] In the PISA framework mathematical literacy is defined as an "individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen" (OECD, 2003, p. 24).

mathematical context in which the problems are located (OECD, 2003; Braš Roth et al., 2008). The sources at `www.oecd.org/pisa` give detailed information about PISA assessment, including the assessment results.

In 2006 Croatia participated to PISA assessment for the first time as a partner country and was ranked 36th in the domain of mathematical literacy with the mean score 467. In the next cycle 2009 Croatia was ranked as the 40th country in the domain of mathematical literacy with the mean score 460 (Braš Roth et al., 2008; 2010). These results present Croatia again being statistically below the OECD average.

Analysis of PISA items

In 2009 the major domain was reading literacy, so there were only 35 mathematical items in PISA 2009. The PISA testing items are not publicly available, but the OECD published a set of released PISA items (OECD 2006), that give examples of unreleased PISA mathematical problems. Analyses on the publicly available PISA mathematical items for Croatia are, for example, conducted in Glasnović Gracin (2009) and in Glasnović Gracin and Vuković (2010). This paper brings the results on PISA 2009 unreleased items[2]. The analysis encompassed all PISA mathematical items involved in the PISA assessment in 2009 (35 items).

The research objectives of PISA items in the domain of mathematical literacy were: to determine what mathematical contents and activities are required in mathematical PISA 2009 items, to compare Croatian PISA results with the results of other countries in the region, and to compare PISA and curricular requirements in Croatia. Basis for content, activities and complexity requirements are taken from the Austrian Standards for Mathematics (IDM, 2007).

Results and discussion

The results are presented according to contents: Numbers, Geometry, Algebra and functional dependences, and Descriptive statistics and probability.

Numbers

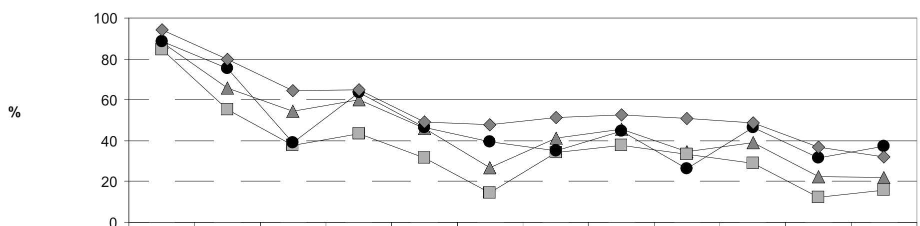
According to my research instrument criteria, there are 12 out of 35 PISA items within the field of Numbers and Measurements. Regarding content, they refer to natural numbers, decimals, percents or measurements. These contents fully correspond to the contents from the Croatian mathematics curriculum (MZOS, 2006). The analysis showed that in PISA 2009 the following activities were required:

[2] These items are analyzed courtesy of PISA Center in Zagreb, Croatia. The data given in this paper fully respect the PISA confidentiality. Since the analyzed items are not publicly available, the analysis results refer only to the item requirements, but do not expose them publicly.

2 out of 12 items required modeling or presenting activities; 11 out of 12 items required computation or operation activities; none of the items from the numbers field required interpretation or argumentation activities. Analysis also shows that 7 out of 12 items are on the simplest complexity level, 3 out of 12 items are on the connections level, and 2 out of 12 items are on the reflection level. The results show that half of the items in the field of numbers were closed answers, and the other half multiple-choice answer types[3]. This confirms that there were no open-answer items within the field of Numbers and Measurements in PISA 2009. These findings suggest that in its big idea of numbers PISA puts emphasis on simple items with requirements of computation and operating with natural or decimal numbers.

The results (Figure 1) show that the Croatian results and OECD average are similar in 8 out of 12 items. I took the difference of 7 maximal points (in percentages) to define the similar results. In 3 out of these 8 items Croatia has a slightly higher percentage than the OECD average. In 4 out of 12 items Croatian students had poorer results in comparison to the OECD averages. These items are all *not typical* for mathematics education in Croatia.

Since Croatia is not an OECD participant, perhaps it would be more interesting to compare Croatian results to those of some other countries, such as Slovenia, Serbia and Montenegro. These neighboring countries shared the same school system with Croatia in former Yugoslavia until 1990. Figure 1 shows that Slovenia has significantly better results in the numbers field in than the other three countries. In most of the items Croatia had better results in the field of numbers than Serbia and Montenegro.



	1	2	3	4	5	6	7	8	9	10	11	12
OECD	88,97	73,12	69,01	65,67	51,51	47,85	46,39	45,76	44,78	43,49	38,38	34,78
SRB	88,17	65,80	54,26	60,09	45,86	26,67	41,29	45,82	34,75	39,04	22,15	21,81
MNE	84,71	55,38	37,75	43,42	31,65	14,42	34,15	37,68	33,28	29,00	12,38	15,61
CRO	88,67	75,33	38,93	63,72	46,56	39,39	35,14	44,77	26,28	46,53	31,43	37,35
SLO	94,11	80,02	64,44	64,83	49,18	47,9	51,23	52,76	51,09	48,56	36,65	32,12

Figure 1. PISA 2009 Numbers.

[3] In this research the closed answer items require one correct (mostly short) answer. The open answer items require a more free expression of students' ideas about mathematics. The closed answer items put more emphasis on the final solution, while the open answers are more concerned with the process and the way of solving a particular problem. Multiple-choice items offer a limited number of already defined responses.

Figure 1 shows[4] that in 2009 Croatia had poor results in two items in particular compared to the OECD average and other countries from the region. Both items are not commonly found in mathematics education in Croatia:

- The item M446Q01 (item 3 on Figure 1) was successfully solved by 69,01% of OECD students (on average) but only 38,93% of Croatian students. The results from PISA 2006 show that Croatian students had the same percentage three years ago (38,62%). This item requires simple mathematical requirements with a closed answer form, but the item also requires modeling activities. Also, the rich text and the context unusual for Croatia surely affected the results. Its mathematical requirements are simple, but the text has to be read carefully with full concentration, and the students need to be confident in their abilities.
- The item M411Q02 (item 9 on Figure 1) was successfully solved by 44,78% of OECD students on average, but only 26,28% of Croatian students. This item is very rich in text (more than 100 words in Croatian translation) and requires students' engagement on the reflection level. Besides dealing with numbers according the given rules, it required some basic knowledge of statistics. Such an item is not common in mathematics education at all. In the 2006 assessment this item was successfully solved by even less Croatian students (21,41%). A lot of text, mathematics in the service of problem solving and *reflective thinking* requirements surely influenced these poor results.

Geometry

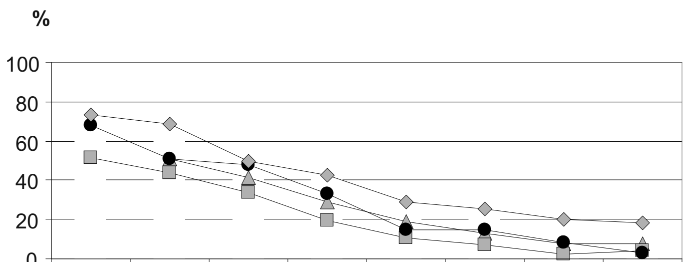
The mathematical literacy domain in 2009 consisted of 8 (out of 35) items from the geometry field. These items require the following mathematical topics: 5 out of 8 items refer to plane shapes, 2 items refer to spatial ability, and 1 item refers to isometric mappings. The analysis shows that 6 out of 8 items fully match the contents from the Croatian mathematics curriculum (MZOS, 2006). Only two items which refer to spatial ability competencies are not an explicit part of Croatian curriculum. The analysis shows that 3 out of 8 items involved modeling or presenting activities, 5 items required computation activities, 2 items involved interpretation activities[5], and none of the items involved argumentation activities. The complexity analysis brings interesting results: none of the items required reproduction, 5 items are on the connections level, and 3 items are on the reflection level. These findings suggest that PISA 2009 put emphasis on more complex requirements in the field of geometry than in arithmetic. Only 1 out of 8 items is intra-mathematical. All context-based items have at least one scheme or picture attached. According to my research, the analysis shows that 5 items require closed answer, 3 items are multiple choice forms (2 regular multiple choice, 1 complex multiple choice), and none of the items require open answer.

[4] Source: Compendium for the cognitive item responses, `http://pisa2009.acer.edu.au` (available on Jan, 5th 2011)

[5] One item may have required more than one activity.

The analysis shows that only 2 out of 8 geometric items in PISA 2009 are considered as *typical* for geometry education in Croatia. It is precisely the reflection and more complex connection requirements, as well as interpretation skills and problem situations (in text format) that make it untypical for geometry education in Croatia. Such items are not often found in mathematics education in Croatia (Glasnović Gracin, 2011).

The results show that Croatian students showed worse results than the OECD average in all geometry items, even in the ones marked as *typical* for mathematics education in Croatia (items 6 and 8 on Figure 2).



	1	2	3	4	5	6	7	8
OECD	75,29	67,45	52,72	42,37	26,72	23,24	16,67	11,37
SRB		50,89	41,41	29,17	18,85	12,91	7,81	7,66
MNE	51,62	43,71	33,76	19,51	10,37	6,85	2,38	4,18
CRO	68,20	50,98	48,09	32,88	14,52	14,74	8,22	3,02
SLO	73,25	68,54	49,49	42,86	28,94	25,36	19,85	18,38

Figure 2. PISA 2009 Geometry.

The results show that Slovenia has better results than Croatia in all geometrical items. Croatia has better results than Serbia and Montenegro, but there are two items in which Croatian students showed significantly poorer results in comparison to other countries. These items call for deeper analysis:

- Item M406Q01 was successfully solved by 26,72% of OECD students on average but only 14,52% of Croatian students (item 5 on Figure 2). Serbian students solved it in 18,85% cases. The results from PISA 2006 show that Croatian students had a similar poor percentage (15,40%) three years ago on this item. This item requires more complex computational requirements, an understanding of wider geometrical ideas, concentration in reading rich text, and understanding the situation. These parameters are not common for mathematics textbooks requirements.

- Item M462Q01 (item 8 on Figure 2) was successfully solved by 11,37% of OECD students (average), 18,38% of Slovenian students, 7,66% of Serbian students, and 4,18% of Montenegro students, but only 3,02% of Croatian students in 2009. In 2006 it was solved by 5,13% of Croatian students.

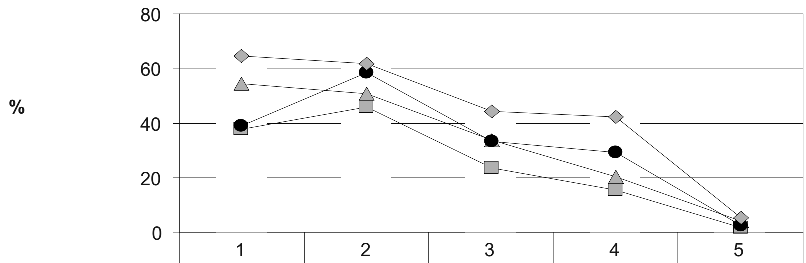
Interestingly, this item is intra-mathematical and is marked as *usual* for mathematics education in Croatia because it was found in textbooks during the textbook analysis in Glasnović Gracin (2011). Therefore it is important to examine it more closely. First of all, the translation should be clearer to students, it is very likely that some students did not understand the problem properly. In their report, the Croatian PISA 2006 expert group for mathematics mentions this particular problem. "One item contained a language problem, so we think that the students did not understand the question properly" (Braš Roth et al., 2008, p. 161, translation: D.G.G.): The other reason for poor result could be the requirement of grasping the whole idea of the triangle, not just following some computation or operation procedures.

Functions and Algebra

According to my research instrument criteria, there are 5 out of 35 PISA items in the field of Functions and Algebra. One of these items belongs to both Numbers and Functions. Comparing these findings to the other fields, the analysis shows that the contents of Functions and Algebra are the least required in PISA assessment. Three out of 5 items required non-linear functions or relations, which are not part of the Croatian curriculum for mathematics (MZOS, 2006), and 2 items required familiar knowledge about proportionality and linear function. The analysis shows that in PISA 2009 the following activities were required: 3 out of 5 items required modeling or presenting activities, 3 items required computation activities, 2 items required interpretation activities, and none of the items required argumentation activities. There were some items that required two activities. In terms of complexity, the analysis of PISA 2009 items showed that 1 out of 5 items are on the simplest complexity level, 1 item is on the connections level, and 3 items are on the reflection level. Two items require closed answer, 3 are multiple choice (2 multiple choice and 1 complex multiple choice), and none of the items require open answer in the field of Functions and Algebra. All the examined PISA 2009 items from the field of Functions and Algebra are put in context.

These findings suggest that in the functions field PISA puts emphasis on the reflection knowledge items with requirements of modeling, computation and interpretation and mostly with non-linear contents. These requirements are not predominant in Croatian mathematics textbooks in the field of functions and algebra (Glasnović Gracin, 2011). The analysis shows that *not one* of these five items in PISA 2009 is considered common in mathematics education in Croatia. None of them are typical textbook items, primarily because of the content (non-linear functions) and reflection requirements.

The results show that Croatian students have worse results than the OECD average in all five items from the field of functions and algebra (Figure 3). One of the items shows significantly worse results compared to other countries (item 1 on Figure 3). This item (M446Q01) is already analyzed before, because it requires knowledge of both Numbers and Functions. The results show that students find functions and algebra requirements in PISA 2009 assessment more difficult than numerical items.



	1	2	3	4	5
OECD	69,01	63,31	46,58	41,12	7,07
SRB	54,26	50,56	33,58	20,46	4,26
MNE	37,75	45,81	23,39	15,39	1,43
CRO	38,93	58,56	33,48	29,25	2,29
SLO	64,44	61,90	44,22	42,43	5,43

Figure 3. PISA 2009 Functions.

The comparison with other neighboring countries (Figure 3) shows that Serbian students achieved better results in 3 out of 5 items compared to Croatian students. The results from the other neighboring countries are similar to the geometry results.

Statistics and Probability

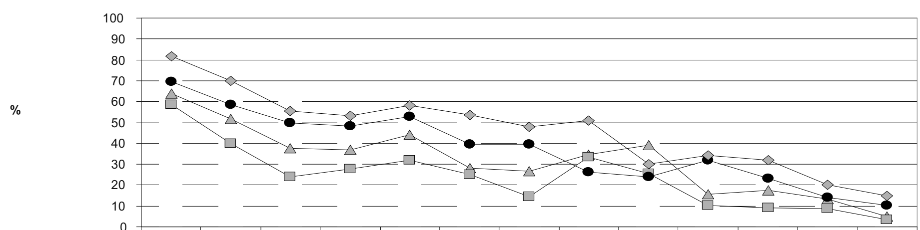
According to my research instrument criteria, there are as many as 13 out of 35 PISA items from the field of Statistics and Probability. Out of these 13 items 4 involve bar charts, 4 involve arithmetic mean, 3 line charts (which are not covered in the Croatian mathematics curriculum), and 3 involve probability contents (they are shown with the dashed pattern on OECD grey bars in Figure 4).

One item required presenting activities, 8 items required computation activities, 9 items required interpretation activities, and none of the items required argumentation activities. There were some items that predominantly required two activities. Two items are on the simplest complexity level, 6 are on the connections level, and 5 are on the reflection level. The analysis shows that out of 13 items 8 require closed answers, 5 require multiple choice (2 simple multiple choice and 3 complex multiple choice), and none require open answers. All the examined PISA 2009 items from the field of statistics and probability are put in real life situations.

The analysis shows that 10 out of 13 PISA 2009 items are considered as *not usual* for statistics and probability education in Croatia. They are not usual mostly because of the reflection requirements, interpretations and rich textual problems.

The results show that Croatian students showed poorer results than the OECD average in all 13 items from the field of statistics and probability (Figure 4). These items contain contents usual for mathematics education in Croatia, but other parameters such as activities, complexity, context or answer type surely affected the Croatian results:

- Item M411Q02 (item 8 on Figure 4) is already analyzed here, because it requires knowledge of both Numbers and Statistics.
- Item M408Q01 (item 9 on Figure 4) is considered as not usual for mathematics education in Croatia. It is from the field of probability and Croatian students did worse in this item not only compared to OECD, Slovenia, Serbia and Montenegro. The item requires interpretation activities on the reflection level. Since probability contents are poorly covered in Croatian compulsory education, the lack of practice and not placing emphasis on understanding the ideas could be also affecting this result.



	1	2	3	4	5	6	7	8	9	10	11	12	13
OECD	79,14	66,30	61,55	56,04	54,88	50,56	47,85	44,78	40,24	32,30	28,48	27,30	18,49
SRB	63,98	51,58	37,76	36,86	44,03	28,26	26,67	34,75	39,00	15,67	17,50	13,14	4,99
MNE	58,62	39,99	24,05	27,93	32,12	25,15	14,42	33,28	25,59	10,26	9,26	8,65	3,35
CRO	69,59	58,38	49,78	48,14	52,82	39,71	39,39	26,28	23,96	31,93	23,16	14,16	10,18
SLO	81,70	69,84	55,59	53,39	58,01	53,65	47,90	51,09	29,98	34,18	31,95	20,09	14,81

Figure 4. PISA 2009 Statstics and Probability.

It is interesting to note that the statistics and probability became official part of the Croatian curriculum no earlier than autumn 2006 in the 7th grade of compulsory education (13-year olds). This means that the 15-year old students that participated in PISA assessment in 2006 were *not* taught Data and Chance in mathematics education at all, and the participants in 2009 had statistics in school. Still, the results from 2006 are no worse than the results from 2009. Only approximately 1–2% of the total mathematics education is devoted to statistics and probability, which is not enough time to practice these contents and to reflect on the ideas in statistics and probability. So, it may be that for Croatian students real life experience is still the main source of understanding statistic and probability concepts. PISA 2009 required statistics and probability in 13 out of 35 items (37%) according to my instrument.

Conclusions

About one third of PISA 2009 mathematical items belong to the big idea of numbers, one third is statistics and probability and one third belongs to both geometry and functions. The overall results show that 77% of mathematical items in 2009 required computation activities, 26% presentation, 37% interpretation and 0% argumentation activities. This lack of argumentation requirements is not fully in accordance with the definition of PISA mathematical literacy, because argumentation is an important part of everyday mathematics. Numbers and statistics required more reproduction and simpler connection items, while geometry and functions required more difficult items. Reflection requirements are present in all overarching ideas, but they are not connected to argumentations in PISA 2009, but to interpretation activities.

The research results show that the PISA 2009 items mostly required closed answers (19 out of 35). The rest were 16 multiple choice items. According to my study, PISA 2009 did not require any item with open answer. Almost all PISA items are very rich in text and are put in a realistic or authentic context. That makes them completely different from the textbook items in the fields of numbers, geometry and functions. Croatian textbook items mostly have closed intra-mathematical contexts in these three fields (Glasnović Gracin, 2011). All these results suggest that the mathematical requirements from the textbooks and plan and program (MZOS, 2006) are different to a great extent from the PISA 2009 requirements. The differences between school and PISA requirements most probably influenced the results, particularly in some items.

In comparison to other countries from the region (Slovenia, Serbia, Montenegro), Croatia has poorer results than Slovenia in PISA 2009, slightly better than Serbia and better than Montenegro. These results may be the basis for further analyses and researches of the mathematics education in the region.

At the very beginning of this study, an important question arose: *What are the things that we should take from PISA in order to improve mathematics education?* Although the objectives of mathematics education can not be fully compared with PISA objectives, some of the requirements that could be taken from PISA are, for example: connecting mathematical contents with realistic and authentic contexts to a greater extent, greater usage of mathematics within problem situations (not just to practice a routine, but to really solve a particular *problem*), more focus on reflective requirements and understanding mathematical concepts within textbook exercises, introducing a thorough approach to the idea of function as well as implementing it in authentic situations, having more focus on statistics and probability in compulsory education, especially through everyday interpretations.

These suggestions should be implemented not with the aim of getting better results in PISA assessment, but to truly improve mathematics education and to encourage students in reflecting on mathematical ideas.

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Regionalna usporedba rezultata iz matematičke pismenosti na PISA istraživanju iz 2009. godine

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Sažetak. Ovaj rad prikazuje rezultate analize zahtjeva u PISA istraživanju 2009. godine iz područja matematičke pismenosti. Analiza se temelji na svih 35 matematičkih zadataka koji su se pojavili na PISA testiranju 2009. godine. Ciljevi istraživanja su bili odrediti koji matematički zahtjevi su traženi u PISA zadacima, usporediti rezultate hrvatskih učenika s onima iz zemalja regije (Slovenija, Srbija, Crna Gora) te usporediti zahtjeve iz PISA studije sa zahtjevima iz udžbenika i nastavnog plana i programa za matematiku u Hrvatskoj. Rezultati pokazuju da se zahtjevi iz PISA 2009 istraživanja u velikoj mjeri razlikuju od kurikulumskih zahtjeva u Hrvatskoj. Članak donosi i detaljnu analizu zahtjeva u zadacima u kojima su hrvatski učenici pokazali naročito slabije rezultate u odnosu na rezultate zemalja iz regije, kao i obzirom na prosjek zemalja članica OECD-a.

Ključne riječi: nastava matematike, PISA istraživanje 2009., matematička pismenost, zahtjevi

Levels of geometric thinking in the second triad of elementary school

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Abstract. Dutch mathematician Pierre M. van Hiele developed a theory how to determine a level of geometric thinking of an individual on the basis of solving geometrical problem. He designed and accurately described five levels of geometric thinking. His findings encouraged several researchers to deal with this issue. Van Hiele's findings also affected changes of the curriculum in many countries. We decided to research this problem because in Slovenia no significant research dealing with levels of geometric thinking in elementary school has ever been made. 782 pupils from grades four, five and six of elementary schools solved a test that we had prepared. We found out that the majority (60.7%) of pupils in the second triad are between level 0 (visual) and level 1 (descriptive) of geometric thinking. Nearly a third (31.7%) of pupils aged 9-11years are on the first level and 4.3% are on level 0. According to the whole test, only 1.4% of children were on the second (informal deduction) level of geometric thinking. We found out that pupils know certain properties of geometric figures. But these properties are not logically arranged and moreover, pupils are not able to apply knowledge to new situations. Among other things, we confirmed that there is a difference among classes. Older pupils are at a higher level of geometric thinking than younger ones. We did not notice any difference between genders. We also confirmed the assumption that pupils who have a higher grade in mathematics are at a higher level of geometric thinking. We found out that they are using different levels of thinking, when they are solving problems with different concepts. Using appropriate vocabulary was the biggest problem that the pupils had. They are still confusing basic geometric concepts such as edge and side. This paper briefly presents a model of teaching that aims at improving geometric vocabulary and at the same time promoting higher levels of geometric thinking.

Keywords: geometry, van Hiele, levels of geometric thinking, second triad, encouraging higher levels of geometrical thinking

Introduction

Geometry is an abstract science, where the relations among different concepts are controlled through an appropriate use of language. Researchers across the world concluded that students have many difficulties with geometric issues. Pierre M. van Hiele dealt with the connection between language and geometric concepts. He developed a theory how to identify levels of geometric thinking based on solving geometric problems.

Levels of geometric thinking

Van Hiele (1999; van Hiele, 1957, in Fuys, Geddes and Tischer, 1984) analyzed how students solved geometric problems and, based on solutions, formed the following levels of geometric thinking:

- **Visual level** (level 0) where students identify shapes according to their appearance.
- **Descriptive level** (level 1) where shapes are not judged by their appearance but rather because they have certain properties.
- **Level of informal reasoning** (level 2) where properties are logically ordered.
- **Level of formal reasoning** (level 3) where an individual uses deduction to prove the principles. It deals with the axioms and the necessary and sufficient conditions.
- **Rigorous level** (level 4) where the comparison between different geometric systems is based on axioms.

Van Hiele (1957, in Fuys, Geddes and Tischer, 1984) claims that the level of an individual depends more on learning than on age or maturity. This means that the way of teaching can accelerate or retard the development of geometric thinking. Burger, Shaughnessy (1986), Wu and Ma (2006) also confirmed that the levels of geometric thinking are not strictly defined by age or by the class students attend.

Van Hiele (1959, in Mayberry, 1983) also states that students cannot work properly at some level, if they do not have any experience allowing them to think at this level. He emphasizes that if the language used by the teacher in teaching geometric concepts is on a higher level than the level at which the student is, then the student does not understand the subject matter (van Hiele, 1959, in Mayberry, 1983). Van Hiele (1957, Fuys, Geddes and Tischer, 1984) believes that individuals must pass the levels by type and that it is possible to identify the level on which an individual is located by the way they solve geometrical problems.

The latter was confirmed by Mayberry (1983), Burger and Shaughnessy (1986). Burger, Shaughnessy (1986) and Wu and Ma (2006) also agreed with the statement from Mayberry (1983) that individuals are on different levels when they are solving problems of various concepts and that some students never get to the level of formal reasoning.

Like other researchers, Usiskin (1983) had troubles determining the level of students. He conducted a study, which included 2699 students aged from 14 to 17 years. He came to a conclusion that at the end of the school year after being taught contents of geometry, 20% of the students were on level zero, 26% on level 1, fewer than that (22%) on level 2, and on the third level there were only 6% of the students. 12% of respondents were below level zero. He could not classify 14% of the students.

Wu and Ma (2006) investigated, at which level of geometric thinking were 5581 students from first to sixth grade. They found out that all the pupils of the first and second grade are below or on level zero (Wu and Ma, 2006). The majority of pupils in the third grade are on level zero, while the maximum number of pupils in fourth, fifth and sixth grade are on the first level of geometric thinking (Wu and Ma, 2006). Only in the fifth and the sixth grade there are around 20% of pupils on the second level (Wu and Ma, 2006).

Yildiz, Aydin and Köğce (2009) came to a conclusion that after being taught according to the new curriculum, only 9% of pupils in the sixth grade were at level zero and 41% on the second level. In seventh grade, the percentage of children on the second level did not change; what changed was the percentage of children on level 0, from 8% to 0% and on the first level from 38% to 46%, always in favour of the new curriculum (Yildiz, Köğce and Aydin, 2009). In other classes, the difference among different classes is not noticeable as to the new or old curriculum.

Empirical part

It is evident that van Hiele's findings prompted several researchers to deal with this issue and at the same time affected the teaching of geometry in the world. While in Slovenia such a study has not been done, we decided to explore this topic.

Problem definition and methodology

With this research we tried to determine at what level of geometric thinking in the content of shapes are the pupils who attend grades four (9-year-olds), five (10-year-olds) and six (11-year-olds) of primary school. We also wondered if the way of teaching geometric shapes in Slovenian primary schools, promotes higher or lower levels of geometric thinking in comparison to other studies conducted.

Research questions

The research questions that we set are:

- At what level of geometric thinking are pupils aged from 9 to 11?
- Are there any differences in levels of geometric thinking among male and female pupils who are in different classes of primary school?

- What is the relationship between grades in mathematics and the level of geometric thinking on which the pupils are?
- Are the pupils on the same level of geometric thinking when doing various tasks?

Sample description

The test papers which we used to determine at which level of geometric thinking are the pupils in the second triad, were completed by 782 children, of whom 385 were boys (49.2%) and 387 girls (49.5%). The proportion of pupils from each class was almost the same.

Data collecting procedures

In order to determine at what level of geometric thinking are pupils, we randomly selected 20 Slovenian schools. From each school, one section of the fourth, fifth and sixth grade took part. The vast majority (95%) of parents agreed that their child could participate in the study. Then we sent test papers and detailed instructions to the schools. The instruction was that the students solve the tests during their classes in no more than 45 minutes, and that the teachers should encourage the students, but not help them. The tests were taken in May and June 2010.

Results and interpretation

The exam consisted of eight tasks, each of which consisted of several parts, altogether the students had to solve 19 tasks. Six tasks verified the second level of geometric thinking. It should be noted that the tasks that we assigned to the second level, were possible to solve also on the first level. On average 17% of the pupils did not solve an individual task. What follows are the results of those students who solved the tasks.

On level zero of geometric thinking we included the answers which suggest that the pupils responded on the basis of appearance of the shape, as well as those that from the perspective of mathematical correctness are nonsense. On the first level of geometric thinking we assigned the answers, where it was noted that the students knew the characteristics of the shapes, which among other things means that their answers were correct in mathematical terms. The second level of geometric thinking was assigned to the pupils, where it is evident from the responses that not only did they know the properties of geometric figures, but these properties were also logically arranged.

In determining the levels we, like many other researchers, had difficulties (Burger and Shaughnessy, 1986; Fuys, Geddes and Tischer, 1988; Usiskinu, 1982). It turns out that a large number of students are in the transition between the first level of geometric thinking and level zero (these students do not look at shapes as a whole, but take into account the specific characteristics of the shape, and at the same time they make a lot of mistakes) as well as the transition between the first and the second level. That is why we decided to form another two levels, 0.5 and level 1.5. On level 1.5 we classified those pupils who are very familiar with the characteristics of individual shapes, but these properties are not logically arranged.

In the first exercise the students had to name different geometric shapes that were drawn for them: a rectangle, a square, a hexagon, an 'upright' and a 'narrow' triangle and an 'upright' and a 'narrow' rectangle. Pupils did not have any problems with naming the shapes. The only exception were a narrow and an upright rectangle for which almost a third of the fourth grade pupils wrote that it was a quadrilateral.

The second exercise consisted of three parts (2.a, 2.b and 2.c) with drawn shapes. Pupils had to eliminate one shape and justify their decision.

Table 1. Levels achieved at the second exercise.

LEVEL	**EXERCISES**		
	2.a	**2.b**	**2.c**
0	12.6%	12.5%	8.4%
0.5	22.7%	9%	9%
1	64.8%	78.4%	82.5%

Table 1 shows that most pupils circled and properly explained why the selected shape does not belong among the other shapes.

The instruction for the third exercise was: draw a quadrilateral that has only one pair of parallel sides. More than half (54.8%) of the students drew the appropriate shape. This was assigned to the first level of geometric thinking. 35.7% of the students drew a shape which had two pairs of parallel sides. These pupils were assigned to level 0.5, while other pupils drew a shape without parallel sides. We assigned them to level 0.

In task 4.a we examined how many pupils solved the exercise on the second level of geometric thinking. Pupils had to draw three different triangles. The second level was assigned to 41.7% of the pupils, 31.5% were assigned to level 1.5. The first level was assigned to 25.6% of the pupils, and level 0.5 to 1.2% of the participating pupils.

In exercise 4.b the pupils were asked to write down how many different triangles could be drawn. 24% of the students were assigned to level 2, 30.9% to the first level, and 45.1% to level 0.5.

In the fifth exercise the pupils had to write letter S into the squares and letter R into rectangles. Nine quadrilaterals were drawn. Among them were two squares, three rectangles, two parallelograms and two trapeziums. In exercise 5.b they had to list as many differences as possible and in exercise 5.c as many similarities between a rectangle and a square as they could. In the following exercise (5.d) the first question was whether the rectangle is a square, then they had to write why they thought so. Exercise 5.e contained the question if a square was a rectangle, and then they had to justify their answer.

Table 2. Achieved levels at fifth exercise.

LEVEL	EXERCISES				
	5.a	5.b	5.c	5.d	5.e
0	46.9%	27.9%	7.6%	36.3%	39%
0.5	29.1%	13.2%	23.8%	33.9%	24.7%
1	24%	58.9%	68.7%	13.8%	3.1%
1.5				0.3%	1.5%
2				15.7%	31.7%

Table 2 shows that we observed the second level of geometric thinking in exercises 5.d and 5.e. The pupils were more successful in exercise 5.e where 31.7% reached the second level and in 5.d with 15.7%. In the first three parts of the fifth exercise pupils were most successful in exercise 5.c. They had to write the similarities between a rectangle and a square. In exercise 5.a the students reached the worst results as one fourth of the participants did not reach the first level, which means that among different quadrilaterals the pupils did not recognize the rectangle and the square.

The sixth exercise requested that the pupils observe different quadrilaterals and encircle the shapes that have a common property. Then these shapes were drawn twice and each time they had to encircle shapes with the common property, but this property had to be different from the previous one. Each time they also had to write the property of the encircled shapes.

Table 3. Levels achieved in the sixth exercise.

LEVEL	EXERCISES		
	6.a	6.b	6.c
0	20.4%	29.9%	32.8%
0.5	42.9%	51%	46.4%
1	36.7%	19.8%	20.8%

Pupils had difficulty with the sixth exercise, which is reflected in the low percentage of the students who reached the first level. In exercises 6.b and 6.c only a fifth of the pupils properly encircled the shapes and explained their decision. Most pupils were in the transition between level zero and the first level of geometric thinking. We found out that they had a problem using appropriate vocabulary.

In the seventh task pupils were asked to write the name of the shape which has two pairs of parallel sides. This did not present a lot of problems, because 88% of the pupils were assigned to the first level, 4.3% to level 0.5 and 7.6% of pupils to level zero.

The last (eighth) exercise was composed of several parts. First, some rhombs were drawn. It was also explained that we had invented a name for them, they were called purps. Then there were different quadrilaterals, including the rectangle and parallelogram. Below them it was written that they were not purps (rhombs). In exercise 8.a there were different quadrilaterals and pupils had to to encircle those purps (rhombs). In exercise 8.b the pupils had to draw the purps. Below (8.c) they had to describe the purps. In the last part of the eighth task (8.d) the pupils had to write if the square is also a purp and justify their answer. The following table shows the results of the eighth exercise.

Table 4. Levels achieved at eighth exercise.

LEVEL	EXERCISES			
	8.a	8.b	8.c	8.d
0	6.2%	20.3%	2.6%	14%
0.5	32.6%	17.4%	24.1%	13.6%
1	61.2%	62.3%	50.9%	50%
1.5			0%	0.5%
2			22.4%	21.9%

Table 4 shows that most of the pupils are in the first level of geometric thinking in all of the eighth exercise. Exercises 8.c and d checked the second level of geometric thinking. We found out that about a fifth of the pupils were on the second level.

We were interested in the average level of geometric thinking. We calculated it by adding up the percentages of achieved levels of individual exercises and divided the sum by the number of exercises. We found out that the majority of pupils (38.9%) are on the first level of geometric thinking, but the fewest of them are on level 1.5, namely 6.7%. On level two are 22.2%, at level zero 15.4% and at 0.5 20.9% of pupils. Because we were interested in the average level of the whole exam, not only in the individual tasks, we numerically evaluated individual levels. When we assigned a pupil level zero, we gave him 0 points. Level 0.5 pupils were assigned 0.5 points, 1 point to the first level, level 1.5 got 1.5 points and 2 points were given for the second level. The pupils who did not solve the exercise were assigned level zero (0 points). We have found that the second level of geometric thinking was attributed to only 1.4% of respondents. Most pupils are at level 0.5 that is 60.7%, on the first level there are almost half less (31.7%). At level zero are 4.3% of pupils. From the results it can be concluded that the majority of pupils in the second triad know properties of geometric shapes, but these properties are not logically arranged. We found that pupils have difficulties using the appropriate geometric vocabulary and that they are not able to use this knowledge in new circumstances.

For each task, we looked for statistically significant differences between classes and achieved levels of geometric thinking, the final grade in mathematics as well as the difference between the sexes. We confirmed that students in higher grades are at a higher level of geometric thinking, which was also written by Wu and Ma (2006). However, we also agree with the finding which was written by Burger and Shaughnessy (1986), namely that the level of geometric thinking is not strictly defined by age or class which pupils attend. We confirmed the assumption that the pupils who have a higher score in mathematics are at a higher level of geometric thinking. Gender proved to be significant only in three exercises. Girls were better twice and boys once.

Conclusion

By making this study we confirmed the opinion of Burger and Shaughnessy (1986), Wu and Ma (2006) and Mayberry (1983) that according to various contents to do with geometric shapes, individuals are on different levels.

If we compare the obtained results with those obtained by Wu and Ma (2006), we find that the pupils in our study were slightly worse in achieving the first level. In our study the first level, on average, was reached by 38.9% of pupils, in Wu and Ma (2006) the percentage was 52.2% of the pupils who attend the fourth, fifth and sixth grade. On the second level of geometric thinking, there were 22% of the pupils, while in the study performed by Wu and Ma (2006), there were 0% in the fourth, 15.3% in the fifth and 6.3% in the sixth grade.
Most students (26%) aged 14 to 17 years, whom Usiskin (1982) taught for 140 hours, were on the first level, 22% were on the second, and 20% on level zero. Percentages of students at each level who participated in our study, are better, because there are 38.9% on level one, 15.4% on level zero, while the percentage of pupils on the second level is almost the same.

After the use of the new curriculum Yildiz, Aydin and Köğce (2009) found out that in the sixth grade 9% of the pupils were at level zero, and as many as 41% (and only 20% before the implementation of the new curriculum) at level two. In our study, 36.8% of the sixth-grade students were on the second level, which shows no significant deviations, but only 16% were on level zero, which does not show large deviations either.

The results and other findings that we obtained with this survey are therefore comparable to the results of other researchers. We were surprised that the pupils do not recognize rectangles and squares among different quadrilaterals (task 5.a). Only a quarter of pupils correctly solved the task mentioned (which is one of the basic tasks) that checks the first level of geometric thinking. Among other things, we found out that pupils aged 9 to 11 years have major problems with forming and using the appropriate geometric vocabulary. In addition, they still replace the basic geometric concepts, such as edge and side, or they do not use the basic concepts which indicate the level zero. Therefore, when teaching geometry contents, a particular emphasis should be paid to the development, consolidation and use of geometric terms, which is the basis for the formulation of thought and knowledge and a proof of the actual level of geometric thinking.

In the review of textbooks used in teaching mathematics in the second triad, we found out that they lack promoting the use of different materials, that tasks are not diverse and that there are only a few or no tasks that can be solved at the second level of geometric thinking. In order to promote higher levels, specific contents should be explained, described and understood as for example the hierarchy between concepts. On the basis of information given a new conclusion should be formed, new knowledge should be used and their own solution justified.

In order to develop an appropriate geometric vocabulary and encourage the transition to higher levels of geometric thinking, the contents of geometry should be taught step by step, as they were formulated by Dina van Hiele-Geldof and Pierre M. van Hiele (inquiry, directed orientation, explication, free orientation and integration). It is necessary to use a diverse material and give students a number of practical tasks. It is also important to include discussion. Pupils need to be properly motivated for work. This can be done by creating a variety of games, such as memory, Ludo or Black Peter, that are adapted to particular contents. Teaching should often be carried out in pairs or in groups, while little time is devoted to frontal interpretation. In addition, pupils should justify their solutions. It would be necessary to address the hierarchy of concepts, and often use the acquired knowledge in new situations. They should be given problems that can be solved on the second level of geometric thinking. This would promote thinking with pupils who are not yet on the second level and pupils who would have the opportunity to use the achieved level. It is therefore necessary to create suitable circumstances for the students to be able to get as much experience as possible and as many challenges as possible.

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Nivoji geometrijskega mišljenja v drugem triletju osnovne šole

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Izvleček. Nizozemski matematik Pierre M. van Hiele je razvil teorijo, pri kateri se na podlagi reševanja geometrijskih problemov določi nivo geometrijskega mišljenja, na katerem se posameznik nahaja. Oblikoval in natančno je opisal pet nivojev geometrijskega mišljenja. Njegove ugotovitve so spodbudile številne raziskovalce, da so se ukvarjali s to tematiko. Van Hielove ugotovitve so med drugim vplivale na spremembe učnih načrtov po svetu. Ker v Sloveniji ni bila opravljena raziskava, s katero bi se ugotavljalo, na katerem nivoju geometrijskega mišljenja so učenci, ki obiskujejo osnovno šolo, smo se odločili raziskati to problematiko. Oblikovani preizkus znanja je izpolnilo 782 učencev četrtega, petega in šestega razreda osnovne šole. Ugotovili smo, da se največ učencev (60,7 %) drugega triletja nahaja na prehodu med nivojem nič (vizualnim) in prvim (opisnim) nivojem geometrijskega mišljenja. Na prvem nivoju je skoraj tretjina (31,7 %) učencev, na nivoju nič pa 4,3 % učencev, starih od devet do enajst let. Le 1,4 % učencev je skozi celoten preizkus znanja kazala drugi nivo geometrijskega mišljenja oz. nivo neformalnega sklepanja. Ugotovili smo, da učenci drugega triletja poznajo določene lastnosti geometrijskih likov, vendar pa te lastnosti niso logično urejene, poleg tega znanja niso sposobni uporabiti v novih okoliščinah. Med drugim smo potrdili, da obstaja razlika med razredi (učenci višjih razredov so na višjem nivoju geometrijskega mišljenja kot učenci nižjih razredov), medtem ko razlik med spoloma skoraj nismo opazili. Potrdili smo domnevo, da so učenci, ki imajo pri matematiki višjo oceno, na višjem nivoju geometrijskega mišljenja. Učenci so pri reševanju različnih vsebin uporabili različne nivoje geometrijskega mišljenja. Največ težav so imeli z uporabo ustreznega geometrijskega besedišča. Med drugim še vedno zamenjujejo osnovna geometrijska pojma, kot sta rob in stranica. V prispevku je kratko prikazan model poučevanja za izboljšanje geometrijskega besedišča in hkrati za spodbujanje višjih nivojev geometrijskega mišljenja.

Ključne besede: geometrija, Pierre van Hiele, nivoji geometrijskega mišljenja, drugo triletje, spodbujanje geometrijskega mišljenja

Didactic model of development of multiplicative concept in mathematics in elementary school

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Abstract. Didactic model of the mathematical concepts based on thoroughly analyzed key points of development, links and development cycles, didactic and methodical approach designed model. Goal is to precisely define all aspects of mathematical concepts, approaches to the realization of these concepts, taking into account the age and cognitive capabilities of the child. Globally didactic model describes the approach to mathematical concepts through two aspects, substantive and procedural. The substantive aspect will list all the key points, concepts, level, learning outcomes, a situation that represents a particular class situation, key issues, representation, possible misconceptions, the age at which it expected to be, a description of the cycle and connectivity, while the procedural aspect based on description facilities through which it will develop competence in the spectrum of mathematical processes. Example of didactic model of development of mathematical concepts in this paper will describe the multiplicative concept through lower elementary grades, and the analysis of how the textbooks used in Croatian schools presents multiplicative concept comparing to the described model.

Keywords: mathematical concepts, multiplicative structures

Introduction

Thinking about teaching mathematics we aim to find the most efficient procedures in approaching mathematical content. Tasks of teaching mathematics include a broad range of competencies for which it is important to develop the students to reach the objectives set by the national curriculum, but also a personal vision of the teacher. Perhaps more demanding than other subjects, mathematics is a subject which aim is interpolation of a broad spectrum of competencies which include skills mastery and understanding of mathematical concepts to competencies that include a wide range of behavior in the handling of mathematical situations. It is difficult even at the most elementary level, omit the part of the competency or select competencies from basic structure to more complex, because they are all equally important and intertwined. Numerous studies have provided insight into the misconceptions and misunderstandings of basic mathematical concepts for future mathematics teachers. And when working with future teachers, it is often possible without deep insight to spot the gap between the perceptions of a mathematical concept. Complexity and intertwining of mathematical competence areas it's clearer for people who have adopted these concepts in its depth and scope, but for the teachers in the lower grades of elementary school, who by profession are not mathematicians, and the math is only one of a spectrum of areas which they teach, sometimes it is difficult to provide insight into all the elements that comprise a concept. Working with teachers in lower grades of primary school showed us the need for a systematic presentation of mathematical concepts according to the elements of the concept that is not taught in elementary school, but concept rely on the elements adopted in this period.

For the foregoing reasons, we designed a model approach to elementary mathematical concepts that aim to present all aspects of a particular mathematical concept, the complexity and interconnectedness and all elements that make the concept meaningful and upgradable in integrating it with other concepts. As it has grown out of the need for coherence in the approach to mathematical concepts, this model in its nodes have defined elements of methodological approach, but it is not aim of a model at finality in defining appropriate teaching methods, but only giving representative examples that describe a class of important situations in the concept. Not reaching of all aspects of a concept results in lack of intertwining of mathematical knowledge and understanding of concepts in its entirety, but as isolated and unrelated facts that are difficult to remember.

Because teachers in their work use standard textbook, we'll give a mathematical representation of a concept by components of didactical model of development of mathematical concepts and their representation in the textbooks currently used in teaching mathematics in Croatia.

Theoretical background

The initial consideration is the problem of describing certain mathematical concepts under consideration of the existing theories that were created with the aim of describing the nature of mathematical concepts and mathematical definition of the concept. The largest contribution to this was given by Vergnaud (1996) with his works on the nature of mathematical concepts, and a systematic attempt to show what is a mathematical concept. Didactic model of the mathematical concepts follows Vergnauds (1996) definition of mathematical concepts. Vergnaud (1996) start his theory from a definition of the concept:

Definition. The concept is the three-tuple set $C = (S, I, R)$.

S is a set of situations that make the concept useful and meaningful.

I is a set of operational invariants used by people when dealing with these situations.

R is a set of symbolic representations (diagrams, graphs, algebra,...), which can be used to represent relationships, talk about them and generally that help to manage the situation.

Already in the definition of the concept Vergnaud highlights situations that make the concept meaningful, as the backbone of the concept of a concept, a set of symbolic representations that are methodical part of describing the concepts and operational invariants that in didactic hatches should not be neglected in the teaching of mathematics. Vergnaud has not dealt with the detailing of the approaches to the concept, but he cited examples to illustrate the meaning of mathematical concepts. His works are more epistemological attempted to describe the nature of mathematical concepts. Significant contribution to its consideration is linking related concepts in the conceptual fields. Conceptual field is a set situation, requiring mastery of several interrelated concepts. It is at the same time a set of concepts, with different properties, the meaning of which stems from the different situations. (Vergnaud, 1996) Vergnaud cites some examples of conceptual fields related to mathematical concepts, and describes them as examples of conceptual fields. These are conceptual field of additive structures (Vergnaud, 1996), conceptual field of multiplicative structures (Vergnaud, 1996), which in some papers is called the multiplicative conceptual field (Vergnaud, 1994), and conceptua the field of elementary algebra (Vergnaud, 1996).

Vergnaud (1994) defined multiplicative conceptual field as the simultaneous set of situations and a set of concepts. Concepts become meaningful through different situations and different aspects of the same concept. At the same time the situation can't be analyzed with the help of only one concept, it is necessary to include at least a few concepts. For this reason, in his first paper examines multiplicative conceptual field with a conceptual point of view and situational standpoint. From the conceptual point of view multiplicative conceptual field is the set of situations and solutions which involves a multiplication, a division, or a combination of such operation. The conceptual field of multiplicative structures is also a set of interconnected concepts: measure, scalar, ratio, quotient and product of dimensions, fraction, rate, rational number, vector, space, linear and n-linear function, constant coefficient, linear combination and linear mapping, and of course multiplication and division. (Vergnaud, 1996).

While the situational conceptual standpoint multiplicative conceptual field includes a very large group situations that need to be classified and carefully analyzed, so that they would describe a hierarchy of competencies that students must learn both inside and outside of school.

In his work on the nature of mathematical concepts Vergnaud (1996) briefly and symbolically represents diagram multiplicative structure as follows:

Table 1. Multiplicative structures.

MULTIPLICATION		PARTITIVE DIVISION		MEASUREMENT DIVISION	
1	a	1	□	1	a
b	□	b	c	□	c

Although seemingly simple look, the multiplicative structure students become more complicated when it is turned situational context and numbers which may not be mastered. Especially when it comes to relations between two measures, or when the relationship is expressed in a rational number. Different authors emphasize the different situations, so it is necessary to specify at least a representative.

Problem situations for multiplication and division, and strategies for their solution are investigated by Mulligan and Mitchelmore (1997), and many others. Specifically, there are a number of different problem situations whose solution is multiplication and division as a computational operation and that it is not a trivial consequence of the model of repeated addition or partitive or measurement model. We have to understand the fact that in general there are two fundamental situations models for division (partitive and measurement), but there's a whole range of different problem situations for division by its semantic structure and that the child uses a variety of strategies to solve them. Children need to be offered a whole range of problems so that they can properly develop the concept of multiplication and division. Mulligan and Mitchelmore bring several different problem situations for multiplication and division situations that differ in their semantic structures and therefore are classified into different groups, classes of problems. Values and types of numbers in a certain class of problems can vary, and the topic or task context, however, the basic structure that includes actions and connections and relationships remains the same. For all uses of context conditions must be such as to have mathematical properties that allow us to analyze the system and situation. Situation within a class include the same actions over sizes or distances. Children recognize the structure of the task with the help of intuitive model by which it selected the relevant strategy for its solution. One and the same problems, students can solve a variety of strategies, but they are one component of that strategy, the same intuitive model. While researching for 24 mathematical problems that are solved students second and third grade, Mulligan and Mitchelmore saw 12 distinct strategies for solving. They have sublimed it into four categories and of solving strategies: direct counting, repeated addition, repeated subtraction and multiplicative operation. Logically, the strategies over time, in parallel with the child's cognitive development are becoming more complex and abstract. As well as strategies and class situations are changing the use of rational numbers. Examples of classification problem situations multiplication and division are given by various authors Greer (1992), Coats (1985), Nesher (1988), Schmitd and Weisser (1992) and Vergnaud (1983). Greer is now the most common classification in the analyzes and studies the development of multiplicative thinking.

Table 2. Greer's classification of multiplicative situations.

Class	**Multiplication problem**	**Partitive division**	**Measurement division**
Equal groups	3 children each have 4 oranges. How many oranges do they have altogether?	12 oranges are shared equally among 3 children. How many does each get?	If you have 12 oranges, how many children can you give 4 oranges to?
Equals measures	3 children each have 4,2 liters of oranges juice. How much orange juice do they have altogether?	12,6 liters of orange juice is shared equally among 3 children. How much does each get?	If you have 12,6 liters of orange juice, to how many children can you give 4,2 liters?
Rate	A boat moves at a steady speed of 4,2 m/s. How does it move in 3,3 seconds?	A boat moves 13,9 meters in 3,3 seconds. What is an average speed in meters per second?	How long does it take a boat to move 13,9 meters at a speed of 4,2 m/s?
Measure conversion	An inch is about 2,54 cm. About how long is 3,1 inches in centimeters?	3,1 inches is about 7,84 cm. About how many centimeters are there in an inch?	An inch is about 2,54 cm. About how long in inches is 7,84 cm?
Multiplicative conversion	Iron is 0,88 times as heavy as copper. If a piece of copper weights 4,2 kg, how much does a piece of iron of the same size weight?	Iron is 0,88 times as heavy as copper. If a piece of iron weights 3,7 kg, how much does a piece of copper the same size weight?	If equally sized piece of iron and copper weight 3,7 kg and 4,2 kg respectively, how heavy is iron relative to copper?
Part/whole	A college passed the top 3/5 of its students in an exam. If 80 students did the exam, how many passed?	A college passed the top 3/5 of its students in an exam. If 48 students passed, how many students sat the exam?	A college passed the top 48 out of 80 students who sat an exam. What fraction of the students passed?
Multiplicative change	A piece of elastic can be stretched to 3,3 times its original length. What is a length of a piece 4,2 meters long when is fully stretched?	A piece of elastic can be Stretched to 3.3 times its original length. When fully stretched it Is 13.9 meters long. What was its original length?	A piece of elastic 4.2 meters long can be stretched to 13.9 meters. By what factor is it lengthened?
Cartesian product	If there are 3 routes from A to B, and 4 routes from B to C how many different ways are there of going from A to C via B?	If there are 12 different routes from A to C via B, and 3 routes from A to B, how many routes from B to C are there?	
Rectangular area	What is a area of rectangle 3,3 m long by 4,2 m wide?	If the area of rectangle is 13,9 m^2 and the length is 3.3 m, what is the width?	
Product of measures	If a heather uses 3,3 kW of electricity for 4,2 hours, how many kWh is that?	A heather uses 3,3 kW per hour. For how long can it be used on 13,9 kWh of electricity?	

Although systematic the class situation is defined mentioned is insufficient in didactic form of teaching mathematics, and need to be split into sub-classes depending on the type of numbers, and the application of measures or facilities.

Didactic model of mathematical concepts

Didactic model of development of mathematical concepts is designed as a spiral with prominent nodes that are connected by a few common characteristics. Each node is defined by the following elements: a representative situation, visual-didactic representation, key concepts, elements of teacher notes and learning outcomes. It is important to emphasize that no one element of each node concept is not trying to give a "recipe" in teaching, but the frame of the main aspects of a particular concept that can't be ignored for the quality of teaching. The assumption of this deliberation and setting of such a model is inadequate representation of all aspects of the concept in the teaching of mathematics, which will be shown through the analysis of textbooks.

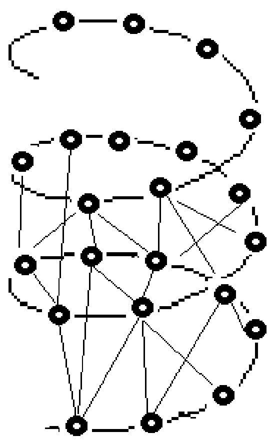
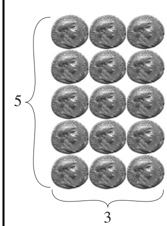
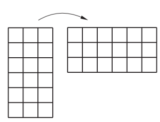
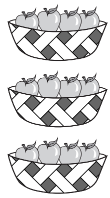
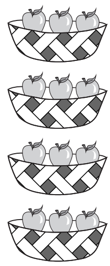


Figure 1. Visual representation of nodes in helix for didactic model of development of concepts.

Helix is a visual representation of the didactic model broth for reason to evoke the level of adoption of certain aspects of the mathematical concepts and links between nodes. Connecting nodes from different levels of acquisition of knowledge is essential to upgrade the quality of knowledge and build mathematical concepts, not an isolated facts. The different levels of adoption can refer for example to apply the concept in different number systems. In general didactic model of development of mathematical concepts is applicable to all mathematical concept, and will be featured here and part of the multiplicative concept its development in order to obtain insight into the intention of introducing such a model.

Table 3. An example of individual nodes in didactic model of multiplicative concept at an elementary level.

Nod of concept	**Key points**	**Situation**	**Visual (oral) – didactical representation**	**Teacher instructions**	**Learning outcomes**
Multiplication as successive addition – equal groups	Times, factors, product	Ivo, Ana, Mira, Stipe and Frane are collecting old coins. Every one of them has up to three coins. How many pennies have together?		This form of multiplication is asymmetric. Class equal groups. This class leads to the classic situation of equal measure, the class situation is conversion units and the product of physical quantities.	Realized that repeatedly addition of the same number. Calculate result of multiplying with the successive addition. To understand the meaning of the first and second factors. Linking visual representation to the multiplication.
Rectangle array – equal groups	times	Ivo, Ana, Mira, Stipe and Frane are collecting old coins. Every one of them has up to three coins. How many coins they have all together?		Although situation is equal to the priore situation, a set of visual elements are diaplayed in a rectangular shape which leads to the commutativity of multiplication. Rotation of the image change the situation. This class is led by class rectangle array – equal measures and the concept of area of the rectangle.	Linking visual representation to the multiplication.
Commutativity of multiplication	Factors changing places	Tina and Anna are playing with cubes. Tina puts 3 cubes in 5 rows. Anna puts 5 cubes in 3 rows. Which one used more cubes?		Introduce tasks where it is necessary to use commutativity for ease of computation.	Spotting commutativity property and apply it to the new situation is to facilitate the multiplication computation.
Multiplicative change – equal groups – whole numbers	Times more	Ana has 3 pennies. Iva has five times more than Anna. How many pennies has Ana?		Visual display showing the linkage from multiplying. Numerous studies have pointed to the problems of students with this kind of situation.	Apply multiplication in these kind of situations.

Partitive division – equal groups	Dividend divisor and quotient	On the table there are 12 apples. Apple should be put in three baskets, so that in each are equal apple. How many apples will be in each basket?		Firstly, put one apple in each basket, then put the another apple in each basket and so on.	Determined by dividing the quotient in the multiplication table using given model. Identify a model of the situation and link it with dividing numbers.
Measurement division – equal groups	Dividend divisor and quotient	On the table there are 12 apples. Apple should be put in baskets, so that in each are three apples. How many basket we need?		Put three apples in the first basket, three apples in the second one and so on.	Determined by dividing the quotient in the multiplication table using given model. Identify a model of the situation and link it with dividing numbers.

Thus, for example, we show that the two nodes of the concept of multiplication, rectangular array and multiplication as successive summation associated situational context. This was not unequivocally determined, but the teacher should initiates and develops it. Feature of the model is the underlining of connection and necessity of presenting all aspects of concepts through the teaching mathematics. Often teachers expect from students that the presentation of only one model of a concept is enough that I'm a student establish links with all other aspects, but this is possible only for those with higher cognitive abilities. As the aim of this paper is not a detailed description of the model, which requires a larger volume and time in the table are listed and briefly described only certain parts to facilitate the presentation of idea. Some nodes are given for the purpose of describing links and a display of inadequacy classification situations mentioned above, for the didactic design and development concepts.

That rectangular array covers more situations than subclasses listed in Greer classification, with didactic aspects, because it introduces the commutativity of multiplication and students getting used to visualize rectangle forms as representations of multiplication. This node associate multiplicative concepts with the area of rectangles and multiplying with equal measures, as well as developing an understanding of dimensionality. Certainly this form of multiplication has significant consequences in later adoption as multiplicative concept and application in various fields of science.

Multiplicative comparison, also the aspect of multiplicative concept, should be presented to students layered and nurture of manipulating in the level of natural numbers to the application of the rational numbers, when it only makes sense in the handling with the measures. In this class situation is also worth pointing out the linguistic context, which in the English language has the same form for the natural and rational numbers, while in the Croatian language, it is not the case. It's so hard to say that iron has 0.88 times the mass of the same volume of copper, because it is a lesser value, is “more time” is a phrase that should be used only if you get a higher value, which for 0.88 times as much and is not the case.

Also interesting in the progression through the multiplicative model is concept of division, which for different number systems and situations that used measures student look different and unrelated. The task of teaching is exactly the same model fit through the different classes of situations and thereby develops the concept of division as part of the multiplicative concept. Otherwise, students create misconceptions and isolated parts of knowledge in such a way that they are not able to survive (Cindrić, Mišurac Zorica, 2012). Unequal representation of different aspects of the concept in the lower levels of education, and inadequate connection of concept nodes at different levels of education lubricates the construction of quality in mathematical knowledge. Thus, the imposition of one aspect of the concept, such as the partitive division – class equal groups, limited understanding of other aspects of the concept as well as parts of one whole. If the numeric expression 12 : 3 is presented to the child, they will associate it with problem situations in which 12 elements (eg, candy or apples) are shared among 3 groups (eg, three children), because the division of whole numbers was imposed through the partitive model of equal class group. If you fail to represent, during teaching, the measurement model of division, or they are less represented, division of rational numbers and division of whole number would not automatically be connected in child mind. Numeric expression $12 : \frac{1}{2}$ student hardly fits in the partitive model of equal groups, because they can’t imagine how the 12 apple split in half child. In further consideration child not to go, but comes to the conclusion that the division of rational numbers is not division he knows. The assumption in this paper is that situation involving the measurement model of division are disadvantaged, which creates the perception in children’s minds about division like actions in which a certain amount of elements are divided into equal subsets. This fosters partitive division model, while the measurement model has been neglected and is rarely associated with division.

In our research we will do an analysis textbooks to show what kind of representation for division is brought in teaching of mathematics. The analysis includes all textbooks and accompanying manuals approved by the Ministry of Science, Education and Sport for teaching mathematics from second to fourth graders. Some studies (Glasnović-Gracin, 2010.) showed that in the teaching of mathematics in the middle school teachers monitor processing method presented in the textbook and use examples and assignments generally presented in the textbook. Although this is a mathematics teacher in the upper grades, the assumption is that with equal probability accompanying textbook and class teachers in the teaching of mathematics. Therefore, the presentation of the concept in a textbook, it can be associated with the presentation of the concept to children in school. The emphasis in the analysis will be the representation of problems in relation to the computational tasks and problem-oriented tasks, including the determination of the measurement and partitive divisions. In tables 4, 5 and 6 are presented data which indicate the ratio of the amount and proportion partitive and measurement division in the tasks in relation to the total number of tasks that involve division, which appear in the textbooks for the teaching of mathematics.

Table 4. Analysis of division task in the second grade textbooks.

Textbook	Number of partitive division tasks	Number of measurement division tasks	Total number of division tasks	Percentage of partitive division tasks in total number of tasks in textbook	Percentage of measurement division tasks in total number of tasks in textbook
1.	34	5	793	4,3%	0,6%
2.	65	21	880	7,4%	2,3%
3.	78	19	842	9,27%	2,23%
4.	75	20	834	9%	2,4%
5.	37	2	862	4,3%	0,2%

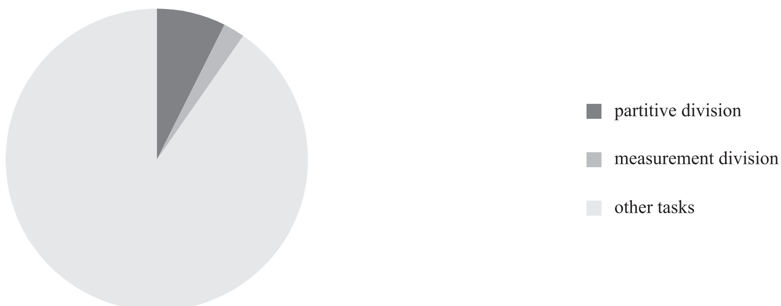


Figure 2. Number of partitive division and measurement division tasks in the textbook for the second grade.

Table 5. Analysis of division task in the third grade textbooks.

Textbook	Number of partitive division tasks	Number of measurement division tasks	Total number of division tasks	Percentage of partitive division tasks in total number of tasks in textbook	Percentage of measurement division tasks in total number of tasks in textbook
1.	11	2	478	2,3%	0,4%
2.	9	4	605	1,5%	0,7%
3.	55	9	518	10,6%	1,7%
4.	48	1	548	8,8%	0,2%
5.	56	6	502	11,2%	1,2%
6.	30	3	286	10,5%	1,1%
7.	21	2	329	6,4%	0,6%

Figure 3. Number of partitive division and measurement division tasks in the textbook for the third grade.

Table 6. Analysis of division task in the fourth grade textbooks.

Textbook	Number of partitive division tasks	Number of measurement division tasks	Total number of division tasks	Percentage of partitive division tasks in total number of tasks in textbook	Percentage of measurement division tasks in total number of tasks in textbook
1.	7	1	356	2%	0,3%
2.	5	7	704	0,7%	1%
3.	27	5	455	5,9%	1,1%
4.	11	2	166	6,6%	1,2%
5.	61	13	500	12,2%	2,6%
6.	42	6	295	14,2%	2%

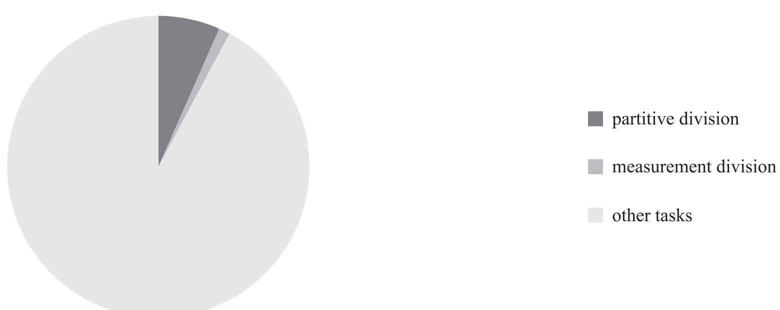


Figure 4. Number of partitive division and measurement division tasks in the textbook for the fourth grade.

As it is obvious textbook tasks involving real situation and division as a computational procedure in textbooks are represented in a small percentage of 0.2% to 14.2%. But in this, the partitive division tasks are numbered in relation to the proportion of measurement division tasks. These data confirm our hypothesis about the uneven representation of all aspects of the concept of division.

Conclusion

There are many factors that affect quality of the teaching, but in the center of attention is access to educational content. Content of teaching math, math concepts, are the key element in teaching, because of their complexity, connectivity and complexity can lead to a complete neglect of mathematics by students as subjects of human knowledge. Deteriorating student achievement on external evaluations point out the marginalization of mathematical knowledge as a real matter of concern.

The aim is to detect any research proposal and the current state in the direction of improvement. This brief analysis of the work of only one small segment indicates weak linkages of various aspects of mathematical content and only at an elementary level and in small part, raising questions about the coherence of all the other parts, aspects, approaches, and in particular the correlation of different educational levels. Greater contribution of this paper we consider the proposal to build a unique didactic model, which is intended to show unity mathematical content in all its fullness. Concretization of such a model should not be restrictive in its description, to act as a recipe for doing their share of lessons because the teaching process is determined primarily by students, their previous knowledge, interests, abilities, personality, teacher, and numerous material - formal factors. The task of defining a didactical model of development of mathematical concepts and their links is highlighting the importance of equivalent representation of all aspects of the concept, structured the teaching content and perceiving standards in national and world wide math teaching.

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Didaktički model razvoja multiplikativnog koncepta kroz nastavu matematike u nižim razredima osnovne škole

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Sažetak. Didaktički model razvoja matematičkih koncepata temelji se na detaljno opisanim ključnim točkama razvoja, poveznicama i prikazom ciklusa razvoja, te didaktički i metodički osmišljenim pristupu modelu. Cilj joj je precizno definirati sve aspekte matematičkih koncepata, pristupe ostvarivanju tih koncepata, vodeći računa o dobi i kognitivnim mogućnostima djeteta. Globalno gledajući didaktički model opisuje pristup matematičkim konceptima kroz dva aspekta, sadržajni i proceduralni. Sadržajni aspekt navest će sve ključne točke, pojmove, razinu, ishode učenja, primjer situacije koja predstavlja određenu klasu situacija, ključna pitanja, reprezentacije, moguće miskoncepcije, dob u kojoj se očekuje realizacija, opis ciklusa i povezivanje, dok će se proceduralni aspekt bazirati na opisu pristupu sadržajima kroz koji će se razvijati kompetencije iz spektra matematičkih procesa. Primjer didaktičkog modela razvoja matematičkih koncepata u ovom radu opisan je kroz primjer multiplikativnog koncepta kroz niže razrede osnovne škole, te je prikazana analiza udžbeničke literature korištene u hrvatskim školama u svjetlu opisanog modela.

Ključne riječi: matematički koncepti, multiplikativni koncept

Use of navigation devices in teaching mathematics at primary schools

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Abstract. GPS (Global Positioning System) is one of the modern technologies which have been expanding in our everyday life. Not only can the navigation devices be used in industry, agriculture or during outdoor free-time activities, but also in teaching certain subjects at school.

The purpose of this article is to outline the possibilities of using navigation devices in teaching mathematics. We introduce an activity which was carried out at primary schools in Hungary. During this activity, the pupils have learned how to use a hand-held GPS receiver, and then solved measuring tasks around the school using this receiver.

Keywords: teaching mathematics, GPS, navigation device

Introduction

GPS (Global Positioning System) is one of the modern technologies which have been expanding in our everyday life. Not only can the navigation devices be used in industry, agriculture or during outdoor free-time activities, but also in teaching certain subjects at school.

The purpose of this article is to outline the possibilities of using navigation devices in teaching mathematics. We introduce an activity which was carried out at primary schools in Hungary. During this activity, the pupils have learned how to use a hand-held GPS receiver, and then solved measuring tasks around the school using this receiver.

The GPS receiver

The GPS is a space-based satellite navigation system that provides accurate navigation data in all weather conditions, anytime and anywhere on the Earth. For positioning the GPS receiver uses the exact time and the current position of the satellites orbiting the Earth. To obtain two dimensional position data (latitude and longitude) it is sufficient to have the current position of three satellites, while to obtain three dimensional position data (latitude, longitude and altitude) a GPS receiver needs to receive good signals of at least four satellites. GPS receivers require an unobstructed view of the sky, so they cannot provide navigation information in enclosed, covered areas (tunnels, inside buildings). GPS receivers receive satellite signals only passively, they do not transmit signals of any type [1].

The services of the GPS are freely accessible to anyone with a GPS receiver. Even the simplest receiver is capable of:

- determining and storing current position with 3–15 m accuracy,
- determining the direction of the desired spot, its distance, the velocity of approach and the estimated arrival time,
- determining instantaneous velocity,
- determining the exact time.

Apart from all this a GPS receiver (Figure 1) may have a built-in electronic compass, barometric altimeter, camera and is capable of displaying map.

Figure 1. GPS receiver Garmin Oregon 450t (Source: [3]).

With the help of the GPS receiver, therefore, different outdoor measuring tasks can be carried out (distance, time and velocity measuring). The saved data can be stored in many forms, one of the most popular file type, the GPX (GPS eXchange Format), which has been created for the transmission of data between users and computer applications. Such information includes the *waypoints*, *routes* and *tracks*.

Waypoint – a Waypoint can be defined as a marked position somewhere on the earth, with known coordinates. We can save these coordinates as a Waypoint to the memory of the GPS receiver and it can be selected as a coordinate to navigate to. Waypoints can be given names and they can be assigned a symbol (e. g. red flag, blue pin etc.). Waypoints can be created by GPS receiver outdoors, this way the altitude, the exact date and time will be automatically stored. If a software is installed in our computer (e. g. MapSource) which allows you to display and manage GPS data and maps, it can also be used to create waypoints, and they can be uploaded to the GPS receiver.

Route – series of waypoints, which are supposed to be matched in a specified sequence, so that the road to the starting point and the target can be clearly identified. The best way is to create them using software, and then upload to your GPS receiver.

Track – series of waypoints like routes, except that these waypoints are not created by the user, but are saved by the GPS receiver at regular intervals. Tracks are very useful since they help you see which way you have moved in the field. If alongside a hiking trip the GPS receiver has been kept switched on, then at home you can download the track into your computer and you can see the route you have taken, where you have made a stopover, at what speed, the altitude and other data (Figure 2).

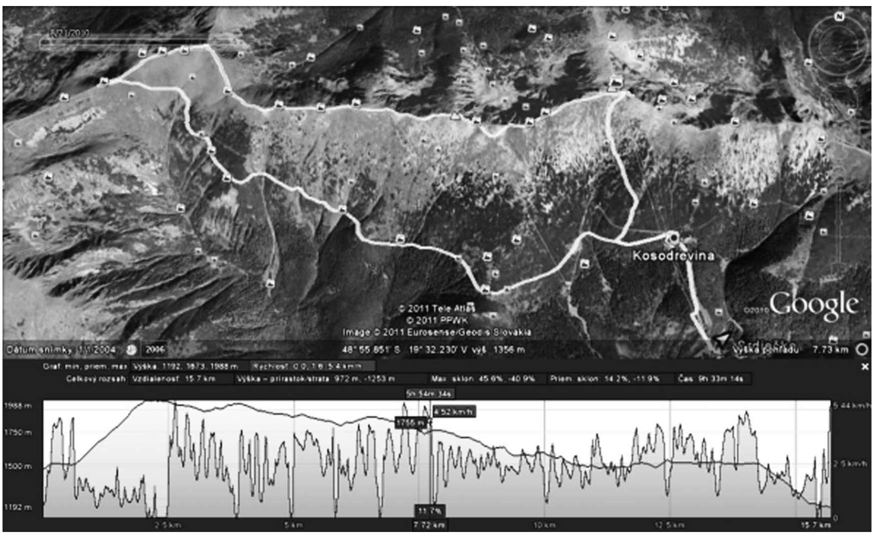

Figure 2. The track of the hike in the Low Tatras visualized in Google Earth programme.

Navigation devices in teaching mathematics

In the following we present an activity conducted in primary schools in Hungary, in which students carried out measuring tasks around the school with a GPS receiver. The participants of the activity were 7th grade students of Baja Eötvös József Főiskola Gyakorló Általános Iskolája (Eötvös József Training Elementary School) (11 students) and Budapest Mustármag Keresztény Óvoda, Általános Iskola és Gimnázium (Mustard Seed Christian Preschool, Elementary and Secondary Grammar School) (9 students). The activity included a theoretical as well as a practical part.

The theoretical part of the activity (45 minutes) was conducted in the classroom, the following tasks were carried out:

- recalling some basic knowledge (linear measurement, proportion, latitude and longitude, geographical coordinates),
- introducing the principles of GPS operation,
- demonstration of how to handle the GPS receiver.

The practical part of the activity (60 minutes) was held in the open air. The students worked in groups of 2–4, each group was given a Garmin Oregon GPS receiver (Fig. 1), which was available for a total of 5 pieces. At the beginning of the activity, students learned to handle the most important functions of the GPS receiver (see the accuracy of the device, switch the view between map and compass page, creating, editing and deleting waypoints). Then, the groups received their tasks and began working. During the occupations each student completed a worksheet on which they provided data and calculation results as well as feedback on how they felt during the activity, what new information they acquired, what was the most interesting and least interesting part of the activity.

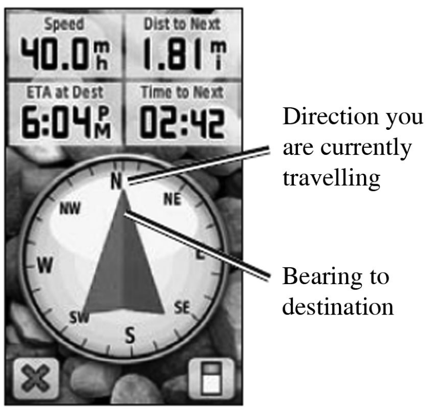

Figure 3. GPS receiver Garmin Oregon 450t – compass page (Source: [2]).

Figure 4. GPS receiver Garmin Oregon 450t – elevation plot page (Source: [2]).

During the practical part of the activity, the following tasks have been performed:

- identification of the compass-points using electronic compass (Figure 3),
- determination of altitude using barometric altimeter (Figure 4),
- determination of geographical coordinates of a selected spot,
- the distance of a selected object from a pre-determined spot,
- finding the perimeter of a particular area (yard, park)
- geocaching games around the school (navigation to spots with pre-determined coordinates and the search of hidden boxes of different sizes).

The following task was appointed for students in Budapest: **find the perimeter and the area of the school yard with a GPS receiver!**

`Solution`: The shape of the school yard is approximately a rectangle, it is therefore sufficient to determine its width and length. The task can be figured out easily using the waypoints. During the navigation to a specific waypoint the GPS receiver will show the distance of the waypoint from our current position.

So stand in one corner of the yard (e. g., northwest), and save geographic coordinates of this location as a waypoint (WP01 – N47° 36.071 E19° 02.754, see Fig. 5).

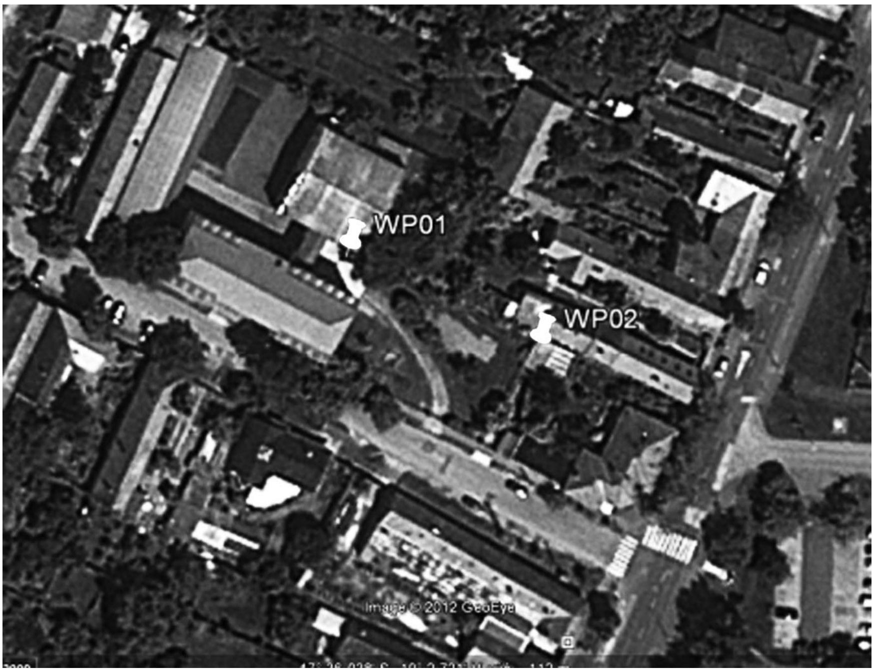

Figure 5. Location of waypoint WP01 and WP02 on the map.

Then walk to the northeast corner of the yard so that we navigate to waypoint WP01 with our GPS. Moving towards the northeast corner the distance will increase as we move away from waypoint WP01. Reaching the corner of the park, stop and record the value shown by our GPS receiver (42 m).

Now save the northeast corner of the geographic coordinates as a waypoint (WP02 – N47° 36.061 E19° 02.784, see Figure 5). Walking to the southeast corner of the park, determine the distance in the above mentioned method (22 m).

In possession of the measurement data, it is easy to calculate the perimeter (128 m) and the area 924 m^2 of the school yard. The following the data were obtained by the groups:

`Group 1`: length 43 m, width 16 m, perimeter 124 m (miscalculation!), area 688 m^2

`Group 2`: length 45 m, width 22 m, perimeter 134 m, area 990 m^2

`Group 3`: length 48 m, width 23 m, perimeter 142 m, area 1104 m^2

The first group probably made an error in the measurement of the width of the school yard (did not save the waypoint in the right place), and therefore received the inaccurate results.

We should note that the values are approximate, depending on the accuracy of the GPS receiver. Ideally, the accuracy is 3–5 m, but different disturbing factors (tall buildings or trees) can affect the detection of satellites negatively (even 15–20 m). The accuracy of the GPS receiver can be checked on the website which illustrates the current position of satellites (Figure 6).

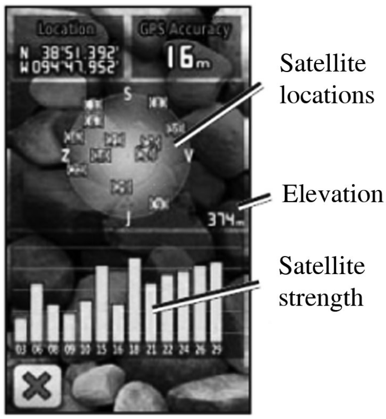

Figure 6. Satellite page of GPS receiver Garmin Oregon 450t (Source: [2]).

After the measuring tasks were carried out, the geocaching game began. In the beginning students were given coordinates of four boxes which were hidden in the yard. Originally it was planned that students would have been expected to determine the last three digits of the coordinates. The coordinates could have been determined by solving mathematical tasks, but it had to be disregarded due to time deficiency. Three boxes were small each containing stickers with 1-2-3 numbers on them. The last box was bigger in which there was a small logbook, a pen, and some chocolate hidden. The groups' task was to find as many hidden boxes as possible for the rest of the time (approx. 20 minutes).

After finding the large box each member of the team could have a chocolate, and then they had to write something in the logbook. The numbered stickers which were in the small boxes indicated the order in which they were found: the group which found the relevant box first, could take sticker number 1, the following group could take sticker number 2, and the third group got the last sticker with number 3 on it. The game was a great success amongst all the students in Baja as well as students in Budapest.

At the end of the activity students were asked to fill in a worksheet, stating what was the most interesting and least interesting for them. The percentage distribution of students' responses is shown in Table 1 and 2.

Table 1. The percentage distribution of responses to question "Today the most interesting thing was when. . . "

	geocaching game	finding the big box	all
Baja	70%	20%	10%
Budapest	100%	0%	0%

Table 2. The percentage distribution of responses to question "Today the least interesting thing was. . . "

	putting down coordinates on paper	had to walk a lot	no such answer
Baja	10%	10%	80%
Budapest	45%	0%	55%

Conclusions

The use of GPS receivers in education requires outdoor activities that are prepared thoroughly and accurately. Before the date is set for the activity, it is recommended to check the site as its features may affect the tasks. At the same time we can measure the important objects and coordinates of the hidden boxes.

According to the received feedbacks from the students we can state that the use of GPS devices for educational purposes opens the door to big possibilities. The worksheets were filled in by all students as "I feel good", confirming thereby the motivating effect of the work with a GPS receiver. The students mastered the handling of the devices quickly, and skillfully completed the measuring tasks. Several students also noted "if only all math lessons were like this."

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Navigációs készülékek használata a matematikaoktatásban az általános iskolákon

Gubo István

Selye János Egyetem, Komarno, Szlovákia

Összefoglaló. A GPS (Global Positioning System – Globális Helymeghatározó Rendszer) egyike azon modern technológiáknak, melynek használata az utóbbi években egyre szélesebb körben terjedt el. Navigációs készülékeket nemcsak az iparban, mezőgazdaságban vagy kültéri szabadidős tevékenységek során lehet használni, hanem bizonyos tantárgyak iskolai oktatásánál is.

A cikk célja bemutatni miként lehet alkalmazni navigációs készülékeket a matematikaoktatásban. Leírunk egy magyarországi általános iskolákon lebonyolított foglalkozást, melynek során a tanulók megtanulták kezelni a GPS vevőkészüléket, majd segítségével mérési feladatokat oldottak meg az iskola környékén.

Kulcsszavak: matematikaoktatás, GPS, navigációs készülék

The role of activity in teaching axial reflection

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Abstract. The basis of learning geometric transformations, thus axial reflection as well, is inductive cognition based on gaining empirical knowledge. Starting out from the concrete and gathering experience from various activities, such as folding, clipping, drawing, and the use of mirror will finally lead to the formulation of general relationships.

In the junior classes pupils observe the reflected image of different geometric figures in space and plane by using plane mirror by gaining experience in a playful way. They construct reflections of geometric solids and they produce the reflections of simple plane figures and axial reflections my means of folding, clipping and drawing.

Activities have an equally important role in teaching axial reflection in junior high school as well. Producing axially reflected shapes is possible in several ways: by means of moving, using transparent paper and drawing on grid and by construction.

Keywords: teaching mathematics, axial reflections, activity

It is already in the kindergarten that children are introduced to geometrical transformations, including axial reflection. Children in kindergarten are mainly involved with playing games, thus they primarily meet geometrical transformations through games and gather experience from various playful activities. Both in kindergarten and the lower primary grades the playful way of using mirror, folding, clipping and drawing related to reflections can be found. Gathering experience gained in these concrete activities will lead later on to the formulation of general relationships.

When teaching geometrical transformations it seems to be more practical to start with the manipulative examination of solids, because it is more natural to gain experience in space than in plane. Teaching axial reflection in plane is as a matter of fact is preceded by reflection on plane in space.

In the lower primary grades while constructing mirror images children observe the various characteristics of reflection, such as:

- the mirror image is of the same shape and size as the original construction

- the distance of the mirror image from the mirror is the same as that of the original construction
- the original construction, for instance a chimney on the left of a house, will be on the right of the mirror image after reflecting.

In this task children are asked to construct a mirror image:

You are working in pairs. The pencil placed in the middle of the desk designates the position of the imaginary mirror. One of the members of the pair should build something from three matchboxes, and the other should construct the mirror image. Check it with a mirror. Compare the mirror image with the original.

Children are also asked to find objects in their environment which are the mirror images of each other, such as the left and right shoe. They try to turn around the left shoe and they realize that there is no way to place the left shoe into the right shoe. Activities like this will demonstrate to the learners that constructions normally cannot be moved to their mirror image, unlike in plane where plane figures can always be turned over on their mirror image. The constructions, which are symmetrical, thus they have symmetry plane, can be turned over on their mirror image. At the beginning the shapes put together for reflection in space and plane should be asymmetric in order that their mirror image could be distinguished from them.

Children can observe the rotation of the pinwheel made in technology class and they can notice when the direction of going round of the pinwheel has reversed.

Having taught the reflections in plane we turn to the symmetry of solids, and symmetry is seen as the characteristic of shapes. The examination of the symmetry of of solids related to plane is connected with defining the place and number of symmetry planes. See the next task:

Hold a matchbox in your hands, then draw the position of the cut, where if the matchbox was cut and one half of it along the cut would be placed on a mirror then the whole matchbox could be seen. Draw the place of the mirror plane. If you find more than one, mark each of them with various colours. How many did you find?

When checking the task the three different cuts can be easily shown on rectangular sponges or potato, or play dough.

In lower primary grades producing mirror images in axial reflection in plane can be carried out by means of various activities and tools. Producing mirror images can be achieved in the following way:

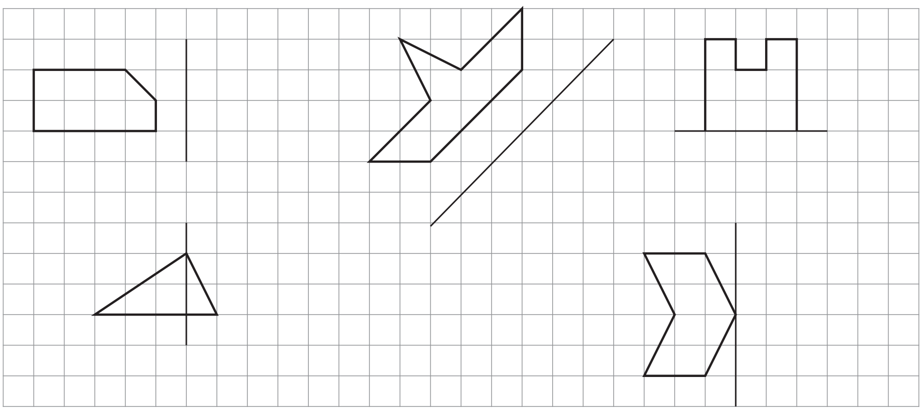
- *folding*, for instance folding coloured sheets of paper. On a folded black photographic paper making a given pattern by running pins on it, then having a look at it against the light.
- *clipping*, for instance cutting a given shape from a folded sheet of paper
- *putting together* congruent planes: The two sides of the congruent planes are painted with different colours. If the plane is symmetrical then the mirror image can placed in a way that the other half of the paper is not turned over.

The mirror image of the non-symmetrical plane can be put together if the other half of the plane is turned over, thus the colours of the the two planes put together actually do not match. Mirror images are normally produced by turning over.

- *copying, moving*: The plane figure to be reflected, the reflection axis, and its any dot are copied on cellophane or carbon paper, then by turning exactly the cellophane and fitting the axes exactly on each other we have produced the mirror image. We should pay attention to the fact that the dots of the axis should cover each other exactly.

- *Drawing* on squared grid, on regular triangle grid by counting the vertexes on the grid. Set tasks in which the axis of reflection coincides with the straight line of the grid, a slanted straight line, a side straight line of the shape to be reflected, it goes through on one of the vertexes of the shape to be reflected, it goes inside the shape to be reflected.

For instance: *Make a reflection of the polygons on the symmetry axis with bold type. Draw the mirror images with a ruler.*

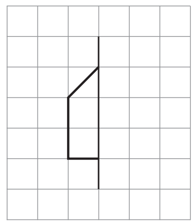


Learners can check the mirror images with a mirror. Having produced the mirror images, learners should notice the characteristics of axial reflection. The formulations should be understandable and accessible for the children. Technical jargon should be used according to the cognitive level of learners in the lower primary grades.

Axially symmetrical plane figures can be produced by folding sheets of paper and clipping or tearing. Fold a sheet of paper, make the edge smooth, and cut some kind of pattern, such as a half heart, half butterfly, half leaf, half mushroom, so that a part is cut also from the line where the paper is folded. After unfolding the sheet of paper, it can be seen that the shape produced in this way is symmetrical. This fact can be checked with a mirror placed on the line of folding or folding the sheet of paper again. If we want to check whether a pattern cut from a sheet of paper is axially symmetrical or not, we should fold it so that the two halves should exactly cover each other. If this is possible at some line of the folding then along this line as a symmetry axis the shape is axially symmetrical.

Learners can also draw axially symmetrical plane figures on a grid, if it is completed on one of the side lines of a given shape. For instance:

Make a reflection of the quadrangle on the symmetry axis with bold type.



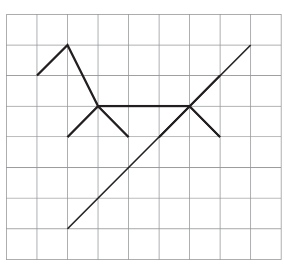
When the axial symmetry of a plane figure is examined, the position and the number of the symmetry axes are also defined. Children are asked to examine regular triangles, isosceles triangles, general triangles, general trapezoids, symmetrical trapezoids, general parallelograms, rhombuses, rectangles, squares, convex deltoid, concave deltoid, general pentagons, regular pentagons, regular hexagons, circles, etc. cut out from paper. Children can define the position and the number of symmetry axes by folding and using mirrors. Another task can be for the children to draw plane figures with exactly one, two or three symmetry axes or without symmetry axis.

The role of activities in teaching symmetry reflection is also highly important in the upper primary grades. Producing axial mirror image can happen in various ways, such as:

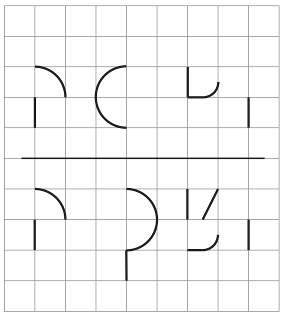
- *copying, moving*, using transparent carbon paper
- *drawing* on squared paper
- *construction.*

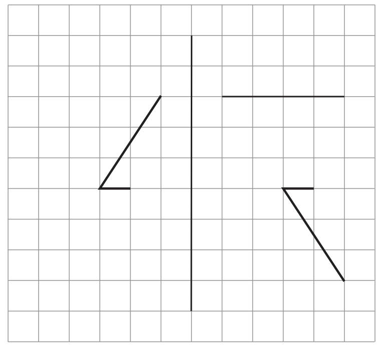
Mirror images produced by folding, moving and drawing on squared paper seem to be more challenging and complicated compared to the tasks set in lower primary grades. For instance see the tasks given in grade six below:

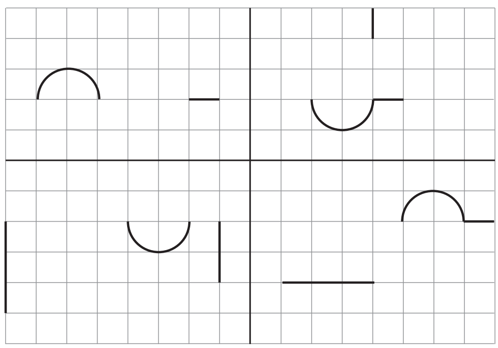
Make a reflection of the dog on the given axis.



The figures were taken to pieces by reflecting. When reflecting on the axes provided, the original figure will be produced. Try to figure out what the original picture was and then check your guess by means of carbon paper.







(*Mathematics course book, volume I. grade 6. Apáczai Kiadó, p. 33*)

It seems to be practical to introduce children to the characteristics of axial reflection using proper technical terms before they start constructing axial mirror image. These characteristics should be presented in a rather expressive way and the mathematical content should be illustrated not only by means of geometrical figures but also references to pictures, patterns from nature or everyday life, e.g. architecture, design, products of technology etc. The easily memorable pictures could be associated with some particular characteristics. For instance, the sections drawn on the wings of the butterfly cut out from transparent paper, which are actually the mirror images of each other, by opening and closing them, the wings will cover each other. In this way it can be demonstrated that the length of the section is identical with that of the mirror image.

When the concepts of geometrical transformations, including axial reflection are established, we should also make children realize that in case of transformations and reflection it is not only a figure but every dot of the plane is concerned with them. In a particular task it is always the picture of a figure that we are looking for, and this is why children tend to think that only this figure is being transformed. This misconception can be corrected by the following task:

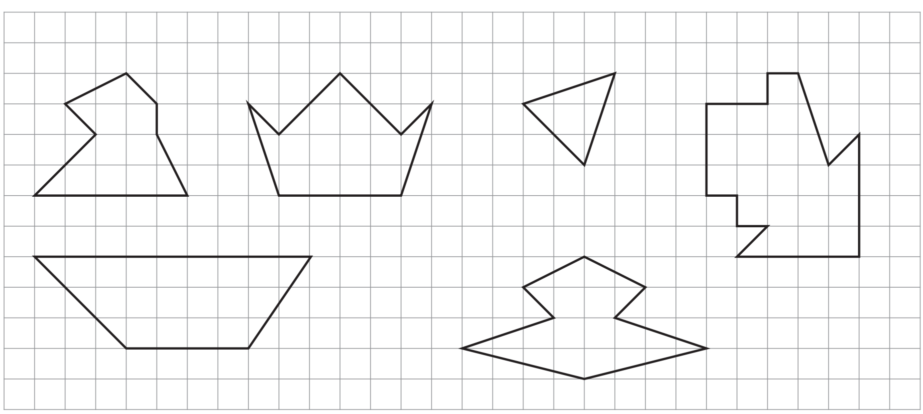
Move the chairs and the desks to the walls of the classroom. Then I draw a square grid on the floor with a piece of chalk and in the middle of the classroom I marked one straight line of the grid red. Everyone should stand on a random dot of the grid, and when the whistle is blown, they should take the shortest way to the red straight line, and then the same length.

Following these instructions children will get to the place of their mirror image. The red straight line is the axis of the axial reflection. Having completed the task children can easily visualize the plane as an enormous floor extending to infinity, whose dots, like the children can be transformed according to various rules.

In lower primary grades three and four learners are familiar with compasses and ruler, and in grade five they will learn how to construct triangles from given sections and also perpendicular bisector. In grade six children learn to construct the mirror image, the bisector and the perpendicular straight line and the symmetry axis of a figure. Learners construct the mirror image of a dot, a section, an angle, a circle, a semi-circle, and various polygons. Thus, they can make a reflection of a rectangle on one of its side straight line, on one of its diagonals, perpendicular bisectors, and on a straight line intersecting the two sides of any rectangular. Making a reflection on the perpendicular bisector of the rectangle can lead to the examination of the axially symmetrical figures and the definition of the position and the number of symmetry axes.

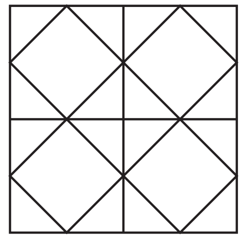
Two tasks involving symmetrical figures are shown below:

Peter wanted to draw symmetrical figures. He did not manage to draw some of them symmetrical. Correct the drawings he did not get right. Mark the axis red on the symmetrical plane figures.



Colour the figures (if it is possible) in a way that the coloured figures should have

1. *exactly one symmetry axis*
2. *exactly two symmetry axes*
3. *exactly three symmetry axes*
4. *exactly four symmetry axes*
5. *no symmetry axis.*



(Mathematics course book, volume I. grade 6. Apáczai Kiadó, p. 151.)

Teaching axial reflection requires the use of quite a lot of tools, and the use of these tools and the preparation of the various learner activities requires more organization and preparation from the teachers. However this extra work is absolutely essential, as without the proper use of tools and individual activities children would not be able to gain personal experience, which is necessary for the development of the proper content of notions. The proper system of notions are absolutely essential for the solution of mathematical problems and understanding concepts, such as the axial reflection.

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A tevékenység szerepe a tengelyes tükrözés tanításában

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Összefoglaló. A geometriai transzformációk – így a tengelyes tükrözés – tanulásának alapja a tapasztalatszerzésből kiinduló induktív megismerés. A konkrétból való kiindulás, a sokféle tevékenységből (hajtogatás, nyírás, rajzolás, tükörhasználat) származó tapasztalat összegyűjtése vezet el az általánosabb összefüggések megfogalmazásáig.

Alsó tagozaton a síktükörrel való játékos tapasztalatszerzés révén a gyerekek megfigyelik a különböző térbeli és síkbeli alakzatok tükörképét. Megépítik a testek tükörképét, hajtogatással, nyírással, rajzolással előállítják egyszerű síkidomok tükörképét, illetve tengelyesen tükrös alakzatokat alkotnak.

A tevékenységnek felső tagozaton is kiemelt szerep jut a tengelyes tükrözés tanításában. A tengelyes tükörkép előállítása többféleképpen történik: mozgatással, átlátszó papír segítségével, négyzethálós papíron rajzolással, illetve szerkesztéssel.

Kulcsszavak: matematikatanítás, tengelyes tükrözés, tevékenység

Formal language of mathematics and logic in elementary school instruction of mathematics

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Abstract. This paper presents a didactic tool for introducing pupils of early school age to the formal language of mathematics, logic and science. The tool is a virtual world on the computer screen. The picture – world consists of a chessboard on which different figures are laid. Properties are attached to every indivudual – figure and their positions on the chessboard are favourable for establishing various relations. The idea is based on a computer aid, *Tarski's World*, designed for the instruction of mathematical logic for philosophy students on American universities. Our world, named *Our Little Village* (the name has been borrowed from the Czech movie of the same title, directed by J. Menzel) is coloured, fun and suitable for creating children' stories, from which a trained teacher can abstract a lot of mathematical and logical principles/validities expressed by mathematical – logical formulas. In the same way the world can be approached reversely, so that, based on a list of formulas – statements – principles, a new configuration is created on the chessboard. This aid is a good tool for the introduction of pupils to mathematical structures such as the naive set theory and mathematical logic and is a good means of solving logical equations or riddles from recreational mathematics.

Keywords: Tarski's World, the naive set theory, mathematical logic, formalisation, mathematical structures

It is difficult to find content pertaining to the area of mathematical logic or naive set theory in the primary school math textbooks used in the Republic of Croatia. However, in other national curricula such as the Italian one, logic and sets are present in textbooks already early on in primary education [9], [13].

In the recommendation of the American committee for improving logic classes from 1993, lessons in logic are recommended for children as young as five [2].

In addition to the textbook *Language, Proof and Logic* by Barwais and Etchemendy [3], enclosed is also a CD containing several computer programmes intended for the users of the textbook – students of philosophy at Stanford University, USA – as an aid in following lessons in mathematical logic. Among these I would like to particularly emphasise a programme titled Tarski's world, in which three sizes of geometrical figures such as cubes, tetrahedrons and dodecahedrons are arranged on one chessboard. The user's task is to create the correct formula based on the position of the figures on the chessboard, or to correctly place the figures on the chessboard in relation to a given list of formulas. Further in their work, Barwais, Etchemendy [1], [4] and their students completed their idea and developed a theory on connecting classical mathemathical logic with visual perception in order to prove logical ([11], [12]) or mathematical [8] theorems and theories.

On the basis of the ideas expounded, this article will present one modified chessboard-based game. It will attempt to elaborate on the methodical potential of the game in using the *didactical phenomenology* of abstract mathematical ideas in line with Freudenthal [6], and utilising them as a teaching tool for introducing young pupils into the world of the fomal language of mathematics and of science in general. The nature of mathematical logic and set theory is close to the everday symbolical and linguistic practice of the young pupils, who are familiar with it even before commencing their primary education [7] it would be a pity not to utilise its great methodical and mathematical potential in class. With a good selection of figures and verbal stories about them, we can communicate in a way that is psychologically familar to children attempting to motivate them to work with the abstract, rich and precise language of mathematics.

The picture – Village 0. presents a small virtual world depicted as life in one small village. This is of course only one proposal for the configuration of this model and many others could also be made, by working in cooperation with pupils to shape the text and image,

A bit of theory

Let me introduce to you us meet the *denizens* of the village. *Every figure* on the chessboard shall be considered a *denizen of the village*.

1. The denizens are located in the squares of the chessboard which we shall mark as:

$$D = \{(i,j) \mid i = 0,1,2,3,4,5,6,7,8,9,\ j = 0,1,2,3,4,5,6,7,8,9\}$$

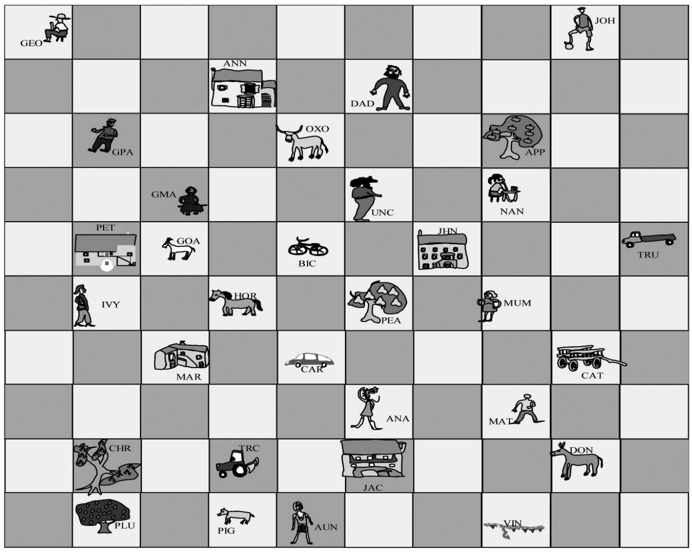
(i,j), with the *positional variable* interpreted as the *position* in row i and column j.

The sentence – formulas are first-order logical formulas with a standard interpretation of logical conjuntions and quantifiers, enriched with the specific constants defined in the list below. The constants $\top$ (true) i $\perp$ (false) are *closed logical sentences* in meaning.

2. The individual constants are:

GMA (grandma), **MUM**, **AUN**t, **AN**n**A**, **IVY**, **NAN**a, **GPA** (grandpa), **DAD**, **UNC**le, **JOH**n, **GEO**rge, **MAT**thew, **OXO**(ox), **HOR**se, **DON**key, **GOA**t (goat),

PIG, **VIN**e, **APP**le, **PLU**m (plum), **CHR**ry, **PEA**r, **TRC**tor, **CAR**, **TRU**ck, **CA**r**T**, **BIC**ycle, **J**o**HN**'s house, **JAC**ob's house, **MAR**k's house, **ANN**e's house, **PET**er's house



Village 0

3. The predicate constants are:

Female, **Mal**e, **Hou**se, **Ani**mals, **Pla**nts, **Veh**icles, **Peo**ple, **Anim**a**t**e (animate), **I**nani**mat**e, **Red**, **Yel**low,

4. the binary relations are:

SameCo**lumn**, **SameRo**w, **SameSo**r**t**, in **Fro**nt, **Beh**ind, **Nea**r, **Right**O**f**, **Left**O**f**

5. the ternary relation is: **Betw**een

6. special relation: $\rangle$ ("to be older than"), $=$ ("to be a same age")

7. constant functions: $\theta(i,j) = \emptyset$ (the position (i,j) is empty or unpopulated). Every figure-denizen has its *name* in relation to the configuration given on the chessboard, in the form of an ordered triple (α, β, γ) in which α is the individual constant, β is the predicate constant and $\gamma \in D \cup \{\emptyset\}$ where $\gamma = \emptyset$ indicates that α is not on the chessboard. For example, the *name* of the figure **OXO** in village 0 is (**OXO**(ox), **Ani**mals, (2,4)).

8. *individual variables*: $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$, $\mathbf{v}$, $\mathbf{u}$, $\mathbf{w}\ldots$, follow the variables of their components $\mathbf{x}_\alpha$, $\mathbf{y}_\alpha$, $\mathbf{z}_\alpha$, $\mathbf{v}_\alpha$, $\mathbf{u}_\alpha$, $\mathbf{w}_\alpha,\ldots$, $\mathbf{x}_\beta$, $\mathbf{y}_\beta$, $\mathbf{z}_\beta$, $\mathbf{v}_\beta$, $\mathbf{u}_\beta$, $\mathbf{w}_\beta,\ldots$, $\mathbf{x}_\gamma$, $\mathbf{y}_\gamma$, $\mathbf{z}_\gamma$, $\mathbf{v}_\gamma$, $\mathbf{u}_\gamma$, $\mathbf{w}_\gamma,\ldots$, $\mathbf{x}_\alpha$, $\mathbf{x}_\beta$, $\mathbf{x}_\gamma$ as the first, second and third components of name $\mathbf{x}$.

By τ used or a given configuration, for example for Village 0. we marke the interpretation of invididual constants, predicate constants, constant functions, *variable names* and complex variants. The basic constants are interpreted according to their full (or abbreviated) name as written in standard font with a standard natural and linguistic meaning. The other constants are interpreted as follows:

9. $\tau(\mathbf{Peo}) = \text{Fem}\cup\text{Mal}$, $\tau(\mathbf{Viv}) = \text{Fem}\cup\text{Mal}\cup\text{Ani}\cup\text{Pla}$, $\tau(\mathbf{Ima}) = \text{Veh}\cup\text{Hou}$, $\tau(\mathbf{Red}(\mathbf{x})) = \top \iff \mathbf{x}_\gamma = (i,j),\ i+j = 2k+1,\ k \in \mathbf{N}$. The figure is located in a *red field*. $\tau(\mathbf{Yel}(\mathbf{x})) = \top \iff \mathbf{x}_\gamma = (i,j),\ i+j = 2k,\ k \in \mathbf{N}$. The figure is located in a *yellow field*.

Interpretation of the relations:

$\tau(\mathbf{SamCo}(\mathbf{x},\mathbf{y})) = \top \iff x_\gamma = (i,j),\ y_\gamma = (k,l)$ and $j = l$. Figure **x** and **y** are in the *same* column.

$\tau(\mathbf{SamRo}(\mathbf{x},\mathbf{y})) = \top \iff \mathbf{x}_\gamma = (i,j),\ \mathbf{y}_\gamma = (k,l)$ and $i = k$. Figure **x** and **y** are in the *same* row.

$\tau(\mathbf{Fro}(\mathbf{x},\mathbf{y})) = \top \iff \mathbf{x}_\gamma = (i,j),\ \mathbf{y}_\gamma = (k,l)$ and $i < k$. Figure **x** is in the row *in front of* **y**.

$\tau(\mathbf{Beh}(\mathbf{x},\mathbf{y})) = \top \iff \mathbf{x}_\gamma = (i,j),\ \mathbf{y}_\gamma = (k,l)$ and $i > k$. Figure **x** is in the row *behind* **y**.

$\tau(\mathbf{LefOf}(\mathbf{x},\mathbf{y})) = \top \iff \mathbf{x}_\gamma = (i,j),\ \mathbf{y}_\gamma = (k,l)$ and $j < l$. Figure **x** is *left of* **y**.

$\tau(\text{RigOf}(\mathbf{x},\mathbf{y})) = \top \iff \mathbf{x}_\gamma = (i,j),\ \mathbf{y}_\gamma = (k,l)$ and $j > l$. Figure **x** is *right of* **y**.

$\tau(\mathbf{Nea}(\mathbf{x},\mathbf{y})) = \top \iff \mathbf{x}_\gamma = (i,j),\ \mathbf{y}_\gamma = (k,l)$ and $\max(|i-k|,|j-l|) = 1$. Figure **x** is *next to* **y**.

$\tau(\mathbf{SamSo}(\mathbf{x},\mathbf{y})) = \top \iff \mathbf{x}_\gamma = (i,j),\ \mathbf{y}_\gamma = (k,l)$ and $\mathbf{x}_\beta = \mathbf{y}_\beta$. Figure **x** has *the same properties as* **y**.

Ternary relation:

$\tau(\mathbf{Betwe}(\mathbf{x},\mathbf{y},\mathbf{z})) = \top \iff (\mathbf{x}_\gamma = (i,j),\ \mathbf{y}_\gamma = (k,l),\ \mathbf{z}_\gamma = (m,n))$: $(i = j = k \vee j = l = n \vee |k-l| = |i-j| = |m-n|) \wedge (|(m+n)-(k+l)| = |(i+j)-(k+l)| + |(i+j)-(m+n)|)$.. Figure **x** is in the same row, column or *diagonal* as figures **y** and **z** and is located *between* them.

10. special relations: $\rangle$, $=$ are a total order and equvalence relation respectevly.

(α) is defined for the set associated with the predicate constant **Peo** as **GPA** $\rangle$ **GMA** $\rangle$ **DAD** $\rangle$ **UNC** $\rangle$ **MUM** $\rangle$ **AUN** $\rangle$ **GEO** $\rangle$ **MAT** $\rangle$ **ANA** $\rangle$ **NAN** $\rangle$ **IVY** $\rangle$ **JOH** or defined for all individuals as:

(β) **PET** $\rangle$ **JHN** $\rangle$ **MAR** $\rangle$ **ANN** $\rangle$ **JAC** $\rangle$ **CAT** $\rangle$ **GPA** $\rangle$ **VIN** $=$ **GMA** $\rangle$ **TRC** $=$ **DAD** $\rangle$ **UNC** $\rangle$ **PLU** $\rangle$ **PEA** $\rangle$ **MUM** $\rangle$ **CHR** $\rangle$ **AUN** $\rangle$ **TRU** $=$ **GEO** $\rangle$ **OXO** $\rangle$ **APP** $\rangle$ **MAT** $\rangle$ **HOR** $\rangle$ **DON** $=$ **ANA** $\rangle$ **CAR** $=$ **NAN** $\rangle$ **IVY** $\rangle$ **BIC** $=$ **JOH** $\rangle$ **GOA** $\rangle$ **PIG**.

With the formula $\mathbf{x}\rangle\mathbf{y}$ it is indicated that *person or denizen* **x** *is older than person or denizen* **y**.

Let us now deal with some specificities regarding mathematics and logic on the chessboard.

The problem with implication

It is well-known that the logical connective $\implies$ or *material implication* resists intuitive understanding more than connectives such as conjunction $\wedge$, disjunction $\vee$ or negation $\neg$, as the latter follow directly from the natural practice of language.

The following psychology test that illustrates the *unnaturalness* of the logical connective $\implies$ has become famous, The Wason Selection Test.

Four cards are placed on a table before the examinee as depicted: A 6 D 7 with the question: how many times must the cards be turned to verify the following rule – if there is a vowel on one side of the card, then an even number must be on the other side? The examinees, even those with a background in mathematics, are usually unsuccessful in solving the test, as at first glance it appears to them that it is sufficient to turn over the 'A' card, while forgetting that the card marked '7' ought to also be turned. [5] However, if an analogous example from everyday life is given, the results are incomparably better. One example for this is the following question – in which manner will stranger A in the unknown city B verify that the pedestrians respect the red light when crossing the street? Most examinees answer that A has to spot that

1. when the light is red nobody crosses the street and

2. when the pedestrians cross the street the light is not red.

We shall try to introduce our young pupils into the world of implication by using the example of *our little village*. Let us approach them with the following story:

The head of the village is the uncle (**UNC**)*, who makes sure that life in the village unfolds harmoniously. In order to stimulate the grandchildren to visit their grandmother* (**GMA**) *more frequently, he parks the car* (**CAR**) *next to her house and promises that every young person that visits grandma will receive the car keys.*

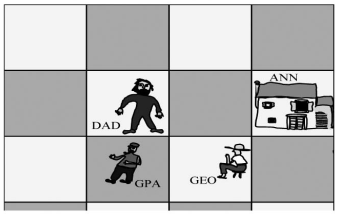
Let us translate the story in terms of the uncle's (**UNC**) promise:

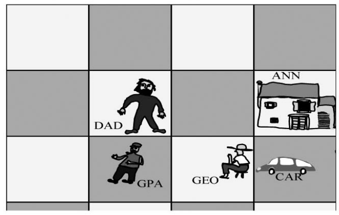
(1) *Every grandchild that visits grandma* (**GMA**) *will receive the car keys.*

Let us interpret this sentence as a formula:

$$(1a)\ \forall \mathbf{x}((\mathbf{Peo}(\mathbf{x}) \wedge \mathbf{Nea}(\mathbf{x}, \mathbf{GMA}) \wedge \mathbf{AUN}\rangle\, \mathbf{x}) \implies \mathbf{Nea}(\mathbf{x}, \mathrm{CAR})).$$

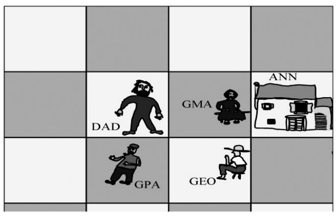
Let us take a look at the diagrammatical solution in *our little village*. We are interested as to in which cases the *village head's proposition* is respected and in which it is not.

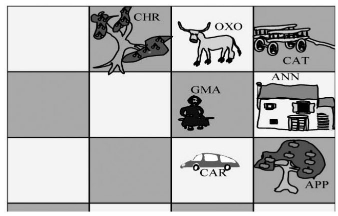




Village 1 *Village 2*

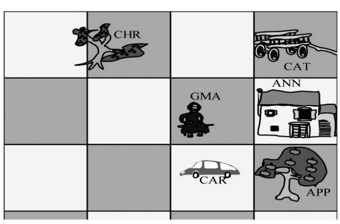
The pupils' task is to ascertain for which of the configurations the uncle's (**UNC**) promise is fullfilled, that is, for which is formula (1a) valid and for which it is not. In Villages 1 and 2 there is no grandma, thus (1a) is trivially correct because the antecedent has not been fulfilled, so it is irrelevant whether grandson George (**GEO**) near the car (**CAR)** (Village 2.) will receive the keys or not.

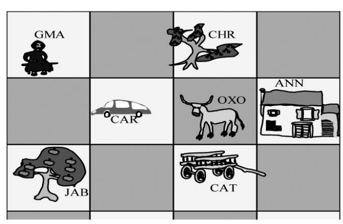




Village 3 *Village 4*

In Village 3 the promise has not been fulfilled for, though grandson George (**GEO)** is near grandma (**GMA)**, he cannot receive the keys as the car is not parked in his vicinity. In Villages 4 and 6 the promise has been fulfilled as in both of the free fields close to grandma (**GMA**) her grandsons can receive the keys of the car (**CAR**) (as they will be in its vicinity), but in Village 5 the grandson can be in grandma's vicinity (*above* grandma) but not receive the car keys, and thus formula (1a) will be invalid.





Village 5 *Village 6*

Therefore, we can verify formula 1a. with various configurations and connect its validity with the verification of the uncle's (**UNC**) promise, which is psychologically more familiar to pupils than an abstract definition of implication. They will probably agree that the promise has not been fulfilled (only) if the grandson has visited his grandma (antecedent fulfilled) but has not received the car keys (consequence not fulfilled), and that in all other cases association of the pair fulfilled/not fulfilled with the pair antecedent-consequence is correct.

Conclusions from the chessboard

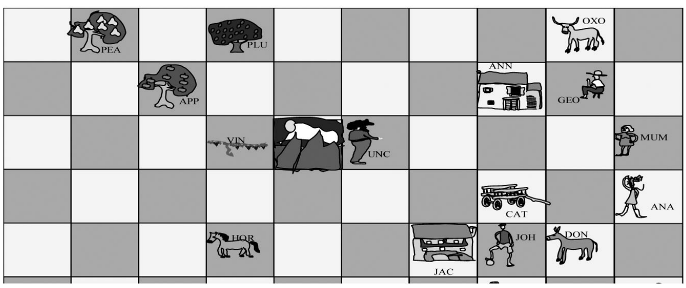
Let us imagine that the village is organised according to three formulas (see Village 7):

1. $\forall \mathbf{z}\ (\exists x \exists \mathbf{y}(\mathbf{Peo}(\mathbf{x}) \wedge \mathbf{Peo}(\mathbf{y}) \wedge \mathbf{Mal}(\mathbf{z}) \wedge \mathbf{Betwe}(\mathbf{z}, \mathbf{x}, \mathbf{y})) \implies (\mathbf{z}\rangle\, \mathbf{x} \wedge \mathbf{y}\rangle\, \mathbf{z}))$. *Every man lives between an older and a younger person if he finds himself between them.*

2. $\exists \mathbf{x} \exists \mathbf{y}(\mathbf{Pla}(\mathbf{x}) \wedge \neg \mathbf{SamSo}(\mathbf{x}, \mathbf{y}) \wedge \mathbf{Nea}(\mathbf{x}, \mathbf{y}))$. *There exists a fruit tree in whose vicinity there is no fruit tree.*

3. $\forall \mathbf{x}(\mathbf{Mal}(\mathbf{x}) \implies \neg \exists \mathbf{y}(\mathbf{Mal}(\mathbf{y}) \wedge \mathbf{Nea}(\mathbf{x}, \mathbf{y}) \wedge \mathbf{x} \neq \mathbf{y}))$. *Men do not like to socialise.*

4. $\exists \mathbf{x} \forall \mathbf{y}((\mathbf{Mal}(\mathbf{x}) \wedge \mathbf{Ani}(\mathbf{y})) \implies \neg \mathbf{Nea}(\mathbf{x}, \mathbf{y}))$. *There exists a man who does not like animals.*



Village 7

Our task is to infer and thus discover what is hidden *behind the hill in Village 7*. On the grounds of 2., behind the hill is $x \in \text{Ama}/\text{Pla} = \text{Fem} \cup \text{Mal} \cup \text{Ani}$. We can see that all men except the uncle (**UNC**) have animals. Therefore, according to 4 there is no animal next to the uncle and thus $x \in \text{Ama}/(\text{Pla} \cup \text{Ani}) = \text{Fem} \cup \text{Mal}$. There is a man *behind the hill*. According to 3 it is not a man thus $x \in \text{Ama}/(\text{Pla} \cup \text{Ani} \cup \text{Mal}) = \text{Fem}$. We can see that the uncle is located between two women, thus according to 1 it follows that the *person behind the hill* has to be older than him because mum (**MUM**) is younger than him. Throwing a glance at the list of the relations $\rangle$ we can see that it can only be grandma (**GMA**) *behind the hill*. Such tasks could be produced endlessly.

Here we demonstrated the construction of a digram of Village 7 based on the formulas given.

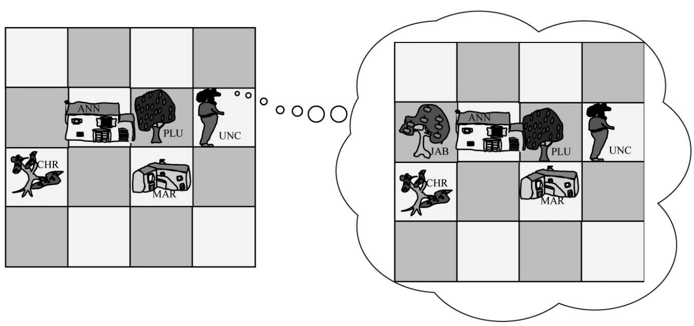
Laws and the world of necessity and possibility

Let us assume that the game on the chessboard allows for the position of the figures to be changed, for their number to be increased, for the relation *to be older than* to be reorganised etc. Let us also assume that the pupil is familiar with *all the denizens of our little village*, regardless of whether they are present on the chessboard or not. The only thing that shall be maintained during the game is the chessboard's format of 10×10 squares.

Together with the pupils, the teacher can discover which *claims* are valid regardless of the game's progress. Let us call such propositions the *laws* of the virtual world, just as normal science discovers and reveals the laws of reality.

Let us continue with stories about the village. For example, *the uncle (***UNC***) wishes to plant an apple tree (***APP***) next to Anne's house* (**ANN**). Therefore, within the configuration of the right village in the image of village 8 he imagines the *village* configuration in the cloud. In order to express the proposition of the story with a formula, we have to add the modal operators of *necessity* $\Box$ and *possibility* $\Diamond$ to the formal language.

In Village 8, picture to the right, it is valid that: $\Diamond\forall\mathbf{x}(Hou(\mathbf{x}) \Longrightarrow \exists\mathbf{y}(\mathbf{Pla}(\mathbf{y}) \wedge \mathbf{LefOf}(\mathbf{y},\mathbf{x}))$. We can see that without the operator $\Diamond$ the formula is not valid for the configuration of the right-side village. The uncle can relocate the denizens of the village or introduce new ones according to his wish, but there are some laws he cannot alter.



Village 8

For example: *It is a necessity that it is not possible that for every x there is a* **y** *located to the left of it*. Or, as expressed in a formula: $\neg\Diamond\forall\mathbf{x}\exists\mathbf{y}\mathbf{LefOf}(\mathbf{y},\mathbf{x})$. The law of the *left border*. Upon observing the image above, it is clear that in every world, and thus also this imaginary one, the uncle (**UNC**) cannot plan to plant the apple (**APP**) left of the cherry (**CHR**). The propositions in the scope of with the operator $\Box$ are thus valid *in all possible worlds*. Let us name the whole proposition a *cosmological law*.

Cocider the sentence: *Everything in the village is old*. This proposition can be interpreted with the formula:

(a) $\forall \mathbf{x} \exists \mathbf{y} (\mathbf{x} \rangle \mathbf{y} \vee \mathbf{y} \rangle \mathbf{x} \vee \mathbf{x} = \mathbf{y})$, the well-known law of *trichotomy*. Let us observe the expanded formula

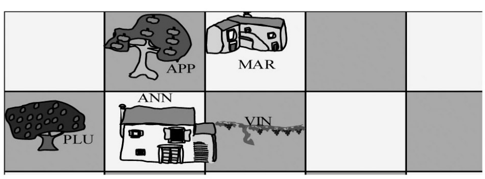
(b) $\Box \forall \mathbf{x} \exists \mathbf{y} (\mathbf{x} \rangle \mathbf{y} \vee \mathbf{y} \rangle \mathbf{x} \vee \mathbf{x} = \mathbf{y})$.

Formula (b) is valid only if we have selected (β) a sequence of relations $\rangle$ and will not be valid for a selection (α) of sequences. Therefore, a law with the prefix $\Box$ is dependent on the *inner structure* of the game. Let us describe these laws as *global*. Let us also add to these laws another class of claims stated by the *head of the village*, which are valid only for the configuration *we are viewing*. For example, the aforementioned conditioned promise on lending the car to young people to go out at night. Let us term them *the laws of the village head. The laws of the village head* can be expressed with formulas with the prefix $\neg \Box$, as they do not have to be valid in some other imaginary world. The possibilities of inventing new laws upon new laws are many, and they are followed by the invention of motivating stories as a background for introducing young pupils into the secrets of logic and science. A competent teacher can also introduce pupils to the crucial theorems of mathematical logic that are the foundation of mathematics, and also human practice and science in general.

Relations as structural determinants

The world depicted on the chessboard, *our little village*, represents a good tool for introducing pupils into the world of mathematical structures. We shall demonstrate several structural relations on the relation **Nea**.

It is easy to ascertain that a relation is symmetrical, that is, that the formula $\forall \mathbf{x} \forall \mathbf{y} (\mathbf{Nea}(\mathbf{x}, \mathbf{y}) \implies \mathbf{Nea}(\mathbf{y}, \mathbf{x}))$ is valid. It is also trivially valid that $\forall \mathbf{x} (\mathbf{Nea}(\mathbf{x}, \mathbf{x}))$ (follows directly from definition of the relation). The relation is thus *reflexive*, as can be plainly seen in the image below. Is **Nea** an *equivalence relation*, that is, is it also *transitive*?



Village 9

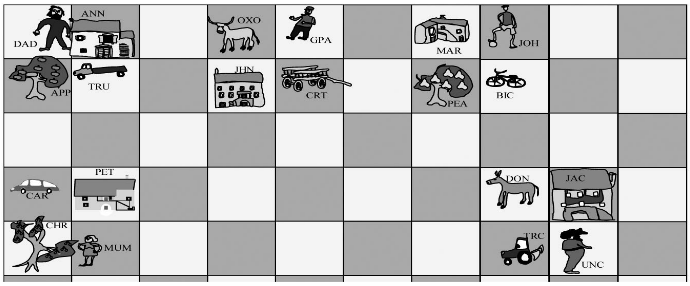
The drawing of the village above shows that this has not been fulfilled, as $\tau(\mathbf{Nea}(\mathbf{APP}, \mathbf{PLU})) = \top$ i $\tau(\mathbf{Nea}(\mathbf{APP}, \mathbf{MAR})) = \top$ but $\tau(\mathbf{Nea}(\mathbf{APP}, \mathbf{MAR})) = \bot$. Therefore, the sentence

$$\forall \mathbf{x} \forall \mathbf{y} \forall \mathbf{z} (\mathbf{Nea}(\mathbf{x}, \mathbf{y}) \wedge \mathbf{Nea}(\mathbf{y}, \mathbf{z}) \implies \mathbf{Nea}(\mathbf{x}, \mathbf{z})) \qquad (*)$$

is not true, that is, the relation **Nea** is not *transitive*.

Let us create such a village in which the relation **Nea** will be transitive, that is, will be an *equivalence relation*.

Let us say that a contagious disease is threatening the village and thus the head of the village, the awe-inspiring uncle (**UNC**), decides to reorganise the village into a string of smaller villages for hygienic reasons: *A garage has to be built next to every inhabited house, a fruit tree must be planted next to the house or a barn must be built, and the small villages must not touch each other.*



Village 10

In village 10 the relation **Nea** is transitive. In order for the relation **Nea** to satisfy sentence $(*)$, we have to form a configuration for which it is valid: $|\{y : \mathbf{Nea}(\mathbf{chf}, \mathbf{y})\}| < 5$ under the condition

$$\forall \mathbf{u} \forall \mathbf{v} \forall \mathbf{z} (\mathbf{Nea}(\mathbf{z}, \mathbf{u}) \land \mathbf{Nea}(\mathbf{z}, \mathbf{v}) \implies \neg \mathbf{Betwe}(\mathbf{z}, \mathbf{u}, \mathbf{v})) \qquad (**)$$

(none of them is between its “neighbors”) in which with **chf** we imagine a *variable-name* of village denizens that represents a set of proximate figures, and let us say that this valuable has the property **Poe**, *head of a small village*. The set $\{y : \mathbf{Nea}(\mathbf{chf}, \mathbf{y})\}$ is an element of the *partition* of the village according to an *equivalence relation* based on the relation **Nea**.

The elements of the partition may contain 1 (reflexive member), 2, 3 or 4 elements at most. If the set $\{y : \mathbf{Nea}(\mathbf{chf}, \mathbf{y})\}$ were to contain more than 4 figures, then $(*)$ will not be fulfilled. That is, for configurations on the chessboard under the condition $(**)$ the pupils will realise that: $|\{y : \mathbf{Nea}(\mathbf{chf}, \mathbf{y})\}| < 5 \iff \forall \mathbf{x} \forall \mathbf{y} \forall \mathbf{z} (\mathbf{Nea}(\mathbf{x}, \mathbf{y}) \land \mathbf{Nea}(\mathbf{y}, \mathbf{z}) \implies \mathbf{Nea}(\mathbf{x}, \mathbf{z}))$.

It is possible to form such games in *our little village* that may lead us into the world of naive set theory, ones based on *properties* (or *predicate constants*) enriched with set operations as a generator of new sets. Therefore, the game presented has the potential to, with the aid of various motivational stories and illustrated depictions, guide young pupils into the abstract world of mathematics and into the world of science in general. Also in doing so, we connect mathematics with topics from the mother tongue and art education.

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Formalni jezik matematike i logike u osnovnoškolskoj nastavi matematike

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Sažetak. U ovom članku prikazat će se jedan didaktički alat za uvođenje učenika u ranoj školskoj dobi u formalni jezik matematike, logike i znanosti. Alat je virtualni svijet na zaslonu elektronskog računala. Slika – svijet sastoji se od šahovske ploče na kojoj su poslagane različite figure. Svakoj individui-figuri pridružena su svojstva, a njihovi položaji na šahovskoj ploči prikladni su za formiranje različitih relacija. Ideja je temeljena na kompjutorskom pomagalu, *Tarski's World*, namijenjenom nastavi matematičke logike za studente filozofije na američkim sveučilištima. Naš svijet, nazvan *Selo naše malo* (ime posuđeno od istoimenog češkog filma J. Mencela), obojen je, zabavan i podoban za sastavljanje dječjih priča iz kojih osposobljen učitelj može apstrahirati mnoge matematičke i logičke zakonitosti izražene matematičko-logičkim formulama. Tom se svijetu može pristupiti i obrnuto, tako da se na osnovi liste formula-tvrdnja-zakona izgradi nova konfiguracija na šahovskoj ploči. Ovo pomagalo je primjeren alat za uvođenje učenika u matematičke strukture kao što su naivna teorija skupova, matematička logika, i dobro je sredstvo za rješavanje logičkih jednadžbi ili zagonetki iz rekreacijske matematike.

Ključne riječi: Tarski's World, naivna teorija skupova, matematička logika, formalizacija, matematičke strukture

Center for mathematics in living room of the pre-school education institution

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Abstract. Pre-school education aiming at development encompasses design of the Center for Mathematics and manipulative games in the living room for children. Materials of these centers support child exploration, imagination and manipulation i.e. these are the means that help children to learn how to compare, coordinate, calculate and categorize. Their value from the aspect of child's development are that the activities in this center help children develop their intellectual abilities, fine motorics as well as their eye and hand coordination. Through negotiating and problem solving they learn social skills as well.

Pre-school children reveal mathematic notions through rhythm and repetitive movement, colour, sound etc. Ready made didactic materials can be used for this purpose but also the means that kindergarden teachers make themselves are equally efficient. The main feature of the materials are that they offer children concrete, systematic experience in counting, enumeration and comparing.

In Centers organized in this way the task of the teacher is to equip the living room with different materials that offer a number of possibilities for developing skills of mathematical reasoning, observe children playing and use every opportunity to explain the notions in order to help children arrive at mathematical conclusion on their own.

The aim of the paper is to get insight into the organisation and equipment of centers for mathematics and pre-school teachers' idea of their role in center equipment.

Keywords: centers for mathematics, developmentally appropriate practice, pre-school child, pre-school teachers

Introduction

The living room in the pre-school education institution is divided into different activity centers. It is dynamic. Surrounding adjusted to development motivates complex playing, independence, socialisation and problem solving. Children are offered to research materials and find out new information as a way of learning. Activity centers contain different materials that children use and teachers choose the materials carefully so as to motivate research and to make them accessible for children to use them independently. By motivating a child to play we promote natural development of their skills and competences. When teachers plan and create activity centers they reconsider their spacial developmental adequacy to interests, furniture at disposal, materials. Activity centers are “little labs” where learning takes place initiated by children themselves i.e. learning through playing and working with various materials. Children choose the materials themselves and play with them as they wish moving freely from one activity center to the other. The time they spend in a center differs from child to child. When children actively focus on games and are offered possibility to use various motivating and creative materials they spend more time in activity centers and spend less time moving aimlessly around. The team of teachers inserts changes from time to time in the centers exchanging known materials with new ones, individualising, designing them upon children’s interests. Teachers evaluate efficacy of the center arrangement and change it as well.

Usual activity centers in the living room of a pre-school institution are: artistic expression centers, centers for mathematics and manipulative games, construction center, music center, nature research center, beginners’ literacy center, sand and water games center, family and drama games center, open air games area. Some other center can be added to the mentioned ones such as doctor’s corner, firemen’s corner, autumn corner etc. depending on current activities and interests of the group.

This paper aims at organising centers for mathematics in the living room of the pre-school institution and the role of the pre-school teachers in their equipment, changing and motivating children to activities. According to Romstein K. (2010) mathematics is a constitutent part of life present in architecture, fine art and music. Mathematical knowledge is taken as a “measure” while evaluating child’s readiness for primary school and it is actually the practical usage of mathematics that matters most.

Further on, the paper will focus on results of the survey done among pre-school teachers about their practice in organising centers for mathematics in the living room of the pre-school institution.

Methodology of the research

The aim of the paper is to get insight into the organisation and equipment of centers for mathematics and pre-school teachers' idea of their role in center equipment.

There were 39 pre-school teachers from 5 Osijek kindergardens taking part in the survey. The survey consisted of 10 questions divided into 3 groups. The first referred to general equipment of the centers for mathematics, the second to the role of pre-school teachers in center organisation and the third referred to competences that are motivated in centers for mathematics. In addition, the paper deals with ideas of pre-school teachers supporting their answers with recent notions by eminent authors as to get insight into organisation and equipment of the centers for mathematics and pre-school teachers' idea of their own role in the equipment of the centers.

General equipment of the centers for mathematics

The majority of pre-school teachers (93%) have a center for mathematics and manipulative activities in the living room of the institution. Those who do not have a separate center usually include the function of this center within some other center e.g. school center. It is obvious herefrom that pre-school teacher connect early literacy and mathematical concepts, which is natural. Center for mathematics is usually distant from some louder centers of activities (construction, family and drama centers). Herewith the center provides peaceful activity and full dedication to planned activities; children can focus on offered activities and materials. Among materials represented in the center there are didactic games such as puzzles, plastic cubes with numbers, various strings, wooden beads etc. Such materials are represented with 78% of pre-school teachers. 40% of the pre-school teachers have at their disposal social games such as ludo, domino, monopoly, memory, also non-structured material such as horse – chestnuts, acorns, pabbles, pasta, grains (50%). 30% of the pre-school teachers have geometrical figures at their disposal. Games and materials that are made by pre-school teachers themselves such as number cards, bank notes, stopples with numbers etc. are represented in 13% of the pre-school teachers whereas measure tape, set square, ruler and pencils in 20%. Some pre-school teachers have a clock, scales, magnets, glass vessels for measuring liquids in their centers although these are single cases. It is important to note that it is essential to pay attention to development of pre-operational phase (Piaget) in cognitive development of children. It is the period when small children play intensively and through playing they manipulate objects, learn about them and get to know the world around them, acquire first notions, develop language, make logical conclusions. At this age children understand only those quantitative and spacial relations that are perceptively given to them, master per-mathematical skills that help them understand the world of numbers, think in a logical and abstract way. The importance of offering and having these materials permanently is therefore unquestionable.

Out of the activities that are most frequently represented in the center for mathematics, so the pre-school teachers, the following are represented as follows: 63% numbering, 46% sequencing, 40% counting, 63% classifying, 66% comparing, 33% measuring. Some teachers state here that they do mathematical stories and word tasks, a few do various work sheets with all these activities. According to the teachers' statements the centers lack material and activities that motivate measuring, quantity establishing, weighing and other developmentally appropriate activities establishing length, height, quantity, mass.

The role of pre-school teachers in organisation of the centers for mathematics

The role of the pre-school teachers in the center for mathematics primarily refers to adding motivating material depending on interest of the children (76%), supporting children in their playing activity (43%) and most of them motivate the higher level of thinking in children in activities (83%). According to Kirsten A. et al (2004) the pre-school teachers help children widen concrete experience by modelling mathematic language eg. introducing concepts such as smaller than, bigger than, equal to by putting questions such as How many napkins are there? How many spoon do we need? The more the teacher considers the choice that children make, the more they are able to use the chosen activities for introduction and establishing of mathematical concepts. According to Vlahović-Štetić (2010) the job of the pre-school teachers is to help children construct the existing knowledge, to give children freedom for their own constructions and strategies, ensure atmosphere in which discussion helps to build child's knowledge, prepare manipulative materials, ensure social interaction and enable rich surrounding, complex situations that will motivate problem solving. Such thinking is in accordance with constructivism in psychology and Vygotsky's theory attributing great importance to qualitative communication among participants of the educational process as one of the basic factors and motivators of mental development.

Change of the position of the centers in space as well as adding and changing material depend mostly on child's interests (53%) and planned topics, activities and projects (23%). The remaining 24% of the pre-school teachers state that material change depends on their duration or cooperation with parents, one of the pre-school teachers mentioned that materials are changed and added when more difficult tasks are put before children and when she notices that they are ready for new activities. According to Manderić Z. (2010) motivating " the next step in development" is a key moment of constructivism (Vygotsky) and its application in practice.

Competences in children motivated in centers

All activity centers in the living room of a pre-school institution and open air areas offer an opportunity for children for mathematical learning. Pre-school teachers usually state that mathematical activities are carried out in the construction center (66%), family center where children share cutlery, "cook", weigh, add and count ingredients (53%). Research centers, where children research nature: fruits, water, soil, plants (43%) are frequently mentioned in this context. One of the pre-school teachers mentions the example of computer games that children bring from home and which have mathematical content such as joining clusters, continue the sequence of numbers etc. She also mentions that these games lessen the interest of children in non structured material and didactic games. Artistic center (26%), music center (10%) and sport games in the gym or outside (13%) have also been mentioned.

We can conclude about the interest of children in diversity and that the existence of different centers in the living room is justified and mathematics is connected with other areas of child's interest.

The answer to the question about evaluation of exceptionally successful activities for development of mathematical competences in children the pre-school children state besides mathematical activities also activities in the morning circle when children are counted, when the number of girls and boys is established, the number of the absent children (20%). Queing is stated as well, standing in pairs, serving the dishes, joining the napkins, taking a walk (50%) and 6% mention games such as when children play shopping, market, library, bank, post office, watch maker.

According to Slunjski (2012) mathematical competences in children depend on competences of the pre-school teachers, possibilities of interaction among children of different chronological age and different competences, freedom of children in self- organisation of activities, existence of enough continuous time for development of the initiated activities, unobtrusive communication style of the teacher with children. The question which aspects of educational context ensures development of mathematical competences in children is answered by the teachers by putting in the first place didactic material, acquired, made and non-structured (90%). In the second place they state freedom of children to organize the activities themselves i.e. activities when children choose who they will play with (53%). Social interaction between children and teachers takes the third place (50%) and mathematical competences of teachers the fourth (43%). The last position is taken by adjustment of time and space to child's needs (33%). It is obvious that sometimes teachers are limited by time because of obligation related to the rhythm of child's meals and daily rest in the kindrgarden but are also aware of the values of variety of materials being the source of child's comprehension, social interaction in group and child's initiative and freedom but also the importance of mathematical competence.

Examples of organised activities in mathematical centers

- Electrical circuit – the task is to connect the picture of the number and the cluster, the lamp glows if correctly solved
- The table for games with light and geometrical figures – children create by the means of geometrical figures, colours, shapes, teacher makes them more difficult according to the age, interest and abilities of the children

- Get the target – target with numbers is aimed at by balls, targets can be on the floor, on a board, with holes, rings etc. The task is to hit the target of a specific number.
- Games to develop motor skills – wooden grooved boards – in this game motor skills are developed by following the groove line by hand or foot.
- Attach a number – shapes in the picture are compared and a picture of a number or a number of dots is attached to it
- Social games – respect the rules, cooperate, recognize the picture of a number
- Sticks – order them in the sequence from the shortest to the longest
- Computer games – adding
- Count the teeth – clay model, counting
- Shadow theatre – planets scene plays
- Writing numbers – finger writing in corn semolina
- Stringing geometrical solids- according to their model children string wooden solids on sticks
- Hop scotch – game by hopping according to numerical order
- Measuring – measuring of different parts of bodies by the means of paper measures
- Measuring liquids – glass measures of different sizes.

Conclusion

These results offer insight into the organisation and equipment of the centers for mathematics and indicate pre-school teachers' idea of their role in this center. The offer of the material and activities in the ceneter for mathematics depends on age, age homogeneity and heterogeneity, implicit pedagogy of the pre-school teachers, programme (groups for sports, English, religion). Integrated programmes prevail in kindergardens carried out through all areas of child development (physical and motor development, social-emotional development, cognitive development and speech, expression and creativity) so that mathematical contents are present in most of the topics and activities.

While organising centers for mathematics teachers are mostly focused on their equipment with didactic material and social games but it is very positive that 50% of the centers have also non structured materials at their disposal that enable children creative research and learning about nature, motivating creative development and possibility for different ways of material manipulation. It is important to point out that according to teachers' statements there is lack of material and activities that motivate measuring – establishing quantity, weighing and other developmentally appropriate activities by which length, height, quantity and mass are established. The role of the teachers in the centers is viewed as thorough although only one of the teachers states in open-questions that she motivates higher level of thinking in children. This could be explained by lack of time to individually work with children, many children in groups and a number of centers in which the activities take place at the same time. Only very experienced and well organised teacher with a small number of children in a group (the number set by Pedagogic standards) can be dedicated to their work and by constant learning and improvement develop their competences and the competences in children.

The purpose of the paper was to present, by examples from practice, the activities that pre-school children take part in in centers for mathematics and how mathematical competences in pre-school children can be developed in all activity centers organised in kindergardens.

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Matematički centar u sobi dnevnog boravka predškolske ustanove

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Sažetak. Razvojno primjerena praksa predškolskoga odgoja obuhvaća i plansko osmišljavanje centra za matematiku i manipulativne igre u sobi dnevnog boravka. Materijali sadržani u tim centrima podupiru dječju eksploraciju, imaginaciju i manipulaciju, odnosno pomoću njih djeca uče uspoređivati, usklađivati, računati i kategorizirati. Promatrajući njihovu vrijednost s aspekta djetetovog razvoja, aktivnosti u ovom centru pomažu djeci da razvijaju intelektualne sposobnosti, finu motoriku i koordinaciju očiju i ruku. Međusobnim pregovaranjem i rješavanjem problema, ona uče i socijalne vještine.

Predškolsko dijete, kroz ritmičnost i repetitivnost pokreta, boja, zvukova i sl., otkriva matematičke pojmove. U tu svrhu se mogu rabiti gotovi didaktički materijali, ali i različita kreativna sredstva koja odgojitelji mogu samostalno izraditi. Glavna odlika materijala koji se djeci nude je konkretnost. To su materijali koji djeci nude zorna, sistematična iskustva brojanja, računanja i uspoređivanja. U tako osmišljenim centrima, uloga je odgojitelja funkcionalno opremiti sobu dnevnog boravka materijalima koji nude različite mogućnosti razvijanja matematičkog mišljenja, zatim promatranje djece u igri kako bi mogao nadovezati se na iskazane potrebe ili pitanja djece, te poticanje razgovora i pitanja koja potiču više misaone procese i omogućuju djeci da samostalno dođu do novih spoznaja.

Cilj ovog rada je dobivanje uvida u organizaciju i opremljenost centara za matematiku te odgojiteljevo viđenje vlastite uloge u opremanju centra.

Ključne riječi: centri matematike, razvojno primjerena praksa, predškolsko dijete, odgojitelji

The joy of mathematics

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Abstract. According to George Pólya once someone has discovered the joy of *mathematics*, they will never forget it.

In this research paper I present a set of activities that were carried out during the previous academic year among second grade student (aged $7-8$) living in Oradea. The purpose of these activities is to make the student enjoy and like studying mathematics and to make them motivated in gaining mathematical knowledge. Moreover I was applying methods that can make math classes interesting and efficient.

Different interactive methods were applied using elements of Drama Pedagogy, didactical games and some components of the cooperative method. Our main standpoint was to let student be a part of the process of solving the mathematical problem and to encourage them to cooperate as often as possible. Our intention was to show student that they should not be afraid of trying to solve the problems and they should not be afraid of failure. Thus they can enjoy solving problems even on their own.

Keywords: experience, skill development, action-oriented and child-centered learning, sense of achievement, intrinsic motivation

Introduction. Alternative pedagogies

In the 60s in Hungary, Varga Tamas and his work team have brought about a new revolutionary method of teaching mathematics, which is a complex system of *curriculum*, teaching strategies, methods and teaching aids. This teaching method is known abroad as the Varga-C. Nemenyi method of teaching mathematics and it has been under implementation in Finland since 2000.

"The playfulness is served by clever, constructive games. The main occupation of a child is the play because it brings along excitement, enthusiasm and enjoyment. At the beginning of the class it can raise the interest or it could close a learning stage. The play may help dissolve the phenomena that inhibit the thinking procedure. (Kikovicsné et al., 2006, 26–27)

We owe to Tamás Varga, the discovery that the young pupils need to be taught not only counting and measuring but need to be initiated into other fields of mathematics, too. He thinks that a prolonged experience in acquiring knowledge is necessary.

“If the children are in an accepting atmosphere and they learn in group meeting problems specific to their age and interests, then they will experience the freedom and joy of independent thinking. We cannot exclude from our schools some teaching procedures such as: amazement, recognition, the pleasure of solving a problem, the experience of surpassing oneself, the group work, being together with pears, thinking together, the joy of creating. Excluding all those we would exclude the joys of life from schools.” (Kikovicsné et al., 2006, p. 33)

“The more teaching methods we use, the more learners we reach. And the more ways we apply to reach them, the deeper the knowledge embeds”. (Spencer Kagan)

Among the main objectives of the alternative pedagogies (such as Waldorf, Montessori, cooperative teaching, teaching through drama) are: the education for an individual thinking, the use of such methods, techniques and plays which develop the child’s imagination and sentimental education.

Instead of filling the child’s mind with a pack of information that he will soon forget anyway, I chose to make use of the possibilities which lie in the circle plays. Thus the time spent playing made learning more effective and our work more successful. The children memorized more things because they were an active part of the learning process.” (Finser, 2005, p. 28)

Finser M. Torin, a Waldorf teacher, who tries to teach mathematics similar to the Varga method to children aged between 7–14 asserts that his aim is not only to make them practise some skills but to develop their talent, too. To develop such talents that may be used in different situations in life. It includes the fact that the children should learn not only how to follow the sequence of tasks and of numbers, but they should develop the ability of flexible thinking, should preserve their imagination, should learn how to solve problems together and to experience things together. (Finser, 2005, p. 84)

The drama in education (the drama pedagogy, DP) arrived in Oradea 15 years ago due to some enthusiastic teachers and since then it has been offering teachers an alternative and a possibility to meet their pears and to teach in a colorful way. The essence of the DP is to touch the feelings and to make children learn through actions and activities. The DP workshop of Oradea (NDPM) is a professional learning community where the members take part actively in the learning process while they share their teaching experience and thus reinforce their identity as members of a community. After some time the members learn to learn from each other, to work together, to take responsibility for each other and to cooperate for the common aims. The professional learning community – besides the fact it assures the professional development of the individual and of the whole community – functions as a supporting community working as a protector factor against the loss of enthusiasm of teacher.

Activities

In my lecture I'm going to talk about a series of activities designed by us and which are taking place during the current academic year among 24 second grade pupils (aged 7 – 8) from Lórántffy Zsuzsanna Reformed Church School, an average class in Oradea. Their Teacher is Idikó Kovács. The activities are held once a week during a lesson.

Age Characteristics: The children aged between 4 and 8 are the most receptive to forming. According to specialists this age is perfect for basing the natural sciences. This is the time to help them build up relations with the world around them, the rules and phenomena of nature.

The age between 7 and 14 is the period of time when the child's sentimental life is preeminent. All that we teach him during this age, making him use his imagination, will remain well embedded in his mind. If we consider the stages of intellectual development, Piaget says that the child of this age is in the stage of *concrete operations* (Piaget, 1970). This is the time when, in the way of thinking, there appear a series of logical grouping: logical relations, connections, the concept of number, time, length, aria, volume, mass, weight. All these are called operative concepts because they are formed based on certain operations. They were named concrete operations because they depend on the recent concrete experience and the logical conclusions are restricted to these operations.

He can understand and do only such operations that do not surpass the models. The children are not able to draw conclusions on a theoretical level. They need a tangible, real object. The operations are always concretes, they are always connected to an activity. They are never abstract. (Ambrus, 2004, pp. 45–46)

Aims: The aim of these activities is to make learning mathematics more interesting, more exciting, more attractive for pupils and to raise their interest in acquiring mathematical knowledge, as well as to try out such methods that may turn the Mathematics classes into interesting and effective classes.

To stimulate pupils to calculate in mind, being able to add and subtract in mind any time without using paper and pencil.

We look for various exercises, more or less difficult, which cannot be found in a school book and which are closer to true to life situations. Sometimes we transform the more abstract tasks into more tangible ones to bring them closer to the pupils and to make them more pleasant for children when solving. We search for or try to invent such activities, exercises, which can captivate their volition so that the children will work to solve it involving all their energy, all their senses of perception, finally their whole body. Thus we can stimulate not only their minds but their whole body, too. During this procedure the teaching material will be better learned.

Our aim is to develop their creative thinking and their problem solving ability, to guide the children in using the acquired knowledge, to encourage them discover connections, to stir the logical thinking, the critical or divergent thinking and the argumentation. We analyze open problems, too, where they meet different approaches to the problem and there are more questions related to the same problem.

We would like to offer a learning environment which makes the discovery possible, stirs the inner motivation, motivates learning and the group work, the exchange of experience and it supports the talent.

We use different active and interactive methods, we try to vary the activities, mainly those which bring mathematics closer to the learners. We prefer the well working Hungarian methods (those of Tamas Varga and Zoltán Dienes). We do not intend to replace them with foreign methods but we prefer to use drama, didactic games and elements of cooperative methods.

The main objective is the learner: to make him part of the problem, involve him in the solving procedure, offer him the possibility of self expression or manifestation, help him to experience success and form him not to be afraid of failure, make him understand that mistake is allowed and lead him towards the pleasures of solving a problem.

"If we fill them with dry, abstract concepts, then no wonder that they will finally exclaim: And what if. . . ? The abstract thoughts, conceptions will shortly become useless and insignificant." (Finser, 2005, p. 40)

Activities: We start the lesson by pushing aside the desks in the classroom making some space. Then we continue with some games as warming up. These are usually skill games mingled with some mathematical elements, designed by us so that they preserve their ability feature but in the same time introduce some mathematics. After two or three games there are several groups of four formed. Examples of such games:

1. Circle game, ball game: the children are in a circle; everybody says a number in an increasing sequence and memorizes his or her number trying to remember the other ones' number, too. The pupil holding the ball starts the game by saying: "I went to the market and I bought 6 kg of apples" then throws the ball to the pupil who is number six. The later one continues: "why 6? why not 12?" and throws the ball to number 12. Those who do not pay attention or fail the others' numbers or do not notice those who fell out of the game and whose numbers cannot be further used, will fall out of the game, too.

2. Circle game: we make up sequences. Somebody says a number and the teacher asks pupils to add to the previous a number another number 1 or 2 or 3. . . by the time everybody had his or her turn, they would realize that this procedure may be continued with one or two rounds or it can be endless.

3. Animal and number game: everybody picks a piece of paper containing a name, an animal name and a number (e.g. Goose Gill 367). They are asked to walk about and find the members of their family by making sounds imitating the animals (those who gaggle find each other). As soon as they find each other they have to sit in each other's lap starting with the eldest to the youngest (e.g. Goose George 895 sits in Goose Gina's lap 657, who then sit in goose Geer'slap 489 who finally sits in Goose Gill's lap 367). The newly formed family will function during the rest of the lesson as a work group.

4. Domino game: everybody gets a colored domino card which has a number on one side and a mathematical operation on the other side. The teacher starts the activity with her own card everybody solves in mind the operation lying on the floor, so that he or she should know when to continue the domino line with his or her own card.

5. The club of hundreds: everybody picks a colored piece of paper containing a mathematical operation to be solved. As soon as he finishes the calculation he can pick from the blackboard another colored piece of paper with a number on it so that finally the sum of the numbers on his cards should be exactly 100. When the teacher calls out "Stop" they should finish the calculations. With the teacher's help they check each other to see who can sign in the club of hundreds (those whose final sum in their hand is 100).

6. Honey cake: everybody gets a card with a number. The pupils are lined in 2 rows, back to back to each other. The teacher reads aloud a receipt containing some mathematical operations. The children should solve in mind these operations. The first to recognize that the result of the operation is the same with the number on his card should start running to the end of the row. There are always two children having the same number and thus the same result to an operation. But who gets there first? By the end of the receipt a pupil will have to run several times.

The receipt: Ingredients for the dough: 25–15 dkg butter or 17–9 dkg grease, 15–8 dkg sugar, 41–21 dkg honey, 35–34 entire eggs, 29–26 dl milk, 13–12 coffee spoons of baking soda, 19–17 coffee spoons of cinnamon or powder clove. We mix all these and then we add 63–13 dkg flour. We bake 18–12 tablets on the back of the baking dish. We leave it in the oven for 56–26 minutes. Then we leave them to cool down for 43–33 minutes.

The cream: we heat up 48–18 dkg sugar until it gets brown and then we pour on it 26–23 dl of water and 16–12 dl milk.

We mix 23–13 dkg flour with 8–4 dl milk, and we add it to the hot caramel then we boil the mixture and leave it to cool down. We add a pack of Rama margarine to the cold cream and 30–28 vanilla. We mix all that together.

We cover the filled plates with chocolate coating: we melt 56–36 dkg chocolate above steam or we can prepare our own chocolate like this: we boil together 34–24 sugar, 15–9 dl water and 57–55 spoons of cocoa. We take off the stove and we add 24–16 dkg butter.

Time of preparation: 78–28 minutes. It will not last more than 23–16 days.

After the warming games, when the teams are already formed, they sit at the tables in groups of four and get the tasks. First task is a handcraft when they have to assemble together certain number cards according to a given sum. Then they pass to the next task. Together with the teacher I check each team, we listen to their arguments, we accept the good results or if it is needed we help them with prompts/ questions that may lead them to the solution. We sometimes act out the problem or we encourage them to draw representation of it.

Tasks

1. Uncle Tom has an orchard with 24 apple trees. He waters them every day this way: he goes to each tree once a day and he never steps twice the same path. How can he get from tree 1 to tree 24?

2. There are some snakes, frogs and two storks in the reeds. They count altogether 9 heads and 12 feet. How many snakes, frogs and storks live there?

3. A king has 4 sons. Each brother has a sister. Into how many parts should he divide his kingdom to give each of his child an equal share?

4. We have got a pot of 25 liters. There is 18 liters of water in it. If we add 10 more liters to it how much water will be in the pot?

5. Tom and his sister have altogether 30 cards of calendar. If Tom gave his sister 5 cards each of them would have the same amount of cards. How many cards has now each of them?

6. Two little elves, Moss and Fern are planting in the garden. While Moss is planting 3 plants Fern is planting only 2. How many plants does each elf plant if they finally plant 30 altogether?

7. A snail fell into a 6 meter deep well. During the day it climbs 3 meters but it slips back 2 meters each night. When does it finally gets out of the well?

8. If a tortoise can cross the road in 60 minutes how long does it take to 10 tortoises to cross together the road?

9. Ludas Matyi is going to Dobrog. On his way he meets 3 men on horseback, 3 men in a carriage and 3 men walking all coming from the opposite direction. How many people are going to Dobrog?

10. Mr Kovacs has 6 children. There is 2 years difference in age between them. How many years of difference is between the youngest and the eldest?

11. Hencida is 12 km far from Boncida. Two men meet on the way between the two towns: one is on foot, the other is riding a bike. Which one is farther from Boncida if they left at the same time from the 2 towns (one from Boncida, the other from Hencida)?

12. A candle burns in an hour. If we light 5 such candles in the same time how long will they be burning?

13. How many 3 unit numbers are there which you can read from left to right and vice-verso and it still remains the same?

14. The miller goes to the mill. There are 4 women coming towards him. Each woman is carrying 3 sacs. In each sac there are 3 cats, and each cat has 3 kittens. How many altogether are going to the mill?

15. There are 100 hundred houses in a street. A craftsman is asked to write the numbers of each house. How many times does he have to write the number 9?

16. You have got a bag of candies. You give half of them to your friend. You drop the half of the rest so there are only 6 left. How many candies did you have in the beginning?

17. I picked a number. I took away 3 and then I added 5 to the rest. Afterward I took away 10 so the rest was 10. Which was the number I thought of?

Teaching experience/conclusions/

During the activities we observed the following:

In the beginning it was very difficult for the pupils to make calculations in mind but they got used to it and got faster (even at catching the ball).

They better pay attention to each other. When one of the team member tries to explain the others the situation, we are amazed how well can he do that. It's true that the children can explain things best to each other. In the same time they have to explain to the teacher their choice and thus they will better understand the task and they can see it through.

They notice many features they have not learned yet: the features of the addition (associativity and commutativity which make the calculation easier). They haven't learnt the multiplication yet, but certain exercises can well prepare it.

They have learned to pay more attention to the text. We do not solve two similar exercises which have an analogue solution. The children must pay great attention because there are any logical problems hidden among the others.

We have even solved equations but step by step, analyzing.

We always ask them to draw, encouraging them to use the representation.

We stimulate a positive attitude towards mistakes, that is we let them do the calculation they have started asking them whether they are convinced about the result. They may realize that they have made a mistake somewhere.

They children enjoy very much these classes. There were pupils who told us before the second meeting that they had been looking forward to this class.

According to György Pólya "solving a mathematical exercise can be so much fun as solving a crossword, because the powerful intellectual work can be as good as a powerful tennis match. If somebody will taste the joys of mathematics he will never forget it. In this case all the chances are given to make the pupils give some importance to mathematics in the future.

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Megízlelni a matematika örömét

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Kivonat. Pólya György szerint "ha valaki egyszer megízleli a matematika örömét, nem fogja könnyen elfelejteni."

Előadásomban egy általunk kidolgozott tevékenységsorozatról számolok be, amely az elmúlt tanévben zajlott, 24 második osztályos (7–8 éves) nagyváradi kisdiák vett részt benne, egy átlagos iskolai osztály tanulói. A tevékenységek célja a matematika tanulását érdekes, izgalmas élménnyé tenni, megkedveltetni a tanulókkal, motiváltakká tenni őket a matematikai ismeretszerzésben, olyan módszereket és eszközöket kipróbálni, alkalmazni, amelyek eredményesebbé, érdekesebbé tudják tenni a matematikaórákat.

Különböző aktív és interaktív módszereket alkalmaztunk, a drámapedagógiai játékokat, didaktikai játékokat, a kooperatív módszer néhány elemét használtuk. A fő szempont, hogy a tanuló részese legyen a problémának, a megoldási folyamatnak, minél gyakrabban és eredményesebben nyilvánuljon meg, legyen sikerélménye, ne féljen a próbálkozásoktól, a kudarctól, és szívesen oldjon meg egyedül is feladatokat.

Kulcsszavak: élmény, képességfejlesztés, tevékenységközpontú és gyermekközpontú tanulás, sikerélmény, belső motiváció

Application of the elements of Vedic mathematics in classes with Roma pupils

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Abstract. Work with Roma children raises special challenges for educators and teachers who during their pre-service training have not been prepared for the specific work within the culture of the Roma community and the Roma language and in particular for the flexibility in work with Roma pupils. Interest of Roma pupils in educational process is a challenge to which we have tried to respond by introducing elements of Vedic mathematics in tuition. This article provides a methodological proposal which shows a connection of an active approach of Roma students' mathematical learning with the introduction of the Vedic way of calculating in additional mathematics classes and extracurricular activities.

Keywords: Vedic mathematics, Roma, additional mathematics classes, extracurricular activities

Introduction

Primary school teachers with many years of experience with Roma pupils are daily facing challenges and difficulties in the realisation of curriculum. It takes more time for processing of maths lessons, and mastering of those lessons is a necessary precondition for continuing with the curriculum. Maths has to be practised and exercised continuously. Roma pupils come from deprived and disincentive surroundings, and whether we like it or not, this has a great influence on the adoption of the curriculum. Pupils do not work at home; do not write their homework or exercise. Students do not work at home, not doing homework, not training. Multiplication table is an example of basic knowledge of mathematics and very often pupils in higher grades have not acquired it. Vedic mathematics offers a different model of multiplication where it is enough to know the multiplication table up to $5 * 5$.

Roma in Međimurje

The Roma community in Croatia is included in census as a nation since 1971. It consists of different ethnic groups. In northern Croatia, in Međimurje, prevails the group which speaks *Ljimba d'bjash*. This is an old Romanian dialect which was adopted by Roma during their stay on the territory of today's Romania. (Novak Milić, 2007; Olujić and Radosavljević, 2007). They have migrated to Croatia during the last decades of 19th century. In 2011, there were Roma, and 5232 of them claim that their native language is Roma, and 77 claim their native language is Romanian. According to data, we can conclude that there are 5 321 Roma in Međimurje (`www.dzs.hr`).

Table 1. Number of Roma population 1971 – 2011

Year	**1971.**	**1981.**	**1991.**	**2001.**	**2011.**
Međimurje	153	1139	1920	2887	5107
Croatia	1257	3858	6695	9463	16975

In the last years, there are many papers, researches, reports and programs about Roma, their position and life conditions in Croatia, whose aim is to draw attention to discrimination and xenophobia against the Roma national minority which already lives on the alarming margin of society. The most important researches are "Social position of Roma in SR Croatia" which was conducted by the Institute for Social Research in Zagreb in 1982, "Social and developmental status of Roma in Croatia", which was conducted by the Institute of Social Sciences Ivo Pilar in 1998, "Structure of Roma families and understanding of parenthood in them" conducted by the National Research Institute for Family, Maternity and Youth in 2002.

The "Report on Roma approach to employment", 2004 by L. Kušan, I. Zoon, and magazines Rural sociology 87/90, 1985 entitled "Studies of the social situation of the Roma" and Social Studies 2–3 (46–47), 2000, entitled "The social status of Roma in Croatia".

Roma aspiration to preserve their identity and to be different is a basic human right. The problem is how to maintain traditional culture and identity in a time of global and rapid modernization. There is the constant question of whether to maintain the traditional differences which contribute to their unequal treatment or to accept the need for change and modernization, which can help them to gain equality, but also to change their identity. Although the desire of Beash Roma to live together with the majority population in Croatia, the most of Roma are spatially dispersed, unconnected with the "typical" type of settlement. They live cities, where they usually inhabit the suburbs, along the edge of the road, on the forest boundaries, on the edge of the village, but they also live in villages, usually in separate "Gypsy settlements" without built infrastructure (sewer, water, garbage collection, roads, etc.). Their social space can be interpreted on three sociological concepts: exclusion, marginality and subclasses (Fassin, 1996). Today they are spatially stable and live in permanent caravan locations. So Roma are spatially, economically and politically marginalized (Šućur, 2000).

Roma education

Most of Roma political and cultural elite is aware that education is the key to modernization. However, education, especially today, requires tangible assets and liabilities. Many Roma communities are lacking both. Since hard living and housing conditions mainly dominate, and Roma do not have the habit of schooling, and have a high percentage of illiteracy among parents, many look with contempt at investments which education requires. Therefore Roma resist to their children's education.

Low level of education has implications for the general quality of life and life conditions. Starting from the reality that today's schools have become meeting places of different cultures, religions, languages and points of view, the development of teachers' attitude towards culturally different students based on intercultural competence becomes crucial when dealing with Roma students. Student is the subject of the process of education, a teacher's partner in the common work, and the most important reason for the existence of the entire school system and the education system (Mijatović, 2002).

Pupil, teacher and parents are responsible for the success of pupils. Roma pupils and parents are mostly illiterate and uneducated people who think that it is enough to send their children to school and that here end their concerns related to education. Attitudes of students towards school obligations are very different. There are students who come to school with clean books, who carry school slippers and accessories, as well as those which only see the as a dry and warm place to stay. Continuous monitoring of the development of pedagogical science becomes fundamental premise of improving the quality and effectiveness of education, professional development of teachers and necessary step towards a society of knowledge (Hrvatić and Piršl, 2007).

In explaining the cognitive development of children it is usually mentioned Piaget's theory. He explains that the inheritance determines the sequence of developmental stages, but that environment gives specific content (Piaget, 1977). This means that environmental influences determine the duration and effectiveness of content and activities. The child's environment at the age of early childhood present parents, siblings and relatives and neighbors. A child learns by experience, often by method of trial and error and imitation. Thus, children will imitate people around them, using all their merits and vocabulary. The earliest and easiest to adopt are the terms that have to do with children's everyday experiences, such as the ideas of things, shapes, relationships between people and objects. Thus, to understand the cognitive development of children in the ages of early development we must be thoroughly familiar with everyday environment in which the child lives. Particular attention should be paid to the development of concepts about life, the concept of space and time, then the social concepts (relations among family members, and relatives and neighbours).

School reform is a social and educational process. It, one hand, radically changes education policy and position, the position of students and teachers in the education and organization of the education system and its contents. On the other hand, educational reform requires a new organization of the school, the application of modern and proven methods, the application of new teaching technologies... The school, then, through its activities focuses the attention to flaws and weaknesses in the work, especially when it comes to students with disabilities. In the past, therefore, came to doubling and paralysis of inclusive education (Lapat, Milenović, 2010).

Society sets up the task of conducting the inclusive education, to ensure systematic knowledge adopted through active learning of each student in relation to his abilities. Theoretical construct known as “lifelong learning” includes learning from birth to death (more in: Pastuović, 1999). This is actually a system of various forms of formal, non-formal and informal learning of young people and adults. Experts will agree that in this lifelong process the most important thing is learning at the very beginning of life, so learning from birth to school age and learning in elementary school. Experts are always asking questions about the factors that affect the learning of the young human being as well as the ability to influence and control this complex process. The answers to these questions as a starting point for the announced scientific evaluation should be sought in the oeuvre of knowledge and understanding of developmental psychology and preschool pedagogy and education science (e.g. Babić, 1991, Babić, 1993; Miljak, 1995; Miljak, 1998; Pastuović, 1999).

In Europe we countries in which compulsory education begins from the age of four (Netherlands) or five (England) and countries in which the primary obligation of compulsory school attendance begins with seven years of age (e.g. Finland). To the question “when the child is ready for elementary school”, famous psychologist John Furlan (1983) answered with the text titled “Does Muhammad come to the mountain or the mountain comes to Muhammad?”. He actually asked the opposite question: “Does the child need to be ‘mature’ for the school or schools should accept all children of a certain age the way they are and provide them with optimal conditions for progress and development.” In educational circles, the prevailing opinion is that we should create a model of compulsory primary school where all children of a certain age can succeed (“pedagogy of success for all”) and can make progress optimally (Baert et al., 1989). Certain improvements in the education of Roma in Croatia, however, do exist. This is partly a consequence of existing measures undertaken with the aim of better integration of the Roma into Croatian society. Education is certainly one of the most important components of the integration of the Roma minority, and as such has an important place in the National Programme for the Roma and the Action Plan for the Decade of Roma Inclusion (Croatian Government, 2003, 2008).

The approach to teaching mathematics given the uniqueness of the Romany culture

When the education of the Romany children is concerned, the specific qualities of such a relationship must be taken into consideration, and that applies to teaching mathematics in the classroom as well. A Romany child approaches the educational process not only by bringing to it his or her individuality and competence, but also as a member of a culture and a language that is, in a way, different from the culture from which most of the children in our schools approach the teaching process. For this reason, the approach to teaching, including teaching mathematics, poses a challenge not only for the Romany student, but for the teachers who are not familiar with the specific characteristics of the Romany culture. That is why it is necessary, when working with the Romany students, to find a methodological approach to teaching mathematics in which a teacher will try to adjust the learning process on some new level, with the aim of better and easier acquisition of mathematical contents.

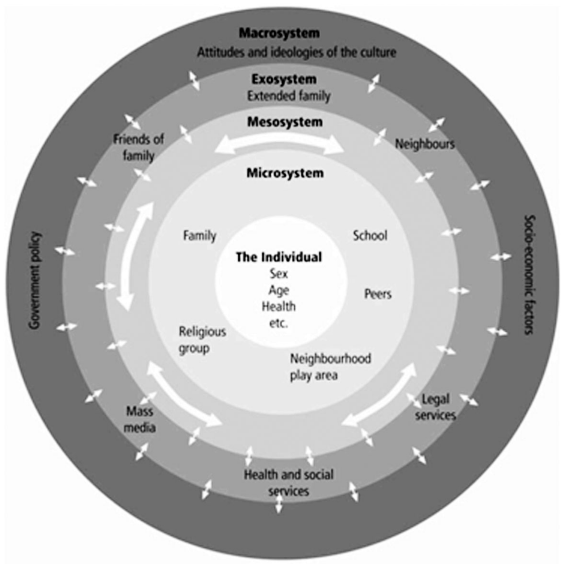

Figure 1. Bronfenbrenner's ecological model of the environment (source: Vasta et al., 2005, 61).

Naturally, the primary focus is on the student's individual skills, competences and aspirations (Eret, 2012, 156), and furthermore, a student has to be perceived as an individual in a particular, and in this case, specific, social and cultural context (Eret, 2012, 157). School, and thus teaching as well, is only a small part of the educational environment; the child as an individual in a society, is influenced by its broader and narrower environment, best described by Bronfenbrenner's model of developmental theory of ecological systems (Figure 1). As we can see, in the closest circle of social influence on upbringing (and education), in the microsystem, are factors of the child's immediate environment, some of which are family and school, while all the way up to the macrosystem there are attitudes and ideologies of the culture in which the child is growing up, and they are shaping him into a social individual, in a way that which each of these influences has the same significance (Eret, 2012, 145).

Vedic mathematics as an alternative answer to a specific methodological problem

As noted in previous chapters, one of the difficulties that the Romany children encounter in their education is the acquisition of mathematical contents in a way that is prescribed by the mathematics curriculum for primary school children. In an attempt to find a better approach to learning mathematics, as an alternative option it is possible to apply calculation by using Vedic mathematics. Vedic mathematics is a calculation system based on 16 sutras (Sanskrit formulas) the basic feature of which is a simplicity of calculation without the written computation, which can stimulate the students' interest in mathematical thinking and creativity in finding solutions to mathematical problems (Miloloža, 2008, 19). In contemporary researches, scientists are trying to find alternative usages and importance of Vedic mathematics apart from its computational aspect. In that process, they are discovering a correlation with various aspects of a daily life, with other sciences, as well as the perspective of observing through the significance of sutras for everyday work and activities; their philosophy and meaning are applicable not only in the mathematical sense, but also to the events and order of social achievements, moral development, interest in individual and social progress (Kandasamy and Smarandache, 2006, 36-39). The 16 sutras, through formulas which are expressed in words and are easily understandable, memorable, and applicable, represent ways of solving mathematical problems in the areas of arithmetic, algebra, geometry and calculus (Miloloža, 2008, 19).

Sutras which are the focus of this work are related to mathematical contents and ways of calculation that could be applied in extracurricular work with the Romany students, particularly in remedial mathematics classes. Precisely because of all the above mentioned characteristics, sutras might be a useful alternative or a supplement to a traditional method of teaching, in cases where such a method of teaching does not achieve the expected results or educational progress. The following chapter proposes the examples of the application of the sutras, that is, Vedic mathematics, in primary school remedial classes with the Romany students.

The application of Vedic mathematics in remedial primary school mathematics classes

The Romany students often experience problems in mastering basic calculations operations and tasks, encountering the problem of insufficient acquisition of the mathematical basics in the prior knowledge necessary for advancement in mathematical work. That is why it is necessary to find an approach to teaching by applying an alternative work method, which would provide the Romany students with a problem solving method that is simpler and faster than the traditional method used in most of our schools.

Two sutras, *All from 9 and the Last from 10*, and *By One More than the One Before*, whose examples have thoroughly discussed Croatian authors as well

(Miloloža, 2008), will serve as a model for methodological examples of the 5th grade primary school remedial classes. The use of Vedic mathematics in these cases is interesting because of the prior knowledge required for the acquisition of the 5th grade content (Set of Natural Numbers), and for mastering basic mathematical operations in general; in the examples which will be outlined, in the Vedic methods of multiplication, it is sufficient to know the basic mathematical operations of addition and subtraction, and 5×5 multiplication table. The main point is, that students can perform calculations by heart and quickly, without the written computation (Kandasamy and Smarandache, 2006; Miloloža, 2008).

Before the calculation, it is necessary to learn and acquire several concepts of Vedic mathematics. By *ten* in Vedic mathematics we mean decimal units, that is, powers of number 10: 10, 100, 1000, 10 000... which is closest to a given number; we call them also *bases*. Likewise, the notion of a number deviation, or to what extent a given number (in a positive or negative sense) deviates from the nearest *ten* (Figure 2), so deviation is either positive or negative, and has a corresponding sign (Kandasamy and Smarandache, 2006; Miloloža, 2008). In the process of multiplication also occurs a *Viculum number* (Belavić, internet source 1) when the digits of a number are 'composed' of positive but also negative numbers obtained by the method. In order to "make" a negative number a digit, it must become positive, so this number is replaced by its complement (deviation), e.g. number -9 by 1, -2 by 8, and from the previous number we subtract 1 (see example 2).

Table 2. Examples of positive and negative number deviation.

NUMBER	BASE	DEVIATION
8	10	-2
17	10	7
73	100	-27
899	1000	-101
1123	1000	123

Examples for practice: Vedic mathematics in the 5th grade primary school remedial classes

The following examples will show how the 5th grade primary school Romany students can master the procedures of multiplication without written calculations, by using the principles of Vedic mathematics. Here we have several examples which can serve as the framework for remedial classes in the acquisition of content and mathematical operations of the *Set of natural Numbers* unit.

Example 1. *Multiply numbers 6 and 9.*

Procedure. (1) The base of both numbers is 10. We calculate deviations of the numbers: deviation of 6 is -4, deviation of 9 is -1.

(2) To one of the numbers we add a deviation of another: $6+(-1)$ or $9+(-4)$; in both cases the result is **5**.

(3) We multiply deviations, $-4 \cdot (-1) = \mathbf{4}$.

(4) The solution, respectively, is 5 and 4, **54** (Figure 3).

Table 3. Multiplying numbers 6 and 9.

6 9	-4 -1
$9 + (-4) = \mathbf{5}$ or $6 + (-1) = \mathbf{5}$	$-4 \cdot (-1) = \mathbf{4}$

Example 2. *Multiplying numbers 103 and 87.*

We approach multiplication as in the above example. However, one of the numbers is higher and another is lower than the base. The procedure is slightly different. It is an example of complementing the rule with the theory presented in sutras.

Procedure. (1) The base of both numbers is 100. Deviations are, respectively, 3 and -13.

(2) To each factor we add a deviation of one another. $103 + (-13)$ or $87 + 3$, in both cases we get **90**.

(3) We multiply deviations. $3 \cdot (-13) = -\mathbf{39}$. As we cannot take a negative number for digits (which in this case would be written $90\overline{39}$), we use the above rule for this Viculum number: the complement of number -39 is $100 - 39 = \mathbf{61}$. We subtract 1 from the number of the previous solution ($90 - 1 = \mathbf{89}$).

(4) The final solution, instead of $90\overline{39}$, is **8961**.

Example 3. *Multiplication by number 9. Multiply numbers 17 and 9.*

We have stated in previous chapters that Vedic mathematics primarily serves as algorithmic thinking in which the student does not have to memorize the multiplication table over 5×5, and the rest of the procedure can be learned by heart. In that way, multiplication by number 9, particularly in the multiplication table 10×10, but also in the case of multi-digit numbers, becomes mathematically logical sequence, not a set of learned information. The example of the 'multiplication table' is shown in Example 1 on the previous page, while multiplication by double-digit numbers is shown here.

Procedure. (1) To the digit of the ten we add 1 ($1 + 1 = 2$) and we subtract the result from the factor ($17 - 2 = 15$). **15** is the first part of the solution.

(2) We append the complement of the unit's digit of the same factor (**3** for 7).

(3) The solution is **153**.

Conclusion

Methodological examples described serve to enrich and complement the traditional practice of methodology of teaching mathematics in primary schools. Primarily, the examples described are applicable to schools with students of the Romany population, referring to the problem of the acquisition of basic mathematical operations, in this case, multiplication.

Of course, Vedic mathematics offers far more examples applicable in practice and in the problem area of methodology of mathematics in working with the Romany students than it is specified in this paper; the sutras theory, as described in the previous chapters, expands through a wide range of the mathematical area. Therefore, the next scientific deliberations might tackle Vedic calculations and mathematical problems of another type, the applicability in other methodological situations, to a different age of (the Romany) students or education level.

Certainly, the purpose of the methodological proposals is to investigate the efficiency of application of the alternative (Vedic) calculations according to the positive developments in the mathematical success of the Romany students. Thus, this paper points to the applicability of the alternative way of thinking and calculating to the traditional system of school education, while on the other hand, we would like to use it as an initiative for further discussion of the same (or similar) issues, and as a model for further scientific deliberations.

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Primjena elemenata Vedske matematike kod učenika Roma

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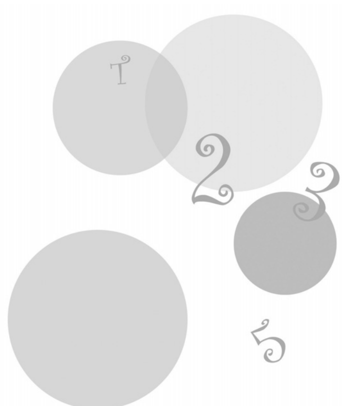
Sažetak. Rad s romskom djecom postavlja posebne izazove pred odgojitelje i učitelje koji nisu tijekom svog školovanja pripremljeni na specifičnost rada unutar kulture romske zajednice i romskog jezika, te na posebnu fleksibilnost u radu s romskim učenicima. Zainteresiranost učenika Roma za odgojno-obrazovni proces izazov je na koji smo pokušali odgovoriti uvođenjem elemenata Vedske matematike u nastavu. Ovaj rad donosi metodički prijedlog rada koji pokazuje povezanost aktivnog pristupa učenika Roma matematičkom učenju s uvođenjem vedskog načina računanja u dopunsku nastavu matematike i izvannastavne aktivnosti.

Ključne riječi: Vedska matematika, Romi, dopunska nastava matematike, izvannastavna aktivnost

2

The role of information and communication technologies in other approaches to teaching high school mathematics



In this chapter the authors analyse the pre-existing conditions connected to using ICT in high school instruction, but also refer to goals and visions for the future. The opposing opinions involved in the “media debate” started in the 1990’s have since been reconciled. Nowadays, the prevailing attitude is that educational goals should precede the selection of appropriate ICT tools in designing instruction. From the papers included in this chapter it is obvious that in the sorrounding countries ICT is recognised as an important factor in high school instruction. Yet, the quality and availability of technical equipment, as well as the professional competence of teachers regarding its application within mathematics instruction, differ among the neighbouring countries. It is argued that providing resources for ICT equipment in high schools is as equally important as investing in life-long training of mathematics teachers for using it. Certain solutions can be achieved by mastering the free ICT tools which contribute to the higher efficiency of mathematics instruction without requiring additional financial investments. Pupils enthusiastically use new technologies on an everyday basis for fun, “surfing” the Internet, or playing games. For this very purpose the role of ICT in the classroom is unavoidable. Yet, the scholars report less willingness in pupils to engage in exploration by means of computer software. The authors highlight the need to encourage active, and subsequently creative approach of pupils/students to computers and computer software, the use of which, however, can cause resistance in pupils.The greatest advantage of using ICT, however, is the adjustment of the environment to the activites of an individual, as well as that of a team. Accordingly, it is important to give pupils the opportunity to make mistakes, think on their own, express dilemmas, look for answers, and independently choose the paths to solving problems. By supporting pupils during specific activities, individually or as a team, using ICT can significantly contribute to the development of their creativity.

Influence of development of computer technologies on teaching in Bosnia and Herzegovina

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Abstract. Our times are characterized by strong changes in technology that have become reality in many areas of society. When compared to production, transport, services, etc education, as a rule, slowly opens to new technologies. However, children at their homes and outside the schools live in a technologically rich environment, and they expect the change in education in accordance with the imperatives of the education for the twenty-first century. In this sense, systems for automated data processing, multimedia systems, then distance learning, virtual schools and other technologies are being introduced into education. They lead to an increase in students' activities, quality evaluation of their knowledge and finally to their progress, all in accordance with individual abilities and knowledge. In this thesis we try to present the advantages and disadvantages of integrating ICT in teaching mathematics, covering the following topics: the need of introducing and the goals of innovative teaching, planning integration of computers in the teaching process – integration models, resistance to integration, ways of applying ICT in education and teaching mathematics (through the classes of acquiring the new teaching material, evaluation classes and testing and evaluation of students' knowledge as well). In order to prove the need of introducing ICT in the teaching process, the survey of students and teachers on the current representation of ICT in teaching was conducted. Furthermore, some mathematics lessons were held supported by the use of ICT and then the research was conducted about computer application in mathematics teaching.

Keywords: education, multimedia systems, distance learning, virtual schools, mathematics and computers, lectures, exercises, integration, survey, research

Introduction

The time we live in is characterized by radical changes in engineering and technology which have come to life in various areas of society. Education, as a rule, slowly opens up towards new technologies in relation to manufacturing, traffic, service industry, etc.

Nevertheless, children live at home and outside of school in technologically rich environment, expecting changes in education in accordance to education imperatives for 21st century. In that sense, systems for automatic data analysis are slowly being introduced in education as well as multimedial systems, long distance learning, virtual schools and other technologies which lead to increase in students' activity, more quality in evaluating knowledge and students' advancement regarding their individual abilities and former knowledge. The guiding thought while conceptualizing and writing this paper was that our education system, particularly in elementary schools is too traditional and should be modified. The school of the present should be replaced by the school of the future where information technologies would occupy an important place, from the aspect of research, as well as multipractical aid in acquiring new and fortifying already gained knowledge through the Internet, various data in digital form, simulation programs, etc.

"Classes should be transformed into a new learning system, which implies less memorizing of facts and more studying of methods and ways through which certain facts and validities can be gained, as well as affirmation of creativity and creation... These and similar changes have been present in the world for a long time, and among other things they are supported and directed by intensive scientific and technological development... " (Hrustić, 2002, p. 11).

Necessity of including innovative teaching and its aims

When talking about pedagogical innovation, we may refer to adapting the school (school system) to social-economic, scientific-technological and other changes, demands of general development in society and our knowledge about development of children (pupils/students). "*The following innovation should be introduced in our schools: individualized classes, multilevel classes, problem targeted classes, computer assisted classes, cabinet, programed, team classes, etc." (Hrustić, 2002, p. 80).*

Computer application as innovation classes would enable the following:

- to reduce and acquire quicker and easier overly wide curricula,
- it would impose modern and contemporary content upon the traditional one,
- memorizing would give place to thinking, and conformity to critical thinking and flexibility,
- uniform formats would provide more space for various studying styles,

- the role of student and teacher would change,
- The teaching process would significantly be embedded with technical, technological, methodical and other novelties, etc.

Models of computer integrating in the teaching process

Considering the current condition in our school system, the following models of integrating personal computers in the teaching process can be abstracted:

The traditional model

The traditional model contains all the elements of the classical classes: fixed place and time, classroom without computer, while the Internet is additional resource which students may use in the computer cabinet during the hours or their free time. The traditional model introduces the Internet in the teaching process and uses it as the alternative information source.

The transitional model

The transitional model keeps the traditional elements of fixed place and time, but the place can encompass regularly planned visits to the computer cabinets.

The distance teaching model

This model surpasses the limits of the traditional classes by embedding the entire material, exercises and resources online. Students do not have to come to school for a classical class; instead, they exchange ideas and information entirely through the Internet, with possible exceptions regarding orientation sessions, official classes or supervised exams.

Computers in inclusive classes

Education system, which is created according to our standards, almost completely neglects children with special needs, forgetting that they too have to bear everyday life. Indeed, they wish to cope with it and be as productive as everyone else.

Information and communication technologies can not only enable these children more accessible learning, but can also by their nature be more accessible for persons with special needs. Computers can greatly expand the horizons of these children, and display options with special accessibilities in order to employ available technologies more efficiently.

Computer in mathematics teaching

In mathematics teaching on all levels: in elementary and high school, at colleges and universities, we are faced with disconcerting situations. Mathematicians are debating and writing about what pupils and students could and should study. At the same time pupils and students are struggling with the basic problems:

— how to learn the language of mathematics,

— how to accept mathematical models and mathematical thinking processes.

There are different manners to influence the overcoming of this uncomfortable situations. Books are being written which provide examples – "patterns" for solving great number mathematical problems. Mathematics teachers give simpler tests and written exams, much easier than those that could be solved based on subject matter prescribed by the curriculum. It is usually considered that the problem in mathematics teaching lies in pupils and students, not in the lack of communication between them and mathematics teachers. It is commonly said that pupils and students do not accept all that is being offered to them through classes or that they are completely disinterested for mathematics classes (MacLane, 1994, p. 24).

This poses the following questions:

- Can mathematics be taught in the manner it is studied?
- Can interest in mathematics be increased, and if there is none, can it be incited?
- What is the role of computers regarding the two previous questions?

Contemplations of the mathematician Davis will be provided in reference to the role of computers in teaching mathematics and possibilities they offer regarding the posed questions. He believes that the possibilities of research in mathematics have significantly improved thanks to computers. Even with great assistance which computers offer in all areas of mathematics, the mathematicians still have the leading role.

It is important to mention that the students' work without computers in a no way neglected by employing computers in mathematics teaching. On the contrary, employing computers stimulates the most important aspect in teaching mathematics, and that is understanding the subject matter. Pupils or students no longer have to learn by heart great numbers of formulas, are no longer afraid making a mistake in calculation, so their entire attention is drawn to understand problems and tasks. A large problem in mathematics teaching without computers, in schools as well as universities, is application to a small number of problems (Davis, 1995, p. 86).

Teacher or professor also meets difficulties while choosing examples of classes and task preparing tasks for knowledge evaluation since it necessary to assemble with "nice solutions". With the use of computers, most of the above mention limitations is entirely eliminated and students acquire better image about the significance of mathematics and its applications.

Current representation of computer application in mathematics teaching

Considering the way of presenting one subject unit in mathematics in an average school in Bosnia and Herzegovina today, the conclusion follows that this way of presenting is in not much different from the way of presenting twenty years ago. If, on the other hand, the students, their environment, lifestyle, aspirations and interests are observed, a significant difference can be notices between students today and twenty years ago. Today, students are exposed to various multimedia contents which can be found everywhere, television, commercials, billboards, mobile phones, computers, the Internet, Today students are overwhelmed with information and their attention is directed towards various contents. In such environment it is necessary to improve and innovate classes so as to make them more acceptable, interesting and useful for students.

Research. How much do we use computers in mathematics teaching today.

Teachers

This research included forty – five elementary school mathematics teachers of different ages, in the Tuzla Canton area. The aim of this part of the research is forming an image about using computers in mathematics teaching process in elementary education, as well as about teachers' attitudes. In reality, the idea is to find out from this part of research how much teachers employ information technologies during classes, but also to examine to what extent they give homework that demands using a computer. Also, the aim is to examine teachers' attitudes about availability of resources in their schools, which are relevant for conducting multimedial classes. Alongside with this the position of teachers, as well as their desire to use computers more everyday teaching process was examined. This research also examined the confidence of teachers towards The Faculty of Natural Sciences and Mathematics as an institution which can provide adequate assistance and support. The research was conducted through survey. The survey consisted of five closed type questions. The survey was anonymous.

Survey results

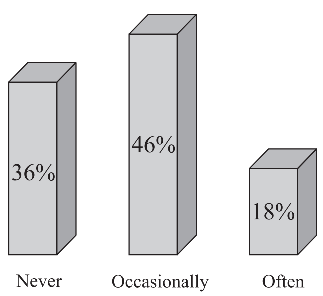

Figure 1. Responses to the question "Do you use a computer in classes?"

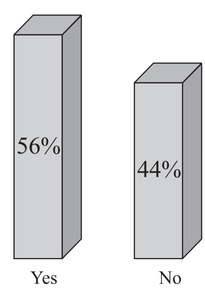

Figure 2. Responses to the question"Would you like to use a computer more in classes?"

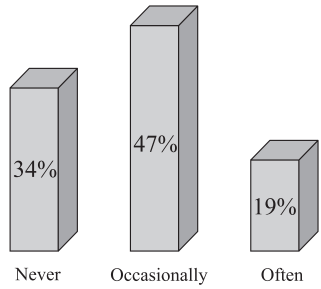

Figure 3. Responses to the question "Do you give homework that demands using a computer?"

Based on the figures a hypothesis can be set that the two previous questions are correlated. Also, based on the two previous graphs it can be concluded resource availability, i.e. the equipment with information technologies in schools largely has no significance for improving classes with the use of computers.

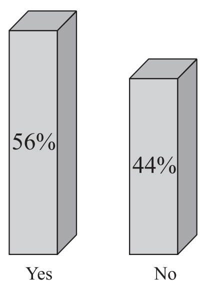

Figure 4. Responses to the question "Would you benefit from the assistance of Faculty for in Natural Sciences and Mathematics as an institution which can provide adequate expert help related to using computers in mathematics teaching?"

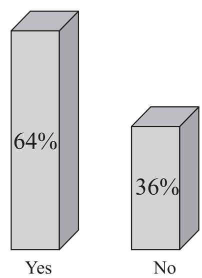

Figure 5. Responses to the question "Are there, schools where you are employed, solid conditions for using computers in mathematics teaching?"

Students

The aim of research

The aim of this part of research is forming an image regarding students' attitudes towards implementing computers in the teaching process. It is expected that the findings of this research will provide information related to the quantity of computer usage by students to which purposes. Students were questioned individually or in pairs. They were encouraged to bring out their opinions and formulate answers to the posed questions. Audio recordings of this part of research were analyzed using content analysis method. The sample encompasses students in sixth, seventh and eighth grade of elementary school. In this part of research thirty students from elementary school "Bukinje" – Tuzla took part. The criterion for choosing students is their expressiveness and communicability.

Students replied to the posed questions:

1. How often do you use a computer?
2. How much time do you spend in front of a computer during the day?
3. Do you use the Internet?
4. What kind of content do you watch on the Internet?
5. Besides the Internet what is your computer useful for?
6. Do you use the computer for educational purposes? How? Which ones?
7. Have you ever processed a teaching unit at school using computers, safe from informatics class?
8. Do you remember what you learned during that class? Was it interesting?
9. Do teachers give you homework assignments which demand using a computer (the Internet)? Are those assignments difficult?
10. Do you have any program on your computer which is useful for school?

Responses resume

Students largely put accent on the Internet as the reason for using a computer. It is interesting that students spend a bulk of their time on the Internet while actually visiting a small number of Internet sites. They are not familiar with many sites which they can use in educational purposes. They spend most of their time on social networks. When it comes to educationally useful software, students mostly express no knowledge about them or gave short and simple answers.

Students were very motivated to answer the posed questions. They were excited to talk about impressions from classes when they use computers. According to students, there were very few of such classes, but they are well-remembered and almost all students had positive impressions.

Students expressed positive attitudes toward including computers into teaching process. They realize many advantages of that manner of learning, but certain students are aware of possible difficulties that can occur while including computers in the teaching process.

The manners of applying computers in teaching mathematics

The manners of applying computers in teaching mathematics are related to representing teaching units or their parts through:

- presentation
- video footage
- programs

- mathematical games
- ready – made software, etc.

These manners of application may be used regardless of the type of class (presentation, revision, and systematizing).

Presentations

Through presentations it is possible to present all subject units in mathematics in a very original or interesting. Preparation of presentations is very simple, so this type of computer usage in teaching mathematics does not require additional training for a teacher.

Video footage

Through video footage it is possible to interpret parts of subject matter and/exercises which, due to their length, difficulty, cannot be performed during class with utmost quality. It is also possible to use short films or other recordings to present historical facts about life and work of certain mathematicians, which are extremely motivational for the children.

Programs – Programed teaching

In programed teaching, the program precisely lays out all the important facts and concepts that students should learn. The subject matter is divided into logically connected units which are easy and small enough, and they are being mastered one after another. In order for students to start mastering the mentioned subject matter, they must acquire knowledge from prior subject units. Subject matter cannot be skipped. During the students' work in class the teacher will follow up the process of completing the tasks, provide additional information and explanation for work, etc.

Games in mathematics

If a student is properly included in the world of mathematical games: In the first stage students should understand the rules of the mathematical game which is analogous to understanding text of the task. In the second stage it is expected that the student should provide a plan of game. Research has shown that during every consequent playing of the same game, students remain longer in this stage so as to find the winning strategy. The third stage comes down to making the correct move in the game. In the final stage, after the game is over, the students are allowed to see the game process. This possibility drags them to think how to discover the winning strategy.

Using mathematical games serves to successfully remove the lack of motivation in teaching mathematics, since the well-expressed affinity of children of playing is used significantly.

Evaluating and grading students' knowledge

Evaluating and grading through computer is mostly designed so that certain tests, papers or exercises are to be completed according to suggested model, even though it can be conducted through other manners of evaluating. Every piece of work should be leveled and have clearly defined measuring object. It must be defined what the work is measuring and with what accuracy, objectivity and consistency. Evaluations of students' knowledge using a computer have their upsides and downsides. Besides being economical, these kinds of works enable more objective and reliable assessment of classwork then with other types of examining. Also, they enable comparison of results between different students as well as comparison of achievements for the same student in different periods of examining.

One of the greatest weaknesses of this type of evaluation is that it measures only what is shown, only that the student managed to write or show at a particular moment, but it does not measure the implicit part of knowledge that subtle understanding of subject matter that a teacher can sense during other types of examining. Another weakness is also the fact that this way it is difficult to measure attitudes and interests of students.

Positive and negative factors in using computers in mathematics teaching

Positive factors

- individual approach enabled,
- offers efficient feedback,
- capacitate student for individual work,
- the position of students and teachers is altered.

Negative factors

- classes are schematized (limited imagination, creativity, originality),
- it is acceptable for students with modest intellectual capabilities,
- limited personal communication – social isolation,

- all parts of subject matter are not equally applicable for presentation via computer
- induces the phenomenon of electronic memorizing in students (children got used to easements when using computer technologies, so they do not even attempt to memorize, calculate something, etc., which leads to ever worse results in mathematics teaching – 60% of eight graders do not know by heart their parents' birthdays and 43% telephone numbers).

Research – computers in mathematics teaching (yes/no)

In June 2012, in elementary school "Bukinje" – Tuzla computer supported classes presentation, revision, and evaluation of subject units in subject mathematics in the seventh grade. The classes took place with a group of 24 students also present during classes were colleagues – teachers for other subjects and the school pedagogue. Their observations are presented in Tables 1 and 2.

Table 1. Results of students' survey.

Questions		Answers 'YES' in percentages (%)	Answers 'NO' in percentages (%)	Other
1.	Did you like the previous class?	95,84	4,16	
2.	Would you like to replace classical classes (teacher, presentation, blackboard, chalk) with this kind of classes?	87,5	12,5	
3.	Do you believe it is is interesting to learn this way?	91,67	8,33	
4.	Should classes in all subjects be conducted this or similar way (meaning other picturesque programs depending on the subject)?	87,5	12,5	
5.	How often should you have classes like this?	All the time	79,18	
		Often	12,5	
		Rarely	4,16	
		Never	4,16	
6.	Do you believe that you encompassed (learned) the subject matter in this class with more quality because it was presented in a different way?	87,5	12,5	
7.	Do you think that the subject matter you learned this way will stay longer in your memory?	91,67	8,33	
8.	Would your success be better if you did written exams (tests, papers, exercises) in this manner?	75,00	25,00	
9.	If you think it is necessary, write your opinion about the previous class!			

Table 2. Results of teachers' survey.

	Questions	Answers 'YES' in percentages (%)	Answers 'NO' in percentages (%)	Other
1.	Did you like the way the previous class was conducted?	75	25	
2.	Would you like to replace your classical classes with these or similar innovative methods of class construction?	75	25	
3.	Do you consider that it is more interesting for students to learn this way?	100	0	
4.	Do you believe that your work would be easier using these methods of teaching?	100	0	
5.	Should classes in all subjects be conducted this way or similarly, even with different and more picturesque programs (depending on the subject)?	75	25	
6.	How often should pupils have this kind of classes?	All the time	25	
		Often	50	
		Rarely	25	
		Never	0	
7.	Do you believe that every student mastered with more quality the subject matter elaborated in the previous class because it was presented using the visualization method (demonstration) more than presentation?	50	50	
8.	In the case that the previous question was answered with 'no', tell us: How many students (in percentages) will in your opinion master the subject matter with more quality?	0–25%	0	
		26–50%	0	
		51–75%	100	
		76–100%	0	
9.	Do you believe that subject matter learned this way will remain in students' memory longer?	75	25	
10	Would you agree to allow students to do written exams in your subject (tests, papers, exercises etc.) this way?	50	50	
11	Would these types of written exams and evaluation be more objective than the classical ones?	75	25	
12	If you think it is necessary, write your opinion about the previous class!			

Conclusion

Well-prepared and expert mathematics teaching with the help of a computer can enable new ways of schooling and significantly facilitate the teachers' job. Therefore, the conclusion derived would be that computer technologies could greatly facilitate the very manner of knowledge transfer, as well as, accepting it, so the results after tests would be much better. In any case, combining the prior manner class conducting and computer technologies would provide excellent results in an easier way of presenting the subject matter, as well as, in better understanding by the students.

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Primjena ICT-a u nastavi matematike u Bosni i Hercegovini

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Sažetak. Vrijeme u kojem živimo odlikuje se snažnim promjenama u tehnici i tehnologiji koje su zaživjele u mnogim oblastima društva. Obrazovanje se, po pravilu, sporije otvara prema novim tehnologijama u odnosu na proizvodnju, saobraćaj, uslužne djelatnosti... Ipak, djeca kod kuće i van škole žive u tehnološki bogatom okruženju očekujući promjene u obrazovanju u skladu sa imperativima obrazovanja za 21. stoljeće. U tom smislu već se polako u obrazovanje uvode sistemi za automatsku obradu podataka, multimedijalni sistemi, učenje na daljinu, virtualne škole i druge tehnologije koje dovode do povećanja aktivnosti učenika, kvalitetnijeg vrednovanja znanja i napredovanja učenika u skladu sa individualnim sposobnostima i predznanjima. Ovim radom ćemo pokušati pokazati prednosti i nedostatke integriranja ICT-a u nastavu matematike obradivši teme: potreba uvođenja i ciljevi inovativne nastave, planiranje integracije računara u nastavni proces-modeli integracije, otpor prema integraciji, načini primjene ICT-a u obrazovanju i nastavi matematike kroz nastavne sate obrade novog gradiva, utvrđivanja, te ispitivanja i vrednovanja učeničkih znanja. S ciljem dokazivanja potrebe uvođenja ICT-a u nastavu izvršena su anketiranja učenika i nastavnika o trenutnoj zastupljenosti ICT-a u nastavi, održani su časovi matematike podržani ICT-om a nakon njih provedeno je istraživanje o primjeni računara u nastavi matematike.

Ključne riječi: obrazovanje, multimedijalni sistemi, učenje na daljinu, virtualne škole, matematika i računari, predavanja, vježbe, integracija, anketa, istraživanje

On the usage of interactive whiteboards in the teaching of mathematics in secondary schools in Primorje-Gorski Kotar County

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Abstract. In the year of 2005 Ministry of Science, Education and Sports of the Republic of Croatia has launched the project procurement of interactive whiteboards for elementary and secondary schools. Accordingly, from 2006/2007 secondary schools have been systematically equipped with "smart boards" and introduced with on-line interactive educational contents. Moreover, the National Educational Standard specifies interactive whiteboard as the required equipment at the school level. We can find numerous statements of teachers who are enthusiastic about the introduction of interactive whiteboards in the classroom, but there are also those who doubt the positive impact of this type of innovation.

In order to determine to what extent is the usage use of interactive whiteboards widespread in teaching of mathematics in PGC secondary schools, we conducted a research among secondary school teachers of mathematics in PGC. Furthermore, the aim was to examine the ways in which the interactive whiteboards are used in teaching of mathematics and what are the possible reasons for not using them. In addition, we have examined how many of undergraduate students of Mathematics in Department of Mathematics, University of Rijeka, have met with the usage of interactive whiteboards in math classes in secondary school. Given the fact that most of them are prospective teachers of mathematics, we were interested in their attitudes, based on their personal experience, towards the usage of interactive whiteboards in teaching of mathematics.

Keywords: teaching of mathematics, interactive whiteboard

Introduction

The usage of interactive whiteboards starts at the beginning of the final decade of the 20th century. The business world was the first one to adopt it and a few years later the same thing happened in education. In addition, pieces of research addressing the usage of interactive whiteboards in teaching of mathematics in secondary schools (Clark Jeavson, 2005), as well as the pupils' results achieved in mathematics due to the teaching based on the interactive whiteboard usage, started to appear in the late 20th century (Clemens, Moore & Nelson, 2001). Moreover, pieces of research on the available literature for the usage of interactive whiteboards in the classroom are also carried out. Ultimately, respective ministries recognize the interactive whiteboard as an integral part of future classrooms (Smith, Higgins, Wall & Miller, 2005).

In the year of 2005 Ministry of Science, Education and Sports of the Republic of Croatia has launched the project procurement of interactive whiteboards for elementary and secondary schools. Accordingly, from 2006/2007 secondary schools have been systematically equipped with "smart boards" and introduced with online interactive educational contents. In cooperation with CARNet, the Ministry executes the project "Application of interactive whiteboards in the classroom" to order develop educational materials on the usage of interactive whiteboards in the classroom and to provide adequate support for teachers in the application of new technologies. (Mudrinić Ribić, 2009). Moreover, the National Educational Standard specifies interactive whiteboard as the required equipment at the school level (Primorac i sur., 2008).

In order to determine to what extent is the usage use of interactive whiteboards widespread in teaching of mathematics in PGC secondary schools, we conducted a research among secondary school teachers of mathematics in PGC and students of Undergraduate University study of Mathematics in Department of Mathematics, University of Rijeka.

Research results

Teachers

In order to learn whether or not interactive whiteboards are used in teaching mathematics a questionnaire was conducted among the present teachers at the Primorje – Gorski Kotar County (PGC) meeting of secondary school teachers of mathematics, held in Rijeka in November 2012. The questionnaire was completed by 54 secondary school teachers of mathematics in PGC, which accounts for 60% of all secondary school teachers of mathematics in PGC. Additionally, the sample comprises of 19 respondents who are teachers in high schools, 34 respondents who are teachers in vocational schools and 1 respondent who is a teacher in both high school and vocational school. Moreover, the conducted questionnaire is comprised of equally represented teachers with different number of work experience: 11 respondents have less than 10 years of experience, 14 respondents have between 10 and 20 years of experience, 13 respondents have between 20 and 30 years of experience, 15 respondents have more than 30 years of experience and 1 respondent did not answer the question.

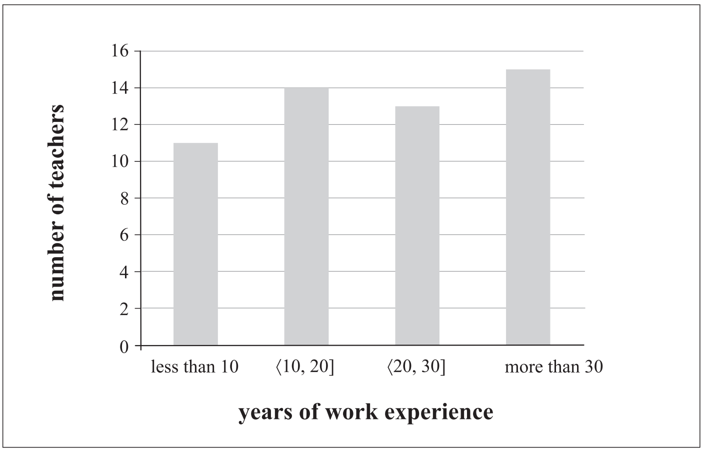

Figure 1. Number of teachers in relation to their work experience.

Among the examined teachers, only four teachers (7.4%) indicated that they use interactive whiteboard in teaching mathematics. Taking into account that 10 teachers (18.5% out of a total number of teachers examined) said their school does not have the interactive whiteboard, we can say that 9% of the examined teachers, among those who have this possibility, use interactive whiteboard in teaching mathematics. Among teachers who report that their school does not have interactive whiteboard, nine (90%) of them were teachers who work in vocational schools.

Out of four teachers who use interactive whiteboard in teaching mathematics two of them work in high school (one has between 20 and 30 years of work experience, and the other one has more than 30 years of work experience), one works in vocational school (with between 20 and 30 years of work experience) and one works in both high school and vocational school (with less than 10 years of work experience). Interestingly, more experienced teachers, in relation to their younger colleagues, report higher tendency of interactive whiteboard usage. Teachers reported that they tend to use interactive whiteboard as a GeoGebri supplement, in teaching and exercising contents of geometry and stereometry, for the exam and for the transmission of educational contents.

If we were to exclude the teachers who state that they do not have interactive whiteboards in their schools, unavailability of the classroom in which it is located is reported as the most common reason for non-usage of interactive whiteboards in the teaching of mathematics. This specific reason is cited by 28 teachers. In other words, there are 70% of teachers of mathematics who do not use interactive whiteboard in the classroom even though they are aware that it exists in their school. This is consistent with the studies which suggest that the availability of interactive whiteboards in one's own classroom is significant for its actual usage in teaching.

Moreover, it seems that there are not as many teachers who tend to plan their lesson plan using interactive whiteboard if they are generally forced to hold a lesson in a classroom where the board does not exist. In addition, teachers are not inclined to disturb the usual routine with casual abandonment of "their" classroom in order to keep their lesson in the classroom in which there is an actual interactive whiteboard, even when it is embedded in the anticipated schedule (Levy, 2002). However, part of the teaching process could be regularly held, at least in some of these schools, in these classrooms if a teacher of mathematics expresses such a desire. Thus, it is possible that the "unavailability" of interactive whiteboards, which is a real problem, is sometimes used as a convenient excuse.

Seven teachers state that the reason for non-usage of interactive whiteboards lies in the absence of the adequate IT support, three of them said they do not know how to work with interactive whiteboards and the same number stated that they do not want to use it. Finally, two teachers did not provide an explanation for non-usage of interactive whiteboards.

Students

We have examined how many of undergraduate students of Mathematics in Department of Mathematics, University of Rijeka, have met with the usage of interactive whiteboards in math classes in secondary school. Given the fact that most of them are prospective teachers of mathematics, we were interested in their attitudes, based on their personal experience, towards the usage of interactive whiteboards in teaching of mathematics.

The questionnaire was conducted during December 2012 and it was completed by 87 undergraduate students of Mathematics in Department of Mathematics, University of Rijeka, which makes 77% of all students in the Department for the academic 2012/2013 year. Out of 87 questionnaires, two were invalid, thus, our sample consists of 85 respondents. Among them, 36 respondents finished secondary school in PGC (24 of them attended high school and 12 of them vocational school), and 49 respondents finished secondary school elsewhere (35 of them attended high school and 14 of them vocational school).

Table 1. Student – sample affiliation in relation to their answers.

Did your school have interactive whiteboard?					
	PGC		Others		
	High schools	Vocational	High schools	Vocational	
YES	9	3	14	4	30
NO	11	8	19	10	48
DO NOT KNOW	4	1	2	0	7

To the question *"Did your school have interactive whiteboard?"* 48 students (56.5%) responded "NO" and only 30 students (35.3%) responded "YES", while the remaining 7 students (8.2%) did not know the answer to this question. With regard to the project of equipping schools with interactive whiteboards, we assume that such a large number of negative responses results from the fact that interactive whiteboards are not used in the classroom. Thus, students are often not aware of their existence in their schools. We can note that students from PGC are equally aware of the presence of interactive whiteboards in schools as their other colleagues. More precisely, 33.3% of students from PGC state that their secondary school had an interactive whiteboard and the same thing claim 36.7% of other students. Overall, approximately 35% of students (12 from PGC and 18 other students) said that their secondary school owned interactive whiteboard. Responses of these students are subjected to further analysis.

To the question *"Did your teacher of mathematics use the interactive whiteboard?"* all 12 students from PGC who confirmed the existence of interactive whiteboards in their schools have responded negatively, while 7 (39%) students from other areas responded positively. Equal number of students from PGC and other areas state that the interactive whiteboard in the classroom was used by a different teacher, so the experience of using interactive whiteboards in other cases had five students (41.7%) from PGC and five students (27.8%) from the other areas. Based on the experience of undergraduate university students of Mathematics in Department of Mathematics, University of Rijeka, we can say that the implementation of the interactive whiteboards usage in the teaching of mathematics in PGC is lagging behind other areas from where our students come.

All students are asked to express their attitudes towards the use of interactive whiteboards in the teaching of mathematics. Out of all students, 14 of them (16.5%) had a positive attitude and 4 of them have participated in such classes. Five students (5.9%) had a negative attitude and 2 of them have participated in such classes. The remaining students did not show any positive or negative attitudes.

Conclusion

Even though it has been more than 5 years since the Ministry started to systematically equipping schools with interactive whiteboards, only 7.4% of respondents-teachers of mathematics in secondary schools in Primorje-Gorski kotar County use them in the teaching process. Moreover, none of the current undergraduate university students of Mathematics in Department of Mathematics, University of Rijeka, who come from the Primorje-Gorski kotar County, had the opportunity to attend such classes. Most teachers who do not use the interactive whiteboard tend to list objective reasons for it (there is no interactive whiteboard at school, interactive whiteboard is in the other, unavailable, classroom, there is no adequate IT support). Furthermore, only six of them (6.9%) report subjective reasons (do not know how to use interactive whiteboard, do not want to use interactive whiteboard), and two of them did not provide an answer to this question. In accordance with these results and results of other studies, we can conclude that there will be no major use of interactive whiteboards in the teaching of mathematics in secondary schools until mathematical classrooms in PGC are equipped with interactive whiteboards and teachers are given an adequate support in the implementation of this type of innovation.

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O uporabi interaktivne ploče u nastavi matematike u srednjim školama Primorsko-goranske županije

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Sažetak. Ministarstvo znanosti, obrazovanja i sporta Republike Hrvatske pokrenulo je 2005. godine projekt nabave interaktivnih ploča za osnovne i srednje škole. U skladu s tim od školske godine 2006./2007. srednje škole sustavno se opremaju "pametnom pločom", a uz to uvode se i interaktivni obrazovni sadržaji *on-line*. Također Državni pedagoški standard navodi interaktivnu ploču kao opremu koju je potrebno imati na razini škole. Možemo naići na brojne izjave nastavnika koji su oduševljeni uvođenjem interaktivne ploče u nastavu, ali i na one koji sumnjaju u pozitivni učinak ove inovacije.

S ciljem utvrđivanja u kojoj je mjeri uporaba interaktivne ploče rasprostranjena u nastavi srednjoškolske matematike u Primorsko-goranskoj županiji, proveli smo ispitivanje među srednjoškolskim nastavnicima matematike. Također cilj je istražiti koji su razlozi nekorištenja interaktivne ploče te kako i s kojim ciljem je upotrebljavaju oni koji se njome služe u nastavi matematike. Osim navedenoga istražili smo koliko se studenata Preddiplomskoga studija matematike Odjela za matematiku Sveučilišta u Rijeci susrelo s interaktivnom pločom na satu matematike u srednjoj školi. Zanimalo nas je kakve stavove o uporabi interaktivne ploče u nastavi matematike imaju studenti, u odnosu na vlastito iskustvo i s obzirom na činjenicu da su mnogi od njih budući profesori matematike.

Ključne riječi: interaktivna ploča, nastava matematike, srednja škola

Influence of interactive boards in improving teaching of mathematics in high school

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Abstract. Interactive (smart) whiteboards are an ideal solution for those who want to increase the quality of teaching, to encourage and motivate the students and help to achieve the desired results. The features and benefits of interactive whiteboards are very large. It can be used as a regular whiteboard, or connect to a computer via a serial cable, USB or wireless, and take full advantage of its interactivity.

The interactive whiteboard management is very easy with an interactive pen, which takes over the function of the mouse which we use every day. On a very easy way we can edit, save, send e-mail or print the contents of all that we input on the interactive board, so there is the possibility of sending materials to students who did not attend the lectures. In a simple way we can approach to all previous processed materials, and held lectures.

With each interactive whiteboard comes accompanying software that includes a wide variety of contents, including content related to teaching and learning in mathematics. With the help of interactive whiteboards and expertise teaching math capabilities are unlimited. The limitation of interactive whiteboards utility is just our imagination. Interactive whiteboard attracts and holds the attention of students creating compelling of interactive lectures where everyone loves to participate.

Interactive whiteboards are a modern revolutionary idea in the field of teaching supplies in educational institutions. The basic concept of the modern ways of teaching and learning is based on the use of modern teaching supplies and technologies that contribute to the new didactic methods.

The purpose of using interactive whiteboards is to motivate students to improve the quality of teaching and learning, combining ease of interactive whiteboards and computer capabilities. We are interested in how smart whiteboard functions in an educational environment (technical description), and how the introduction of such a smart whiteboard in the classroom reflects the subjects of mathematics and teaching methodology (methodological description).

Students through the questionnaire express their impressions about this way of learning, or whether the use of interactive content in the teaching of mathematics contributed it to be their favorite subject. Conducted research suggests that before utility of interactive whiteboards students have not expressed sympathy for the subject of mathematics, and the most of them thought that it was a difficult subject. After installing and applying the same, the results of the final questionnaires show the changes in the positive direction in terms of motivation and learning approach in mathematics, and respectively:

- students demonstrate themselves more independent when acquiring new teaching
- students independently reach conclusions
- slmost all of the students are active in the class
- students express less need for additional explanations from the teacher in front of the whiteboard
- students show more interest and motivation for learning mathematics
- students with their own initiative visit websites with digital materials at home
- students are more satisfied with the mathematics teaching (as they are using the computer more often)
- students can ask each other's help too, and they help each other more

The results of the final interview (the first grade of high school, 150 students, 2012):

Students listed their impressions about this way of learning, whether the use of interactive whiteboards in the teaching of mathematics contributed it to be their favorite subject, etc. The first results (the traditional approach) show that 65% of the students, when they encounter a problem in solving the tasks, address to his/her teacher, and now it is 35%; also 18% tried to find a solution without the help of teachers or parents earlier, and is down to 65% now. From this it follows that provided interactive content increased motivation for individual work and finding solutions without additional help.

The results of the interesting facts of mathematics teaching in the first survey (the traditional approach) were really against existing teaching mathematics, or over 75% of students said they do not like mathematics, and over 80% believed that mathematics is a difficult subject, while nearly 90% thought that the teaching of informatics much more interesting than teaching mathematics. Results presented later modern methods in teaching mathematics are really much better. Even 84% believed that the teaching of mathematics with corresponding interactive contents can be just as interesting as the teaching of informatics, and that over 76% of students said they prefer this way of learning in addition to traditional education.

Eventually, the students were given evaluation for the classes supported by interactive content. Only 8% of the respondents consider and state that it is not satisfied with the aspect of teaching, and 12% of those surveyed students are assessed with grade 2, and 20% with grade

3. With the grade 4 were 26% of respondents and with grade 5 were answered by the most of respondents 34%.

With this issue the science is not sufficiently dealt with in BiH, and what has already been written about it is incidentally and not enough interpreted.

Keywords: interactivity, discovery, GeoGebra, virtuality, constructivism, *multimedia*

Introduction

Development of technology at the same time the need for improvement of education technology occurs. The emergence of computers and other modern teaching tools causes changes and improvement of the existing relations between the participants in the learning process and the teaching process. However, the whole process of improving technology education is conditioned upon the possession of appropriate equipment by the school. Interactive whiteboard is a multifunctional panel that meets all the needs of the presentation, electronic storage, printing or sending e-mail in various formats.

The computerization of the teaching process is necessary for the students in order to become prepared for the challenges of tomorrow, and to go out into the labor market. One of the most important competencies and computer literacy is a reasonable use of technological solutions that make it easier to work and study. Therefore school is a priority focus on computerization of the educational process and the use of modern information and communication technologies in teaching.

Interactive (smart) whiteboards are an ideal solution for those who want to increase the quality of teaching, encourage and motivate students, to help in achieving the desired results. The possibilities and benefits of interactive whiteboards are large. It can be used as a regular panel, or connected to a computer through a serial cable, USB or wirelessly, and utilize of its interactivity fully. Easily managing of interactive whiteboard with an interactive pen, which takes over the function of the mouse from what we use every day. The easy way is to edit, to save, to send an e-mail or print the contents of all that we take the interactive whiteboard, so there is the possibility of sending materials to students who did not attend the lectures (Users Guide, 2012). We can approach in a simple way to all previous processed materials, and delivered papers. With each interactive whiteboard comes accompanying software that includes a wide variety of content, including content related to teaching and learning in mathematics. With the help of interactive whiteboards and proficiency of mathematics teacher the capabilities are unlimited; the limitation of use the interactive whiteboards is just our imagination.

Interactive whiteboards in teaching mathematics

We are interested in how intelligent board functions in an educational environment (technical description), and how the introduction of interactive whiteboards in the classroom reflects the methods of teaching mathematics (method description).

Interactive whiteboard is a device that converts the standard blackboard in interactive. In combination with a multimedia projector board becomes an interactive “touch screen”. Connected to a computer interactive whiteboard controls of numerous applications, such as PowerPoint, Word documents, Excel tables, and an interactive CD-’s (Sanford, 2013).

Interactive whiteboards we want to achieve something impressive, unexpected, and that is exactly easiness of lectures. Therefore, interactive whiteboards are actually toys of teachers, because when one teacher prepares their teaching, the following works are much easier and teaching becomes a more dynamic with a lot of feedback from students. It’s certainly not an easy task when it seeks creativity, willingness of teachers, and certainly the most important, the teacher’s knowledge in informatics.

Interactive whiteboard is not a magic stick and will not be able to do everything that we imagine. Like any new tool we will need some time to work through. The aim of using of interactive whiteboards is to motivate students to improve the quality of teaching and learning, combining the simplicity of interactive whiteboards and computer capabilities.

Students via questionnaires and surveys express their impressions on this educational approach of learning, or whether the use of interactive content in the teaching of mathematics contributed that mathematic become their favorite subject. Implemented research suggests that before applying of interactive whiteboards students have not expressed sympathy for the subject of mathematics, and the majority of thinks that mathematic is a difficult subject.

After installing and applying the same, the results of the final survey changes in the positive direction regarding the motivation and approach in learning in mathematics, respectively:

- students show higher independence in adopting the new teaching facilities,
- students individually reach conclusions,
- almost all of the students are active in the class,
- students reported less necessity for additional explanations to of teachers in front of board,
- students showing higher interest and motivation for learning mathematics,
- students independently at home visiting websites with digital materials,
- students are more satisfied with the teaching of mathematics (as more likely to work with the computer),
- students can turn to each other for help, and help each other more

The results of the final survey (I grade of secondary Economic and Catering High School, 150 students, 2012) Indicate the following:

Students expressed their impressions on this educational approach of learning, whether the use of interactive whiteboards in the teaching of mathematics contributed to be their favorite subject, etc. The first results (the traditional approach) shows that 65% of the students when a problem is encountered is dealing with the assignment addressing to his teacher, and now it's 35% or 18% previously, that tries to find a solution without the help of teachers or parents, and is now down 65%. Referred to in this it follows that providing of interactive contents increased motivation for individual work and in discovering a solutions without assistance.

The results of the curiosities of mathematics teaching in the first survey (the traditional approach) were really against existing teaching of mathematics, or over 75% of students said they do not like mathematics, and over 80% believed that mathematics is a difficult subject, while almost 90% thought that teaching computer science is a lot more interesting than teaching of mathematics. The results after the modern methods were presented in teaching mathematics are really much better. Even 84% believe that the teaching of mathematics with accompanying interactive content can be just as interesting as the teaching of computer science, and that over 76% of students said that they prefer this way of learning in addition to traditional education.

Eventually, the students were given evaluation for classes supported by an interactive content. Only 8% of respondents considered and states that they are not happy with this aspect of teaching, and 12% of the surveyed students are assessed with grade 2, and 20% with 3 grade, with the grade 4 is 26%, and grade 5 responded even 34% of respondents.

Influence of in interactive boards teaching style

The advantages of interactive whiteboards

First of all, the use of smart boards will certainly buy attention of the audience, i.e. our students. They themselves will want us to give them something in order they can use such a board. The atmosphere in the class when is not the same the teacher sitting in front of computer, and students follow what is shown on the wall (Lukač, 2009).

Frequently utilize of interactive whiteboards, will make us to "purify" materials, i.e. to be simple and concise output, and we will begin to "think in the slides". For more frequent usage of interactive materials will make us listeners themselves, because after the initial enthusiasm (the reaction is generally such) will request from us to use it fully, which would imply the use of films, presentations, interactive materials, games, quizzes and assignments, which will give ability and students to appear before the board themselves solve a task or answer the question.

Mathematicians for teaching geometry are available and popular GeoGebra, as well as many other mathematical programs that can be found today on the Internet.

Numerous studies in the world, but in our country, show that interactive teaching greatly improves learning. This way of teaching is particularly desirable in the field of geometry (mathematics), which, unlike other scientific fields requires a specific way of thinking (Banjanin, Vrdoljak, 2008). Access to interactive teaching and learning in geometry (mathematics) should be consistent with the research conducted by renowned mathematics educators Pierre van Hiele and Dina van Hiele-Geldof. These observations are in class, the students learned that pass through the next series of hierarchical levels of thinking: visualization, analysis, informal deduction, formal deduction and strictness.

Standard texts for students to expect from start applying formal deduction. Very little is done to all students with visualization, or to entice them to make assumptions/hypotheses. The main goal of interactive teaching and learning is-students implementation through the first three levels, encouraging the process of discovery, which more closely reflects how mathematics is usually found: a mathematician first ideas and analyze the problem, making assumptions before attempting a proof.

Interactive blackboard as a modern didactic medium

Interactive whiteboards imposing the usage of multimedia presentations in the classroom. Further it leads to so-called segmentation (possibility of implementing multiple smaller modules) school. Teaching content becomes smaller in scale and is processed one by one term, before the conclusion of the whole, and rejects everything that is not necessary to understand the lesson or topic. This makes things easier for the students because the curriculum had been "cleansed" of unnecessary elements. What materials that are considered as essential, is treated with a lot of photographs, films or animations.

With pedagogical and methodological point of view, central to the process of learning and teaching is the student. The figure (Figure 1) shows the didactic triangle which illustrates relationships of students, teachers and teaching facility (Mandić, 2009).

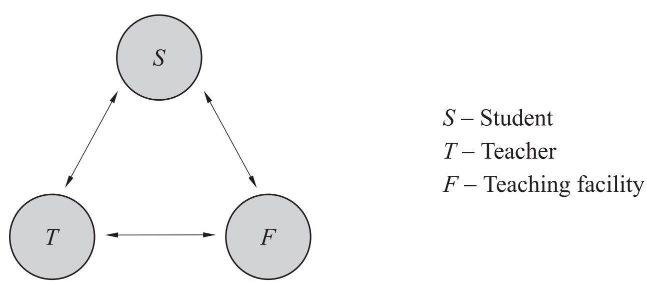


Figure 1. Didactic triangle.

How this approach does it considered with the particular point where the instrument of teaching and learning among other things will be take over technology, we get the complemented didactic triangle (Figure 2).

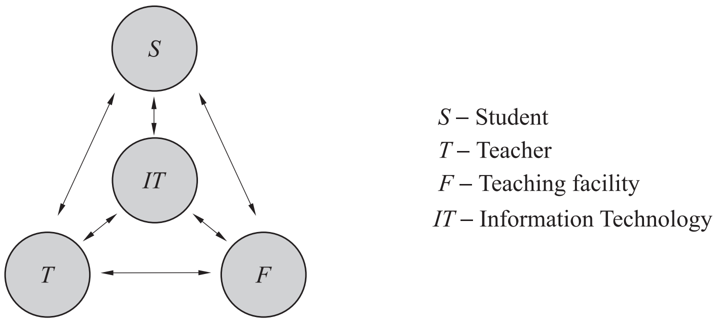


Figure 2.

Previously, teacher took into account the "appearance" of board for the distribution of text on plain boards, and now the material shown through which the slides are not visible at the same time, but it is easy to show again, with or without notes created during the lesson. Teaching material (in digital form) can be easily distributed to students. Improved motivation and interest of the students is evident, especially for those whose academic achievement is weaker.

Acquired knowledge or understanding can be checked by making the frontal tests. It can be concluded that with the interactive whiteboard extended time of the effective teaching and noticed a faster pace of crossing material. That the interactive whiteboards are useful in teaching is the fact that all those teachers who use them in the classroom do not want to give them up into any case.

After a complete analysis the usage of interactive board points out the following: better motivation of students, smart boards attract and retain students' attention, encourage the usage of multimedia presentations, interactive teaching materials used and the higher participation students during lectures, the possibility to tests the ability of students in front of the class with the usage of computers, i.e. board, directly displaying web content throughout class, while it is possible to write with pencil on the web sites, multiple-use charts and graphs during the exercise, multiple use and easy modification of prepared materials for the classes.

In case of using dynamic geometry software (GeoGebra, etc.), they allow the user to construct basic geometric objects such as points, segments, lines, circles and angles. Over them may then be virtual ruler and compass to construct new geometric objects, such as mid-length, long centerlines and angles, vertical, etc. Constructed objects, then, can be mapped, rotated, or stretched. Also, the coordinate system can be displayed graphs of functions. Various measurements are used in dynamic geometry (longer length, volume polygon, polygon area, the area under the graph of the function over a segment or a particular integral, the function of this segment, the size of the angle, the coordinates of points, etc.), moving free points, dynamically monitor changes at the facility. Obtained results can be used in future construction and (or) transformations. Offset of any basic points will accompany corresponding changes throughout the "system".

One of the better digital tools in teaching mathematics is GeoGebra. It covers by the program of mathematics our elementary and secondary schools very good, and may be used by students, especially in engineering, such as building or Mechanical Engineering. More than the other programs connecting geometry and algebra, and has intuitive algebraic inscriptions of equations and functions. His prints high resolution and quality which contributes to the good quality projection. On a very simple way it generates dynamic HTML document with drawings and descriptions of construction, step by step, or a website with a customized Java applet.

With recorder it is possible to synchronize audio recording with records during the lectures, and so create a document that allows to students later observation review with parallel listening to teacher's explanation. This is perfect for distance learning or learning online. As the software has the ability to retroactive viewing, the student can follow multiple lectures.

Influence of interactive boards on teaching mathematics

Modern educational technology, using multimedia systems, creating preconditions for the engagement of all the senses in the process of acquiring new skills, develop student creativity and provides higher activity in teaching and learning. The adoption of information technology and the unique requirements imposed by the today's society, the necessity for adapting education systems innovation, and therefore teachers, who need to take a new and responsible roles and tasks.

The students develop abstract thinking and individual progress in acquiring and expanding knowledge. The teacher is more oriented to the types of students and thus can help them to progress at a pace that suits them and the strong possibility.

Smart board further motivation for students

Choosing colors "chalk", which was once used for highlighting and classification of different unit's lesson in this case is done with one click. Background of table can also be changed and instead of a classic dark green color that could be drawing, picture, chart or slide of in advance prepared presentations, and even dynamic form. Video or live presentation from the Internet site, may contribute to a moment that is far more dynamic, more interesting and informative than usual. The crown system is supporting software that comes with the board and makes it virtually runs on the board and on the computer.

Interactive whiteboard thus making the time one lesson of mathematics utilize maximally. When they do not have dedicated educational software, teachers bring pre-saved lesson and preparation for the class, in standard digital formats. This eliminates the need to dictate or rewriting content from the board, because all changes can be stored in electronic form, and then printed or send by e-mail to students. For the record it can be stored all what one student do to the interactive board in a dedicated folder on the computer, in order to continuously followed the progress of his work and teaching.

In combination with the internet, interactive whiteboard turns into an electronic textbook because every teaching unit can support a text with relevant site and easily relive even most boring matter. In this way it is easy to realize interdisciplinary in teaching mathematics. To use all the possibilities and potential of this technology, it is necessary to continuously improving and teachers themselves.

Advantages and disadvantages of the use of interactive whiteboards in the classroom

The advantages of using interactive whiteboards

- Many examples of related to subjects and real life situations
- Demonstrations of experiments that are difficult or impossible to perform in the classroom
- Interactive simulations for experiential learning
- Visual examples and animations to easier understanding the material
- The ability to getting verification of the learned feedback
- Tools for help in learning
- Temporal and geographical independence (requires a computer and Internet access)
- Streamlining
- Fast transfer of information
- Stimulation of logical thinking
- The deepening of understanding
- Experimentation, discovery, creation and testing of hypotheses

Disadvantages of using the interactive whiteboards

- Simplification can lead to trivialization
- A clear image can become illusion plainness (vividness – a complete sensory experience of the object in order to adopt the facts and forming regular representations)
- Fast transfer of information often brings too much information
- Attractive presentation becomes the show or in the least does not help
- Ignorance of software
- Insufficient number of computers
- Insufficient level of computer literacy (teacher/student)
- It takes additional preparation for lesson

Comments of students and teachers

Students were told:

- “it is easier, there are colorful examples, generally is fun and it seems easier”
- “I got the impression that in this work are included almost all of the students and the interest in work increased in class, also it is simply and clearly at the same time what is important in a material”
- “It is easier when you learn something you can visualize”
- “I liked the with sound and picture content we can prove a theory so the application of it is very interesting to me”
- “This approach is fun and relaxed, so students learn a lot more.”

What the teachers told about:

- “The students are active, motivated and achieving a pleasant and relaxed working atmosphere”
- “clear usage of contents in everyday life”
- “matters is simpler, clearer and vividly explained”
- “The students’ positive comments were very motivating”
- “The students are more satisfied with leaving the classroom after lectures.”

The ten most common mistakes salespeople do when using interactive white-boards in teaching are:

1. Do not turn your back on the students/audience (Set the computer so that you are facing towards the students)
2. Do not dim body interactive whiteboard or screen (If you must point to the screen, use an electronic pointer, otherwise describe the part you want to show or point with the mouse)
3. Do not project a small and low quality image (Adjust projector, screen or panel so that you have something bigger and higher quality image)
4. Do not leave the audience in the dark (Allow at least part of the classroom students are bright that you would not fall asleep)
5. Do not show finger painting (SHOW mouse or some other electronic service indicator)
6. Do not waste time over an hour to set (Well meet the equipment you use, and you can show students how they can help in setting up equipment)
7. Do not assume that the audience sees the same thing as you (use zoom and zoom the screen so that the audience could see what you are saying)

8. Do not be the "main actor" (Use technology to encourage interactivity and the active involvement of students)
9. Do not use the interactive whiteboard or projection screen only as a replacement for a regular plate (If something can be better explained on the regular panel then do so. Consider how well you can take advantage of interactive whiteboards or projection)
10. Do not spend money on the interactive whiteboard if you do not know what to do with it (Give some examples of your teaching where interactive board will enable an increase in the quality of teaching)

Programs (software) in mathematics teaching

Types of educational programs – software for teaching of mathematics through interactive whiteboards:

- Graphic tools – Winplot, Dplot, Graph,. . .
- Software dynamic (interactive) geometry SBS – GeoGebra, Cabra, GSP, Cinderella,
- Computer Algebra System (CAS) – computer software for symbolic calculations: Mathematica, Maple, DERIVE, Sage, Maxima,. . .
- Programs for the spreadsheet – Microsoft Excel, OpenOffice.org Calc, Lotus 1-2-3, Gnumeric,. . .

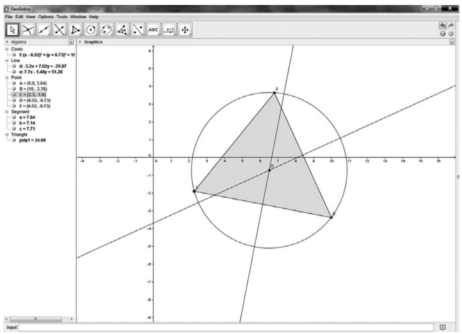
GeoGebra – software of dynamic (interactive) geometry

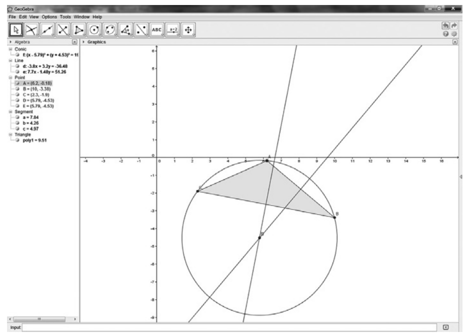
GeoGebra facilitates creation of mathematical structures and models to students by allowing them to interactively explore by pulling objects or changing parameters (Choi K., Motivating students in learning mathematics with GeoGebra, 2010).

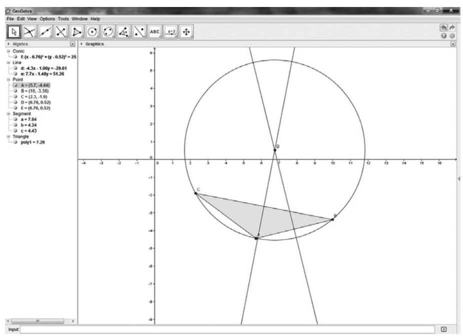
GeoGebra is:

- Computer program of dynamic mathematics
- For teaching and learning at all levels of education
- Connects interactive geometry, algebra, tables, charts, analysis and statistics
- Program has an open source code
- GeoGebra is also an authoring tool that allows teachers to easily produce interactive web pages

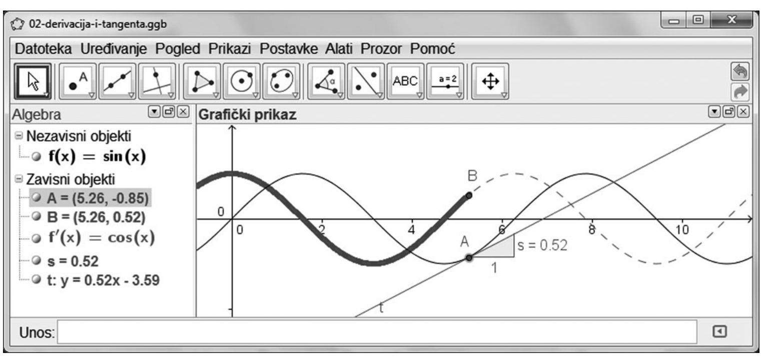
Construction tools on the toolbar using the mouse we can work on the construction of the diagram. In the algebra window displays the corresponding coordinates and equations. The input field is used to enter coordinates, equations, functions, everything which is displayed at the drawing pad immediately after pressing Enter. Geometry and algebra together.







Example. To describe a circle given the triangle and perform the simulation moving the vertices of a given triangle.



Example. Derivative and tangent to the graph of $f(x) = \sin(x)$.

Conclusion

Informatization in the teaching process is necessary process nowadays because it encourages creativity and provides quality performance. Interactive whiteboards are certainly one of the teaching tools, educational and presentation process that we offer interactivity with focus on multimedia. This stems from the technical concept of interactive panels and the specific hardware and software to simulate touch screen.

With the animations and examples raises the quality and diversity of the subject and the general interest and motivation of participants in the education process. Multimedia presentation provides a better understanding and comprehension because the contents are displayed in several ways, which also provides a great educational value. On the other hand, interactive board is the excellent platform for the application of instructional design in the planning and construction of educational facilities. However, the living word of speakers cannot be omitted, while the interactive materials represent a new approach, which will contribute to a better understanding of teaching content, the development of student's the motivation and the spirit.

Same as learning mathematics with computer detection and dynamic geometry software GeoGebra and designed to methodically and didactically designed interactive digital learning materials provides activities for all students in the classroom increases motivation for learning mathematics, encourages independent derivation of conclusions, and cooperation among students. It is important that the student alone or in pairs learns on his computer, and all alone try to discover new insights. This knowledge will remain long in the memory, and the student will learn to think, every new problem much easier to solve, and control the skills of discovery.

However, if this is feasible in teaching practices, largely depends on the school terms. It turned out that the students prepared to cooperate in the implementation of changes in the classroom, but if the teacher of mathematics is not able to use the computer classroom for their classes, then they will not be able to implement the ideas described in this paper. Still, I hope you will read the paper is to encourage other teachers to try to make changes in their pedagogical practices in accordance with the school conditions in which they work.

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Utjecaj primjene interaktivne ploče na povećanje efikasnosti nastave matematike u srednjoj školi

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Sažetak. Interaktivne (pametne) ploče su idealno rješenje za sve koji žele povećati kvalitet predavanja, podstaći i motivirati učenike, kako bi lakše postigli željene rezultate, a mogućnosti i prednosti interaktivne ploče su velike. Može se koristiti kao obična ploča ili se povezati sa računarom preko serijskog kabla, USB-a ili bežično; i iskoristiti njena interaktivnost u potpunosti. Jednostavno upravljamo interaktivnom pločom pomoću interaktivnog pera, koji preuzima funkciju miša s kojim se svakodnevno koristimo. Na jednostavan način obrađujemo, snimamo, šaljemo e-mail-om ili štampamo sve sadržaje koje unosimo na interaktivnu tablu, tako da postoji mogućnost slanja materijala učenicima koji nisu bili prisutni na predavanjima. Pristupamo na jednostavan način svim dotadašnjim obrađenim materijalima, odnosno održanim predavanjima. Uz svaku interaktivnu ploču dolazi pripadajući softver koji obuhvata mnoštvo različitih sadržaja, pa tako i sadržaje vezane za predavanja i poučavanje predmeta matematike. Uz pomoć interaktivne ploče i stručnosti predavača mogućnosti predavanja matematike su neograničene; ograničenje upotrebe interaktivne ploče je samo naša mašta. Interaktivna ploča privlači i zadržava pažnju učenika, kreiranjem zanimljivih, interaktivnih predavanja na kojima svi rado učestvuju.

Interaktivne ploče predstavljaju modernu revolucionarnu ideju na području nastavnih pomagala u obrazovnim institucijama. Temeljni koncept savremenog načina učenja i poučavanja bazira se na korištenju savremenih nastavnih pomagala i tehnologija koja doprinose novim didaktičkim metodama.

Cilj korištenja interaktivne ploče je motivirati učenike na poboljšanje kvalitete učenja i poučavanja, kombinacijom jednostavnosti interaktivne ploče i mogućnosti računara. Zanima nas kako pametna ploča funkcionira u edukativnom okruženju (tehnički opis), i kako se uvođenje ovakve pametne ploče u nastavu predmeta matematike odražava na metodiku i poučavanje (metodički opis).

Učenici putem Anketa navode svoje utiske o ovakvom načinu učenja, odnosno da li je primjena interaktivnih sadržaja u nastavi

matematike doprinjela da to bude njihov omiljeni predmet. Provedena istraživanja govore da prije same primjene interaktivne ploče učenici nisu izrazili naklonost prema predmetu matematika, i većina je smatrala da je to težak predmet. Nakon instaliranje i primjene iste, rezultati završnih anketa pokazuju promjenu u pozitivnom smjeru u pogledu motivacije učenja i samom pristupu predmeta matematike, odnosno:

• učenici pokazuju veću samostalnost pri usvajaju novih nastavnih sadržaja,

• učenici samostalno dolaze do zaključaka,

• gotovo svi učenici su aktivni na satu,

• učenici iskazuju manju potrebu za dodatnim objašnjenjima nastavnika pred pločom,

• učenici pokazuju veći interes i motivaciju za učenjem matematike,

• učenici samoinicijativno kod kuće posjećuju web stranice s digitalnim materijalima,

• učenici su zadovoljniji na nastavi matematike (jer češće rade na računaru),

• učenici se jedni drugima obraćaju za pomoć, te si više pomažu međusobno.

Rezultati završnog anketiranja (I razred srednje škole, 150 učenika, 2012. godina):

Učenici su naveli svoje utiske o ovakvom načinu učenja, da li je primjena interaktivne ploče u nastavi matematike doprinjela da to bude njihov omiljeni predmet, itd. Iz prvih rezultata (tradicionalan pristup) se vidi da 65% učenika kad naiđe na problem u rješavanju zadatka se obraća svom nastavniku, a sada je 35%, odnosno ranije da 18% pokušava da pronađe rješenje bez pomoći nastavnika ili roditelja, a sada je čak 65%. Iz ovoga proizilazi da su dati interaktivni sadržaji povećali motivaciju za samostalnim radom i pronalaženjem rješenja bez dodatne pomoći.

Rezultati o zanimljivosti nastave matematike iz prve ankete (tradicionalan pristup) su zaista bili protiv postojeće nastave matematike, odnosno preko 75% učenika je izjavilo da ne voli matematiku, a preko 80% je smatralo da je matematika težak predmet, dok blizu 90% smatralo da je nastava informatike puno zanimljivija od nastave matematike. Rezultati poslije prikazanih savremenih metoda u nastavi matematike su zaista mnogo bolji. Čak 84% smatra da nastava matematike uz prateće interaktivne sadržaje može biti isto tako zanimljiva kao i nastava informatike, a da preko 76% učenika kaže da im odgovara ovakav način učenja uz tradicionalno obrazovanje.

Konačno, učenici su dali ocjene za nastavu podržanu interaktivnim sadržajima. Samo 8% ispitanika smatra i navodi da nije zadovoljno ovakvim vidom nastave, a 12% anketiranih učenika je ocjenilo sa ocjenom 2, dok je 20% sa ocjenom 3. Sa sa ocjenom 4 je 26% ispitanika, a ocjenom 5 je odgovorilo čak 34% ispitanika.

Na području BiH ovom problematikom nauka se nije dovoljno bavila, a i ono što je dosad napisano je usputno i nedovoljno interpretirano.

Ključne riječi: interaktivnost, otkrivanje, GeoGebra, virtualnost, konstruktivizam, multimedija

MayaVi as a tool for presentation of geometric bodies

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Abstract. It is a common occurrence to talk about the necessity of implementing IC technology within math learning process. Today, learners' generations are dealing with technology from early childhood so, it is essential to adapt teaching methods to their needs and habits.

In this paper, we shall introduce MayaVi – software for animation, modelling and simulation. Through various examples, we will provide just one small segment of the MayaVi usage – the presentation of geometric bodies and intersection of geometric bodies with plane. This area of mathematics is a part of the national curriculum for the 2nd grade of secondary (technical) schools. As in teaching, the body sketches are mainly drawn by hand, we believe that an alternative, more dynamic and interactive approach could refresh and enrich the math teaching process.

Keywords: math, geometric bodies, MayaVi

Introduction

Stereometry is implemented within the math teaching program in the second grade of secondary technical schools. Learners enhance an existing gained knowledge of geometric bodies – the cube, cuboid, prism, pyramid, cylinder, cone and ball. For geometric bodies, volume and surface area are calculated. In addition, the relation between bodies and planes are determined through intersections.

Mathematical education should enable a development of the learner's spatial ability and visualization and also the ability of recognition and usage of geometric properties in real objects.

Spatial ability is the intuitive sense for geometric abstracts and includes the ability of recognition, visual representation and transforming geometric shapes. Geometric thinking, according to van Hiele, is divided into 5 levels, of which visualization is level 0. Visualization in geometric bodies is usually shown through drawing sketches on a board. Interestingly, teachers rarely use technology for the presentation of sketches. In this paper, we present a tool which enables interactive sketches of body which usually captures the learner's interest in the teaching material.

Interactivity enables an overview of geometric bodies from different perspectives that contribute to better comprehension of the relation between two dimensional and three dimensional figures.

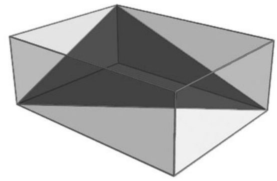
About MayaVi and Python

Python is a free programming language with a high level of abstraction and very readable code. It is very simple even for beginners and yet very powerful for serious projects. Python provides solutions using only a few lines of code, whereas other programming languages require several hundred lines. In solving these kinds of issues, different modules are used that substantially shorten the code's length. Additional modules are subsequently installed as they are usually not standard equipment of Python. One such module is MayaVi, a Python's module for visualization of 3D objects. MayaVi aims to keep a high level of abstraction bearing in mind that in different fields of human activity we often find problems that can be solved by similar methods. However, as Python is powerful programming language with a relatively simple syntax, MayaVi enables users to show complex 3D objects in a more simplified and faster way, without spending further time on writing complicate code. Due to this specification, MayaVi is not only a 3D visualization tool, but has many useful additions not directly related to visualization. Let us mention traits that simplify object programming allowing users to create small applications of their visualizations. So users can input graphic objects in their 3D visualizations, where they can interactively change certain parameters. While the program starts, rotation and zooming of the object are enabled automatically, an area of this complex operation already taken care of. Thus, MayaVi can be used within Python and additionally may be used as an independent application with its own graphic interface. In this way, beginners are able to upload data through menus and represent them without a great deal of programming knowledge.

Examples

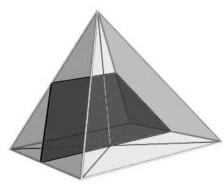
We chose five examples from the workbook for 2$^{\text{nd}}$ grade of secondary technical schools. We will show how geometric bodies and their intersection with planes and other bodies can be presented using the MayaVi programming tool. For each example, we provide a text task, sketches made in MayaVi and code that describes body and intersection.

Example 1. The plane that passes through the vertices A, C and D_1 is suited to the base at 60°. What is the volume of the cuboid if the length of its fundamental edges are 6 cm and 8 cm?



```
#!/usr/bin/python
import numpy as np
from mayavi import mlab
c=5*np.tan( np.deg2rad(30))
X=np.array([0,6,6,0,0,6,6,0])
Y=np.array([0,0,8,8,0,0,8,8])
Z=np.array([0,0,0,0,c,c,c,c])
tocke = np.column\_stack((X,Y,Z))
mlab.figure(figure=None, bgcolor=None, fgcolor=None, engine=None,
size=(800, 600))
#mlab.points3d(X, Y, Z, color=(1,1,0), resolution=15,
scale\_factor=0.2)
mlab.triangular\_mesh(X, Y, Z, [(0,1,2), (0,2,3), (0,4,5), (5,1,0), (1,2,5),
(2,6,5), (5,6,4), (6,7,4), (2,3,6), (3,7,6), (3,4,7), (3,0,4)],
opacity=0.5,color=(1,1,0))
for i in xrange(tocke.shape[0]):
   for j in xrange(i+1,tocke.shape[0]):
      vektor = tocke[i]-tocke[j]
      if np.dot(vektor,vektor) in [c**2,36,64]:
         mlab.plot3d( np.array([X[i], X[j]]), np.array([Y[i], Y[j]]),
         np.array([Z[i], Z[j]]), tube\_sides=8, color=(0,1,1) )
mlab.plot3d( np.array([X[1],X[3],X[4],X[1]]),
np.array([Y[1],Y[3],Y[4],Y[1]]), np.array([Z[1],Z[3],Z[4],Z[1]]),
tube\_sides=8,color=(1,0,0))
mlab.triangular\_mesh( np.array([X[1],X[3],X[4]]), np.array([Y[1],Y[3],Y[4]]),
np.array([Z[1],Z[3],Z[4]]), [(0,1,2)], opacity=0.7,color=(0,1,0))
mlab.show()
```

Example 2. The plane passing foot of an altitude of standard height four-sided prisms is parallel to one's face. The area of intersection is 27 cm^2. If its faces are suited to the base at angle, what is the volume of the pyramid?

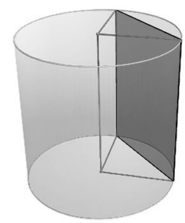


```
#!/usr/bin/python
import numpy as np
from mayavi import mlab
a=2
fi=60
v=a/2.0*np.tan( np.radians(fi))
X=np.array([0,a,a,0,a/2.0])
Y=np.array([0,0,a,a,a/2.0])
Z=np.array([0,0,0,0,v])
#tocke = np.column\_stack((X,Y,Z))
mlab.figure(figure=None, bgcolor=None, fgcolor=None, engine=None,
size=(800, 600))
mlab.points3d([a/2.0],[a/2.0],[0], color=(1,0,1), resolution=15,
scale\_factor=0.03)
mlab.triangular\_mesh(X, Y, Z, [(0,1,2), (0,2,3), (0,1,4), (1,2,4), (2,3,4),
(3,0,4)], opacity=0.5,color=(1,1,0))
#bridovi piramide
for i in xrange(4):
   j=(i+1){\%}4
   mlab.plot3d( np.array([X[i], X[4]]), np.array([Y[i], Y[4]]),
   np.array([Z[i], Z[4]]), tube\_sides=8, tube\_radius=0.01, color=(0,1,1))
   mlab.plot3d( np.array([X[i], X[j]]), np.array([Y[i], Y[j]]),
   np.array([Z[i], Z[j]]), tube\_sides=8, tube\_radius=0.01, color=(0,1,1))
#dijagonale baze i visina
mlab.plot3d( np.array([X[0], X[2]]), np.array([Y[0], Y[2]]),
np.array([Z[0], Z[2]]), tube\_sides=8, tube\_radius=0.005, color=(1,0,1))
mlab.plot3d( np.array([X[1], X[3]]), np.array([Y[1], Y[3]]),
np.array([Z[1], Z[3]]), tube\_sides=8, tube\_radius=0.005, color=(1,0,1))
mlab.plot3d([a/2.0,a/2.0],[a/2.0,a/2.0],[0,v], tube\_sides=8,
tube\_radius=0.005, color=(1,0,1))
```

```
#presjek
Xp=np.array([a,0,a/4.0,3/4.0*a])
Yp=np.array([a/2.0,a/2.0,a/4.0,a/4.0])
Zp=np.array([0,0,v/2.0,v/2.0])
for i in xrange(4):
   j=(i+1){\%}4
   mlab.plot3d( np.array([Xp[i], Xp[j]]), np.array([Yp[i], Yp[j]]),
   np.array([Zp[i], Zp[j]]), tube\_sides=8, tube\_radius=0.01, color=(1,0,0))
mlab.triangular\_mesh(Xp, Yp, Zp, [(0,1,2), (2,3,0)], opacity=0.7,
color=(0,1,0))
mlab.show()
```

Example 3. The right cylinder with a bases radius of 10 cm and a height of 15 cm was cut parallel to the plane of the axis of the cylinder so that the end points to the tendon close to the centre of bases right angle. Calculate the surface area of the larger part of the cylinder which originated in this section.

```
#!/usr/bin/python
import numpy as np
from mayavi import mlab
r=1.5
[u,v]=np.mgrid[0:2*np.pi:100j, 0:3:100j]
[u1,v1]=np.mgrid[0:2*np.pi:100j,0:r:100j]
u3=np.linspace(0.0, 2*np.pi, num=150)
mlab.figure(figure=None, bgcolor=(0.5,0.5,0.5),
fgcolor=None, engine=None, size=(800, 600))
```

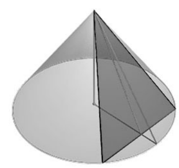


```
#plast valjka
x=r*np.cos(u)
y=r*np.sin(u)
z=v
mlab.mesh(x, y, z, colormap=’summer’, opacity=0.2)
#baze valjka
x1=v1*np.cos(u1)
y1=v1*np.sin(u1)
z1=0*v1
z2=0*v1+3
mlab.mesh(x1, y1, z1, color=(1,1,0), opacity=0.25)
mlab.mesh(x1, y1, z2, color=(1,1,0), opacity=0.25)
#kruznice baza
x3=r*np.cos(u3)
y3=r*np.sin(u3)
z3=0*u3
mlab.plot3d(x3, y3, z3, tube\_sides=8, tube\_radius=0.01, color=(0,1,1))
mlab.plot3d(x3, y3, z3+3, tube\_sides=8, tube\_radius=0.01, color=(0,1,1))
#visina valjka
mlab.points3d([0,0], [0,0], [0,3], color=(1,0,1), resolution=15,
scale\_factor=0.05)
mlab.plot3d([0,0], [0,0], [0,3], tube\_sides=10, tube\_radius=0.01,
color=(1,0,1))
#dodatni elementi
X=np.array([0,-r,-r,0,0,0])
Y=np.array([r,0,0,r,0,0])
Z=np.array([0,0,3,3,0,3])
mlab.plot3d( np.array([X[0],X[5],X[1]]), np.array([Y[0],Y[4],Y[1]]),
np.array([Z[0],Z[4],Z[1]]), tube\_sides=8, tube\_radius=0.01, color=(1,0,1))
mlab.plot3d( np.array([X[2],X[5],X[3]]), np.array([Y[2],Y[5],Y[3]]),
np.array([Z[2],Z[5],Z[3]]), tube\_sides=8,
tube\_radius=0.01, color=(1,0,1))
```

#presjek

```
mlab.plot3d( np.array([X[0],X[1],X[2],X[3],X[0]]),
np.array([Y[0],Y[1],Y[2],Y[3],Y[0]]), np.array([Z[0],Z[1],Z[2],Z[3],Z[0]]),
tube\_sides=8, tube\_radius=0.01, color=(1,0,0))
mlab.triangular\_mesh(X,Y,Z, [(0,1,2),(0,2,3)], opacity=0.4,color=(0,1,0))
mlab.show()
```

Example 4. The height of the cone is 4 cm long, the length of generatrix is 10 cm. What is the area of conic section with plane passing the apex of the cone that is suited at 60° to the plane of the bases?



```
#!/usr/bin/python
import numpy as np
from mayavi import mlab
r=1.5
h=2
fi=60
d=h/np.tan( np.radians(fi))
[u,v]=np.mgrid[0:2*np.pi:150j, 0:1:100j]
[u1,v1]=np.mgrid[0:2*np.pi:100j,0:r:100j]
u2=np.linspace(0.0, 2*np.pi, num=150)
mlab.figure(figure=None, bgcolor=(0.5,0.5,0.5), fgcolor=None, engine=None,
size=(800, 600))
```

#plast stosca

```
x=r*v*np.cos(u)
y=r*v*np.sin(u)
z=(1-v)*h
mlab.mesh(x, y, z, colormap='summer', opacity=0.25)
```

#baza stosca

```
x1=v1*np.cos(u1)
y1=v1*np.sin(u1)
z1=0*v1
mlab.mesh(x1, y1, z1, color=(1,1,0), opacity=0.25)
```

#bazna kruznica

```
x2=r*np.cos(u2)
y2=r*np.sin(u2)
z2=0*u2
mlab.plot3d(x2, y2, z2, tube\_sides=8, tube\_radius=0.01, color=(0,1,1))
```

#presjek

```
xd=np.sqrt(r**2-d**2)
mlab.plot3d([xd,-xd,0,xd], [d,d,0,d], [0,0,h,0], tube\_sides=8,
tube\_radius=0.01, color=(1,0,0))
mlab.triangular\_mesh([xd,-xd,0], [d,d,0], [0,0,h], [(0,1,2)], opacity=0.4,
color=(0,1,0))
```

#visina presjeka, visina stosca, polumjer baze, izvodnica stosca

```
mlab.points3d([0], [0], [0], color=(1,0,1), resolution=15,
scale\_factor=0.05)
mlab.plot3d([0,0,0,0,0], [d,0,0,r,0], [0,h,0,0,h], tube\_sides=10,
tube\_radius=0.01, color=(1,0,1))
mlab.show()
```

Example 5. In the sphere with a radius of 30 cm, a cylindrical hole is drilled whose axis length is equal diameter of the sphere. What is the volume of sphere ring if the hole has a radius of 18 cm?

```
#!/usr/bin/python
import numpy as np
from mayavi import mlab
r=2
#velicina rupe
d1=1.2
d2=np.pi-d1
#polumjer cilindra
r1=r*np.sin(d1)
#polovica visine cilindra
h1=r*np.cos(d1)
[v,u]=np.mgrid[0:2*np.pi:150j, d1:d2:150j]
[u1,v1]=np.mgrid[0:2*np.pi:150j, -h1:h1:100j]
[v2,u2]=np.mgrid[0:2*np.pi:150j, 0:d1:150j]
[v3,u3]=np.mgrid[0:2*np.pi:150j, d2:np.pi:150j]
u4=np.linspace(0.0, 2*np.pi, num=150)
mlab.figure(figure=None, bgcolor=(0.5,0.5,0.5), fgcolor=None, engine=None,
size=(800, 600))
#sfera
x=r*np.sin(u)*np.cos(v)
y=r*np.sin(u)*np.sin(v)
z=r*np.cos(u)
mlab.mesh(x, y, z, colormap='summer',opacity=0.2)
#cilindar
x1=r1*np.cos(u1)
y1=r1*np.sin(u1)
z1=v1
mlab.mesh(x1, y1, z1, colormap='Blues', opacity=0.2)
#kapica1
x2=r*np.sin(u2)*np.cos(v2)
y2=r*np.sin(u2)*np.sin(v2)
z2=r*np.cos(u2)
mlab.mesh(x2, y2, z2, color=(1,0,0), opacity=0.2)
#kapica2
x3=r*np.sin(u3)*np.cos(v3)
y3=r*np.sin(u3)*np.sin(v3)
z3=r*np.cos(u3)
mlab.mesh(x3, y3, z3, color=(1,0,0), opacity=0.2)
#bazne kruznice cilindra
x4=r1*np.cos(u4)
y4=r1*np.sin(u4)
z4=0*u4
mlab.plot3d(x4, y4, z4+h1, tube\_sides=8, tube\_radius=0.01, color=(0,1,1))
mlab.plot3d(x4, y4, z4-h1, tube\_sides=8, tube\_radius=0.01, color=(0,1,1))
#pomocni elementi
mlab.points3d([0,0,0,0], [0,0,0,r1], [0,h1,-h1,h1], color=(1,0,1),
resolution=20, scale\_factor=0.05)
mlab.points3d([0,0], [0,0], [r,-r], color=(0,1,0), resolution=20,
scale\_factor=0.05)
mlab.plot3d([0,0,0], [0,0,0], [-h1,0,h1], tube\_sides=8, tube\_radius=0.01,
color=(0,1,0))
mlab.plot3d([0,0], [0,0], [h1,r], tube\_sides=8, tube\_radius=0.01,
color=(1,1,0))
mlab.plot3d([0,0], [0,0], [-h1,-r], tube\_sides=8, tube\_radius=0.01,
color=(1,1,0))
mlab.plot3d([0,0,0], [0,r1,0], [0,h1,r], tube\_sides=8, tube\_radius=0.01,
color=(1,1,0))
mlab.plot3d([0,0], [0,r1], [h1,h1], tube\_sides=8, tube\_radius=0.01,
color=(0,1,0))
mlab.show()
```

Conclusion

Today, ICT is used in high school education mostly through PP presentations. New generations of teachers are using ICT in learning process more often and more wider.

With this paper, we try to present one of the free available tools that can help us to enrich geometry teaching and improve visualization of mathematic shapes and bodies. We present MayaVi, Python modul for object visualization, through five examples chosen from the workbook for 2nd grade of secondary technical schools. For each example, we provide a text task, sketches made in MayaVi and code that describes body and intersection. We invite readers to try MayaVi through these examples. We also hope, we succeed to interest readers to explore other numerous opportunities that MayaVi provides.

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MayaVi kao alat za prezentaciju geometrijskih tijela

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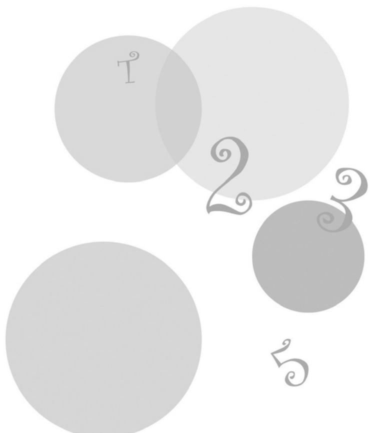
Sažetak. Sve se više govori o potrebi uvođenja IK tehnolologija u proces poučavanja matematike. Današnje generacije učenika susreću se sa tehnologijom od ranog djetinjstva, pa je važno prilagoditi način poučavanja njihovim potrebama i navikama.

U ovom članku predstavit ćemo software za animaciju, modeliranje i simulaciju MayaVi. Kroz primjere, prikazati ćemo samo jedan manji dio, a to je primjena na prezentaciju geometrijska tijela i presjeka geometrijskih tijela ravninom. Ovaj dio matematike sastavni je dio nastavnog programa matematike za druge razrede srednjih (tehničkih) škola. Kako se u nastavi skice tijela i presjeka izrađuju uglavnom ručno, mišljenja smo da bi jedan drugačiji, dinamičniji i interaktivan pristup mogao osvježiti i obogatiti nastavu matematike.

Ključne riječi: matematika, geometrijska tijela, MayaVi

3

Encouraging the development of students' cognitive domain within mathematics instruction



In this chapter the authors underline the importance of encouraging the development of the cognitive domain within mathematics instruction in students of faculties and universities of teacher studies and technical studies. While observing the pre-existing levels of knowledge and the understanding of specific mathematical concepts by students from the sorrounding countries, we detected similar situations and issues. Some of the scholars are convinced that improvements in mathematics instruction at faculties of teacher studies and technical studies could be achieved if instructors managed to master various approaches to learning mathematics with their students. With regards to teacher studies, the inclusion of students in research on the levels of knowledge of their future pupils, as well as various methods for the teaching of mathematical concepts which appear in the latest curricula (probability, data analysis, reading and presentation of data) has proven to be most effective. Furthermore, the emphasis should be placed on encouraging the understanding of and courageous arriving at "different paths to solutions", which presupposes teachers' skills of communicating at different levels of their pupils' knowledge. In the words of Paul Halmos (1916 – 2006), the famous American mathematician born in Hungary, "The best way to learn is to do; the worst way to teach is to talk".

Reflected actionability – acquisition of reflection competences for prospective teachers

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Abstract. An important aim of teachers' education at University of Siegen is to strengthen prospective students in their ability to stay capable of action in uncertainty and conflicting situations in teaching and learning processes at school as well as in their own mathematical education. This could be achieved by building up the reflection competence of students in respect to their image of mathematics, their beliefs on what is important in mathematics and didactics of mathematics and how to teach and how to learn mathematics. From surveys among first year students and interviews with students in mentoring situations it was possible to retrieve fundamental areas of tension such as "form and content", "rigour and intuition" and "process and product", which often leads to the challenge to make the "right" decision in such conflicting areas.

These dialectical concepts of tension are used as orientation marks in lectures and initial points for reflecting processes. Furthermore I will give insight in a case study, showing examples of reflecting essays. Accompanying a university class, students were confronted with statements or questions on mathematics and teaching. They were then asked to write short reflecting essays on their beliefs and opinions on these topics. It is important for students to position themselves in the wide range of possible actions in teaching and learning processes. To know the own position and belief is a crucial basis to achieve skills in decision-making and responsibility for and in teaching and learning situations in mathematics. Furthermore this gives the background to sequentially enhance the ability of reflecting learning and teaching processes and to develop a relaxed attitude in pressure situations in classroom action.

Keywords: reflection competence, teacher education, reflected actionability

Getting started with research questions

The experience in university teaching and interaction with students is that many concepts and didactical achievements are not that present in discussions and school teaching situations as it would be desirable. For that reason we started researching on the effectiveness of teacher education at University of Siegen. What are the reasons for the rejection of didactical theories and ideas from teacher students in their own didactical reasoning and acting? Why do deep beliefs and attitudes (see Törner & Pehkonnen 1995) – formed during the former school experience – dominate that much the performance of teacher students and teachers? In order to better understand the impact and process in teacher education, a new teacher education approach was formed. This approach results from the feedback of opinions of prospective teachers on the relevance of didactical theories for their own learning process. By confronting the teacher students with their opinions and with stimulating questions they start rethinking their attitudes and their image of teaching and learning processes. With this change of attitudes the normative aspect of the research project comes into action by defining what we as lecturers think is important and relevant for a good performance in teaching.

Dialectic concepts

Results from a survey in 2010 among first year students show that teacher students are struggling with the often contradictory goals in learning and teaching processes. The students were asked to describe their view on mathematics and teaching and learning processes with their own words. In the survey 165 first year teacher students for primary and secondary schools have been involved. Looking at some statements of the teacher students discloses beliefs with lots of tension and conflicting potential. On the one side teacher students say basically:

- There is only right or wrong in mathematics.
- There is the one and only solution for a mathematical problem.
- Learning processes need a strong orientation on the product.
- Mathematics is all about strict rules and systematical work.
- Teachers have to lead and control all working steps.
- All kids should learn with the same pace.

On the other side the same students like to have teaching processes in the following way:

- Mathematical problems affecting kid's daily life and having some applications.
- Realigning instructions towards understanding and clearness in teaching mathematics.

- Stimulating communication about different ways and strategies for solving a problem.
- Education being an open and enhancing learning process.

Their view is affected by conventions and rules, strongly regulated learning activities, which leads to internal tensions, if the (prospective) teachers simultaneously reach out for more real life applications and comprehension orientated activities, clearness and more openness in teaching mathematics. These statements could be summarized to certain dialectic concepts (see Helmerich 2012) in teaching and learning processes:

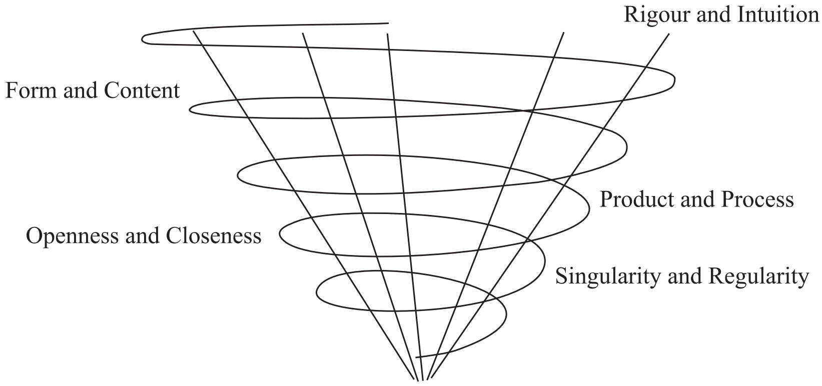
- form and content
- openness and closeness
- rigour and intuition
- product and process
- singularity and regularity

Basis for the clustering of the students' answers was the work of Krauthausen and Scherer (2007) on the foundations of teaching mathematics as "at first contradicting extremes" (translated by MH). The extreme poles mentioned by Krauthausen and Scherer (2007), like application orientation versus structure orientation, student orientated versus subject orientated teaching, individual ways of solving problems versus conventions, open tasks versus closed-ended problems mirror the results of the prospective teacher's survey.

Although we know that teachers encounter conflicting situations, and their professional knowledge and necessary competences are extensively discussed and investigated, studies like TEDS-M still describe areas of tension as a liability for teachers, but not the great potential of activating these tensions in the learning processes as dialectic concepts. Helsper names it "constitutive professional antinomies of teaching" (Helsper, 1996, 2004) (translated by MH), "which entangle the uncertainty of representative interpretation and the simultaneous aspects distance and proximity." (Baumert & Kunter, 2006, p. 471) (translated by MH).

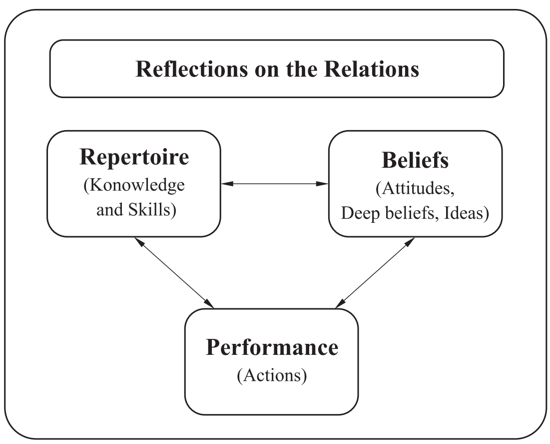
"Taken the antinomious structure of teaching seriously, teachers necessarily will have to make decisions about their actions in teaching, which cannot be in accordance to both conflicting claims of validity at the same time. This situation will only turn out to be bearable and productive, if there is a working agreement of their free will" (Baumert & Kunter, 2006, p. 471) (translated by MH).

In developing a framework for teacher education at university it is crucial to see the antinomies above not so much as areas of conflict but moreover as poles of possibilities for teaching actions and the challenge to find the right balance in teaching and learning processes. In university education these dialectic concepts could be used to stimulate the learning and reflecting process on how to teach and how to learn similar to fundamental and universal ideas in mathematics.



The dialectic concepts could be seen as bars in a spiral curriculum: throughout the learning process teacher students will experience the areas of conflict and tension on different levels of pervasion of mathematical and didactical knowledge and pedagogical fields of action. Learning and Teaching always needs a balanced point of view in these areas of conflict, a "both-and" of the two poles in the right blend.

Normatively speaking the aim of university studies is to enable prospective teachers for reflected teaching in dialectic areas of tensions. This includes a reflective insight in the repertoire of mathematics as well as the sensibility for ones image of mathematics and the correlated belief system in order to come to a high performance in teaching situations.



Reflection should not only refer to repertoire, beliefs and performance itself, but especially also on the relation between these aspects: How do beliefs about the repertoire influence the performance? What tells the performance about repertoire and beliefs? And could you change beliefs for a better by giving the right impulses in lecturers and presenting the repertoire? In the following section I will report on an example for encouraging teacher students to reflect their image of mathematics, their belief system and to adopt an attitude towards teaching and learning processes.

A reflection project in teachers education

This section illustrates how reflections and reflection competence could be encouraged and facilitated in university education for prospective teachers. Examples of reflection essays from students are used in a case study on reflection competence. So far the aim of this project was to get students to a reflected position of their image of mathematics and how to learn and teach mathematics at school.

In the beginning of the project accompanying a lecture, occasions of reflecting were integrated, to start off with just describing opinions on material presented in my lessons, and to continue with writing tasks throughout the whole semester, provoking processes of reflection and enforce my students to take a firm stand on their beliefs, decisions and opinions in teaching and learning processes. The university course on “Learning Mathematics as Construction Processes” for third year students seemed to be the right place for starting with the reflection project. About 200 of the 350 enrolled students took part in the take-home reflection assignments which came in addition to the tutorial work and homework, included some kind of reflection tasks already as for example to discuss advantages and difficulties of different teaching settings or mathematics problems.

With the reflection assignments, shortly called “E-Reflex”, the teacher students are encouraged to reflect their own learning process and to comment on the presented contents, as reflecting is crucial for reasonable and sustainable learning und understanding. In this way of reflecting the teacher students got the possibility to express their observations and thoughts in order to become aware of their own learning process and issues. By working on the reflection assignments the mathematical and didactical content knowledge is engrossed and competence in reflecting is trained and elaborated. Moreover those E-Reflex texts gave me a good feedback on the learning progress of my students and enabled me to respond in further lectures. The student texts had to be uploaded to a learning platform “Moodle”, were revised by myself and handed back with attached short comments. With these comments the students got a feedback on depth and width of their thoughts.

The first E-Reflex task covered the didactical concept of learning by discovery which is based on a constructivist position. The students should reflect on the question whether mathematical discoveries of primary or secondary school students are possible at all, and if so to give a practical example, in which setting or problem discoveries could be made. In addition it had to be discussed, if there are specific mathematical concepts and contents which could not be learned by discovery. Last but not least the teacher students should state their preferred mathematical content for interrogative-developing teaching and teacher centered classroom action, and the reason for their decision.

In this first assignment the students had to reflect on possibilities for discoveries in mathematics, taking a closer look on mathematics in primary school in a mathematical-oriented way, but also with respect to pedagogical concepts and beliefs on how teaching might be. Student Kerstin wrote:

“(. . .) In my opinion, almost everything can be learned by kid’s own discoveries, assumed that one provides specific learning materials. Even concepts could be established by working on tutorial sheets. But I think it is good for kids, if you bring some variety to teaching and don’t withdraw yourself all together. ‘Let the kids just do their own thing’ is not a healthy attitude in discovery learning. (. . .)” (translated student’s text)

In order to reflect on the given question Kerstin has to make her own concept of teaching explicit, moreover she combines her experience and belief of good teaching with lifeworld attitudes and take up a position pro discovery learning. Most of the student’s texts argued for discovery learning as an important principle in teaching, but some students expressed their tendency to go for teacher centered, guided learning methods if it comes to secondary school and putative more abstract and conventional mathematics.

The second E-Reflex assignment challenged the students to justify mathematics in school. With Heymann‘s claim that mathematical instruction should provide a general education (Heymann, 1996) and Winter’s call for important basic experiences with mathematics (grasp mathematics as a certain way of looking at our world, experience mathematics as a structured, well-formed theory and as a certain way of thinking providing problem solving abilities) (Winter, 1995) the students got two positions during lecture to potentially set up their arguments. But the reflection task aimed on their own opinion and their individual justification for teaching mathematics in school, too. It was motivated to make their point of view explicit by giving practical examples or choosing only distinct contents out of the wide range of mathematical knowledge.

Most students argued with the aid of importance of mathematics for every-day life, applying a dimension of lifeworld-oriented reflection. A typical sequence is found in Eva’s text:

“(. . .) Personally I think, that mathematics is sort of basic competence for social life in our culture (in the jungle of course applies something else). But in our industrialized world mathematics is essential. Regardless whether it is shopping, work or leisure we will be confronted with mathematical topics. Some kids say things like: ‘I wanna be a hair dresser, gardener or baker, what do I have to know maths for?’ But you can easily find concrete examples for mathematical topics even in these jobs. (. . .)” (translated student’s text)

The third E-Reflex problem was a rather mathematical-oriented task. The students were confronted with a supposedly ‘illogical spot’ of mathematics. The students then had to understand, what is going on in this problem and complete a dialogue of two school kids talking about this thing and trying to figure out, how it works. By using the dialogue setting the students had to think about the answers to the given problem, but also about difficulties in understanding the mathematical concept underlying the problem, so to think about both parts in the conversation. This method and its positive learning effects are discussed more deeply in Wille (2011). After writing the dialogue it was mandatory to reflect on this kind of task: How did it feel it to work on this assignment? What could be learned by doing this? Is this method a possible starting point for learning processes at school?

Working on this task the students experienced the constraints and opportunities of putting yourself in another's position. Many students found themselves struggling with the problem and so having difficulties to take over the explaining part in the dialogue. But they all acknowledged the high potential for learning and deeper understanding of mathematics throughout this reflecting way. Since the dialogues turned out rather lengthy and would have to be shown in reasonable large sections to grasp the line of argumentation, examples could not be displayed in this article.

In the last E-Reflex task, the students were asked to describe their idea of heterogeneity in classroom action and how they would deal with this diverse situation in teaching and why in this way. To carry it further on, a preference for teaching in a homogeneous (assuming this would be possible to form) or a heterogeneous class should be given. Insisting to take up a stance on the dialectics of learning processes is important to really reach the students in their core beliefs and get a reflection process started on this. A final remark should be made to outline possible actions of differentiation in learning processes and to discuss, if – assuming again that homogeneous learning groups could be put together – differentiation is actually necessary.

In the last task I have noticed a huge step towards a reflection competence in the student's texts. Almost all reflections turned out longer than the obligatory two pages and showed the effort of lay out the reflection in different dimensions. The student got more confident in positioning themselves in areas of conflict demonstrated in sophisticated and deliberative lines of argumentation. Exemplarily the text of Diana shows the progress in belief changes:

"(...) If I had been asked several weeks ago, whether I prefer a homogeneous over a heterogeneous learning group for teaching, I would have probably said yes. Because one cannot imagine anything better for a teacher than teaching kids who are all on the same knowledge level and learning the same content at the same pace and time. (...) The teacher would be able to plan teaching precisely and would cover all anticipated content. (...) I want to be a teacher actually just because of the variety and individuality of the students. Of course I want to teach a heterogeneous learning group. (...) The individuality of learners has to be exploited for a good learning environment." (translated student's text)

Conclusion and further research

The reflection tasks were used in the course to enable prospective teachers to overcome their uncertainty in areas of tension by reflecting on mathematics teaching and learning issues, their own point of view and last but not least their options in dealing with conflicting situations.

It was outlined how the tensions could be turned into encouraging dialectic aspects of teaching and learning processes and therefor become fundamental ideas for structuring university teaching and reflection processes.

In a next step I would like to investigate on the student's texts and carve out types of reflection characters to achieve a deeper insight of persistence or approaches to changes in belief systems and thereby a better understanding of the effectiveness of university teaching on students.

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Reflektierte Handlungsfähigkeit – Aneignung von Reflexionskompetenz von Lehramtsstudierenden

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Zusammenfassung. Ein wesentliches Ziel der Lehrer(innen)bildung an der Universität Siegen ist es, die angehenden Lehrer(innen) in ihrer Handlungsfähigkeit im Umgang mit Spannungsfeldern in Lehr-Lern-Situationen im Unterricht und im eigenen Studium zu stärken. Dafür müssen die Studierenden lernen, ihr Bild von Mathematik, ihre Einstellungen und Überzeugungen darüber, was wichtig und relevant ist innerhalb der Mathematik und ihrer Didaktik, und wie Lernprozesse gestaltet werden können, zu reflektieren. Aus Umfragen unter Studienanfänger(innen) und Interviews mit Studierenden in Beratungsgesprächen wurden fundamentale Spannungsfelder wie "Form und Inhalt", "Strenge und Anschaulichkeit" und "Produkt und Prozess" herausgearbeitet. Angehende Lehrer(innen) empfinden die Entscheidung für das "richtige" Handeln in diesen Spannungsfeldern oft sehr herausfordernd.

Diese dialektisch angelegten Spannungsfelder werden als strukturierende Orientierungselemente und Startpunkte für Reflexionsprozesse in der Lehre eingesetzt. Im Vortrag werde ich Fallbeispiele aus den Aufsätzen zu Reflexionsanlässen zeigen. Begleitend zu einer Lehrveranstaltung wurden die Studierenden mit Standpunkten und Fragen zur Mathematik und Unterricht konfrontiert, und aufgefordert über ihre Einstellungen, Überzeugungen und Meinungen zu reflektieren und kleine Aufsätze dazu zu verfassen. Für Studierenden ist es wichtig eine reflektierte Position innerhalb der vielfältigen Handlungsmöglichkeiten in Lehr-Lern-Prozesse einnehmen zu können. Die eigenen Positionen und Überzeugungen zu kennen ist ein entscheidender Aspekt für die Fähigkeit zur Entscheidungsfindung und der Übernahme von Verantwortung für mathematische Lehr-Lern-Prozesse. Außerdem stellt dies die Grundlage bereit, um Lehr-Lern-Prozesse reflektieren zu können und eine entspannte Einstellung in Stress-Situationen des unterrichtlichen Handelns zu bewahren.

Schlagwörter: reflexionskompetenz, Lehrer(innen)bildung, reflektierte Handlungsfähigkeit

Dependence of the problem solving strategies on the problem context

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Abstract. Of all mathematical processes generalization is considered one of the most important ones. We need to distinguish between two aspects of generalization: seeing the general in the particular, or seeing the particular in the general. In the first case we are speaking of inductive reasoning, whereby the observation of special cases leads one to suspect very strongly that some general principle is true. Deductive reasoning, on the other hand, is a process of inferring conclusions from the known information (premises) based on formal logic rules. For the purpose of our research we would like to distinguish between the following types of generalization: generalization through induction, generalization through perceiving recurrence and generalization through generalizing the reasoning.

In the paper the results of the study on primary teacher students' competences in generalization are presented. Our research was focused on delving into the students' problem solving strategies in relation to the problem type. We compared students' problem solving strategies when solving two different types of problems: a conceptual one and a procedural one in relation to two types of generalisation, i.e. the inductive reasoning and the generalization through generalizing the reasoning. The research results lead to some significant conclusions. We conclude that the type of generalization is related to the type of the problem: if a relationship between the problem and the mathematical concept can be established, the reasoning type of generalization prevails. If, on the other hand, it is not possible to establish clear connection between the problem and a certain mathematical concept, working with mere numbers prevails, and, consequently, the inductive type of generalization.

Keywords: problem solving, procedural, conceptual problems, problem solving strategy, generalisation, inductive reasoning, generalisation through generalizing the reasoning

Theoretical background

Of all mathematical processes generalization is considered one of the most important ones. For some researchers generalization is what mathematics is about (Maj-Tatsis &Tatsis, 2012). We all generalize in our everyday lives. We probably do it considerably more often than we realise (Cockburn, 2012). According to Vinner (2012) generalizations are the driving engine of the concepts in all domains and statements about almost any subject. Upon reflecting on people's thought processes, we can realize that there is a tendency to generalize.

Dorfler (1991) understands generalizing as a social-cognitive process, which leads to something general, and whose product consequently refers to an actual or potential manifold in a certain way. We need to distinguish between two aspects of generalization: seeing the general in the particular, or seeing the particular in the general (Kruetski, 1976).

In the first case we can speak of *inductive reasoning*, which is a very prominent manner of scientific thinking, providing for mathematically valid truths on the basis of concrete cases. Pólya (1967) indicates that inductive reasoning is a method of discovering properties from phenomena and of finding regularities in a logical way. He refers to four steps of the inductive reasoning process: observation of particular cases, conjecture formulation, based on previous particular cases, generalization and conjecture verification with new particular cases. Reid (2002) describes the following stages: observation of a pattern, conjecturing (with a doubt) that this pattern applies generally, testing the conjecture, and the generalization of the conjecture. Cañadas and Castro (2007) consider seven stages of the inductive reasoning process: observation of particular cases, organization of particular cases, the search and prediction of patterns, conjecture formulation, conjecture validation, conjecture generalization, general conjectures justification. There are some commonalities among the mentioned classifications: Reid (2002) believes the process to complete with generalization, whereas Polya adds the stage of "conjecture verification", as well as Cañadas and Castro (2007), who name the final stage the "general conjectures justification". In their opinions general conjecture is not enough to justify the generalization. It is necessary to give reasons that explain the conjecture with the intent to convince another person that the generalization is justified. Cañadas and Castro (2007) divided the Polya's stage of conjecture formulation into two stages: the search and prediction of patterns and conjecture formulation. The above stages can be thought of as levels from particular cases to the general case beyond the inductive reasoning process. Not all these levels are necessarily present; there are a lot of factors involved in their reaching.

The second aspect of generalization refers to *deductive reasoning*. This is a process of inferring conclusions from the known information (premises) based on formal logic rules, whereby the conclusions are necessarily derived from the given information, and there is no need to validate them by experiments (Ayalon & Even, 2008). Although there are also other accepted forms of mathematical proving, a deductive proof is still considered as the preferred tool in the mathematics community for verifying mathematical statements and showing their universality (Hanna, 1990; Mariotti, 2006; Yackel & Hanna, 2003).

For the purpose of our research we would like to present yet another classification of the generalization situations. Krygowska (1979: in Ciosek, 2011) distinguishes between the following types of generalization:

- Generalization through induction: one first establishes the rule $f(1)$, $f(2)$, $f(3)\ldots$ and notices that results can be obtained by applying a general rule $f(n)$ for natural n, which is a conjecture only.
- Generalization through generalizing the reasoning: one notices that the reasoning carried out in a single case will remain correct in a different setting, or minor modifications will be needed, only to get a more general result.
- Generalization through unifying specific cases: a bunch of statements, each referring to one case of a setting, proves able to be replaced by one general statement, the original ones being its special cases. E.g.: Pythagoras' theorem, formulas for the acute – angled, the obtuse-angled triangle can be generalized to the cosine formula.
- Generalization through perceiving recurrence: similar to generalization through induction, but in this case the formula $f(2)$ is obtained by using the formula $f(1)$, $f(3)$ by using $f(2)\ldots$; the recurrence rule $f(n)$ is formed.

We can notice commonalities between these types of generalization and the more widely used terms of inductive reasoning. We are going to refer to the first and the last type of generalization defined by Krygowska, namely the generalization through induction and the generalization through perceiving recurrence, as inductive reasoning. The second type, generalization through generalizing the reasoning, describes generalization as an insight into the problem situation without analysing many cases. For the problem solver one case of generalisation is usually enough. The third type is less bound to problem solving; it serves more to investigate the hierarchies among mathematical concepts.

Further, we are going to focus on inductive reasoning and generalisation through reasoning, in relation to the mathematical problem. We are going to distinguish between procedural and conceptual problems in a similar way as we distinguish between procedural and conceptual knowledge. The research in this area was prominent in the years between 1980 and 2000 (see e.g. Skemp, 1979; Hiebert, 1986; Gelman & Meck, 1986; Tall & Vinner, 1981; Gray & Tall, 2001; Sfard, 1994). We are going to use the definition of procedural and conceptual knowledge, proposed by Haapsalo (2003), who opposed the distinction between the two types of knowledge, perceiving the procedural knowledge as dynamic and rich in nature, whereas the conceptual knowledge as static and poor. He defined *conceptual knowledge* as knowledge of and a skilful "drive" along particular networks, the elements of which can be concepts, rules (algorithms, procedures, etc.), and even problems (a solved problem may introduce a new concept or a rule) given in various representation forms (Haapsalo, 2003). *Procedural knowledge,* on the other hand, denotes dynamic and successful utilization of particular rules, algorithms or procedures within the relevant representation forms. This usually requires not only the knowledge of the objects being utilized, but also the knowledge of format and syntax of the representational system(s) expressing them. Procedural knowledge often calls for automated and unconscious steps, whereas the conceptual one typically requires conscious thinking (Haapsalo, 2003). Considering these definitions, we are going to name procedural problems those which require mere procedural knowledge for their solvin; in this case a problem solver is more focused on procedures, rules and algorithms On the other hand, the conceptual problems are those which require the solver to be familiar with the specific mathematical concepts. We are also proposing that there are no disjunctive categories of problems in this manner: however, one of them (procedural or conceptual knowledge) prevails over the other one at problem solvin; some kind of relation between procedural and conceptual knowledge must be established.

We were interested if this categorisation of problems into the procedural and the conceptual ones would be reflected in problem solvers e.g. would influence their choice of the problem solving strategies. We wanted to compare problem solving strategies when solving two different types of problems: a conceptual one and a procedural one in relation to two types of generalisation, i.e. the inductive reasoning and the generalization through generalizing the reasoning.

Empirical part

Problem definition and methodology

In the empirical part of the study conducted with primary teacher students the aim was to explore their problem solving competences. Our research was focused on delving into the students' problem solving strategies in relation to the problem type: the procedural one or the conceptual one. We were interested whether their types of generalisation would differ in relation to the posed problem.

The aim of the study was to answer the following research question: Which type of generalisation prevails for a certain type of a problem?

The empirical study was based on the descriptive, casual and non-experimental method of pedagogical research (Hartas, 2010; Sagadin, 1991).

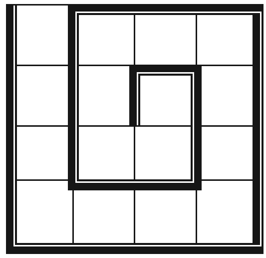
Sample description

The study was conducted at the Faculty of Education, University of Ljubljana, Ljubljana, Slovenia in May 2010 (69 second-year students of the Primary Teacher Education Department), and in March 2012 (89 third-year students of the Primary Teaching Education Department).

Data processing procedure

The students were posed two mathematical problems: the 'spiral' one (the procedural type) and the 'pond' one (the conceptual type). The problems were, as follows:

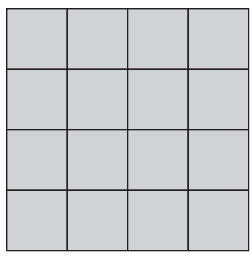
SPIRAL: On the picture below the shaping of the spiral in the square of 4×4 m is presented. Explore the problem of the spiral length in squares of different dimensions.



POND: *Miha works in a horticultural centre, in which different shapes of ponds can be ordered. His job is to advise the customers on the number of tiles to buy in order to make a path around the pond.*

a) *How many tiles* $(1 \times 1\ m)$ *do they need for a path around the pond of* $4 \times 4\ m$*?*

b) *Try to form a rule for Miha to use to calculate the number of tiles for any quadratic shape of the pond.*



The problem of the 'spiral' was addressed in 2010 (see Manfreda Kolar, Hodnik Čadež, 2011), whereas the problem of the 'pond' was addressed in 2012.

The students were solving the problems individually, and simultaneously noting down their deliberations and findings; at the task they were aided with a blank square paper sheet.

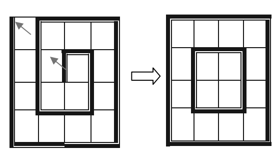
The data gathered from solving the mathematical problems were statistically processed by employing descriptive statistical methods. The students' solutions were analysed from the perspective of the applied strategies and the types of generalisations in relation to these strategies.

Analysis of the problem solving strategies

a) The spiral problem

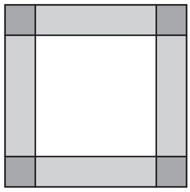
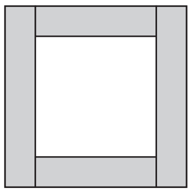
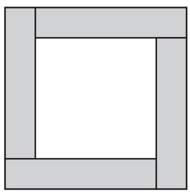
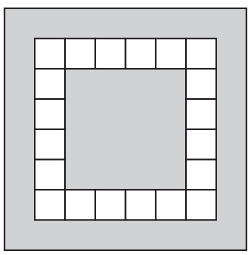
Various solving strategies are presented below, out of which the ones that were encountered among the students' solutions of the spiral problem are highlighted:

Table 1. Description of the applied problem solving strategies for the spiral problem.

Strategy denotation	Strategy description	Generalization record
1 a – "squares" strategy	It is observed that the values of the lengths are obtained by squaring the lengths of the consecutive square (e.g. $15 = 16 - 1$)	$(n+1)^2 - 1$
2 a – "product" strategy	It is observed that the length of the spiral is equal to the product of two numbers that differ for 2 (e.g. $15 = 5 \times 3$)	$n(n+2)$
3 a – "binomial" strategy	It is observed that the length of the spiral is calculated by adding the double length to the square of the square length (e.g. $15 = 3 \times 3 + 2 \times 3$)	$n^2 + 2n$
4 a – "difference" strategy	When observing the differences among the lengths of the spirals, it is obvious that the result is the sequence of odd numbers (e.g. from 1×1 square onwards the lengths of the spirals increase by 5, 7, 9, 11, 13, 15. . . .	The difference between the spiral in the square with $n \times n$ dimensions and the consecutive spiral is $2n + 1$ or in a recursive manner: $d_{n\times n} = d_{(n-1)\times(n-1)} + (d_{(n-1)\times(n-1)} - d_{(n-2)\times(n-2)} + 2)$, whereby the denotation $d_{n\times n}$ stands for the length of the spiral in the square with $n \times n$ dimensions.
5 a – "sum" strategy	It is observed that the length of the spiral can be presented as the sum of individual even sections of the spiral (e.g. $15 = 1 + 1 + 2 + 2 + 3 + 3 + 3$.	$3n + 2(n-1) + 2(n-2) + \ldots + 2 \cdot 2 + 2 \cdot 1$
6 a – "transformation strategy"	It is observed that in cases when the dimension of the square is an even number, spirals can be transformed in squares, the perimeters of which can be calculated.	$4n + 4(n-2) + 4(n-4) + \ldots + 4 \cdot 2;\ n = 2k,\ k \in \mathbf{N}$ 

b) The pond problem

Table 2. Description of the applied problem solving strategies for the pond problem.

Strategy denotation	**Graphical / symbolic representation**	**Strategy description**	**Generalization record**
1 b – ‘corner strategy’		It is observed that the number of tiles is equal to 4 lengths of the side of the square enlarged for the 4 corner tiles.	$4a+4$
2 b – ‘opposite sides strategy’		It is observed that the number of tiles is obtained by adding 2 lengths of the side and 2 lengths of the side enlarged by two.	$2(a+2)+2a$
3 b – ‘equal sides strategy’		It is observed that the number of tiles is equal to 4 lengths of the side enlarged by one.	$4(a+1)$
4 b – ‘area strategy’		It is observed that the number of tiles is obtained by subtracting two areas of the squares.	$(a+2)^2-a^2$
5 b – ‘product strategy’	4×4	$20=$	4	$\times5$ 5×5	$24=$	4	$\times6$ 6×6	$28=$	4	$\times7$ 7×7	$32=$	4	$\times8$	It is observed that the number of tiles is equal to the product of number 4 and a number which is one more than the dimension of the square.	$4(a+1)$
6 b – ’difference strategy’	1×1	8 2×2	12) $+4$ 3×3	16) $+4$ 4×4	20) $+4$ 5×5	24) $+4$ ...	... 10×10	$44=8+4\cdot(10-1)$	It is observed that the differences in the numbers of tiles are always the same: 4.	$8+4(a-1)$

All the students, with the exception off four of them used strategies from 1 to 5 in the table 1 for solving the spiral problem, which are based on generalization from different cases. From their papers it is evident that they drew different spirals and tried to establish a general rule for the n-spiral from those cases. As mentioned before, 4 students did not need different concrete examples for establishing the rule, as they were able to find it on the basis of only one example: one student created the rule (which is true only for the spirals of odd dimensions) by using the transformation strategy. She noticed that the length of the whole spiral equalled the sum of perimeters of squares, which were obtained by transforming the original form of the spiral; three students who used the sum strategy also formed the rule by analysing only one case: they realised that the length of the spiral would be equal to the lengths of the even sections of the spiral.

The pond problem was addressed in the opposite manner. All the students, with the exception of 3 of them used one case and the strategies from 1b to 4b in the table 2 for solving the pond problem. From their paper work it can be concluded that they based their generalisation on the analysis of only one case. Three of them, on the other hand, drew different ponds and defined the rule for any pond from the numerical data, by working with mere numbers.

We can conclude that the spiral problem, which is more a procedural type of a problem, encouraged the students to perform inductive reasoning, whereas the pond problem, which is more a conceptual type of a problem, triggered generalization through reasoning.

Discussion

The research results lead to some significant conclusions. We analysed the type of generalization that was used to solve two different problems: the spiral one and the pond one, with the former being classified as the procedural one, whereas the latter as the conceptual one.

We believe that the type of generalization is related to the type of the problem, or more precisely, to the manner in which an individual perceives the problem situation: if a relationship between the (everyday life) problem and the mathematical concept can be established, the reasoning type of generalization prevails. If, on the other hand, it is not possible to establish clear connection between the problem and a certain mathematical concept, working with mere numbers prevails, and, consequently, the inductive type of generalization. So, the issue is about the context-based approach versus the context-free approach. The problem of making a path around the pond was not difficult to connect to the mathematical context of a perimeter, for which reason, the reasoning type of generalization prevailed among the students. On the other hand, only few students were able to connect the problem of the spiral to a certain well-known mathematical content, and therefore generalization through induction or generalizing through perceiving recurrence prevailed.

It should also be mentioned that the students applied a variety of different strategies for both problems. According to Žeromska (2010) good understanding of a concept becomes visible also through a flexible use of formulas; sometimes it is more practical to use the long-term intuition of the concept than to mechanically apply a formula. This was especially the case with the pond problem, where the students developed different algebraic expressions, based on their different ways of thinking and perceiving the problem and not on mechanically applying a certain mathematical formulas. Different ways of reasoning led them to different algebraic expressions, which could also be a good starting point for a teacher to pose a question: are these expressions equivalent? By comparing them and developing one from another, students can add to their algebraic thinking. Such problems can contribute to establishing better connections between arithmetic and algebraic thinking and to perceiving connections between at first glance different formulas.

Nevertheless, every teacher should ask himself why it is so important to gain an insight into the students' reasoning. We believe that this is the way to discover their misconceptions in the development of a certain mathematical concept (a perimeter – an area); thus, we also allow for different ways of reasoning for the same problem, which students compare and evaluate, which contributes to upgrading their problem solving abilities: only the teachers who have competences in problem solving can create and deal with the situations in the classroom which contribute to the development of those competences in children.

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Odvisnost izbire strategije reševanja problema glede na kontekst problema

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Povzetek. Izmed vseh matematičnih procesov je proces posploševanja eden najpomembnejših. Razlikujemo med dvema vrstama posploševanja: prepoznavanje splošnega pravila v posameznem primeru in prepoznavanje posameznega primera v splošnem pravilu. V prvem primeru lahko govorimo o induktivnem sklepanju, kjer nas opazovanje posameznih primerov vodi do posplošitve oz. oblikovanje splošnega pravila za te primere. Deduktivno sklepanje pa je proces oblikovanja zaključkov na osnovi danih predpostavk, ki temeljijo na pravilih matematične formalne logike. V naši raziskavi bomo razlikovali naslednje kategorije posploševanja: posploševanje na osnovi induktivnega sklepanja, na osnovi uvida pojava oz. zakonitosti ter posploševanje na osnovi posploševanja sklepanja.

V prispevku so predstavljeni rezultati raziskave o kompetencah študentov razrednega pouka o posploševanju v matematiki. V raziskavi smo se osredinili na študentove strategije reševanja problemov glede na kontekst problema. Primerjali smo njihove strategije pri reševanju dveh problemov, konceptualnega in proceduralnega. Rezultati raziskave so nas vodili do pomembnih ugotovitev in sicer, da je izbira strategije reševanja problemov v relaciji s kontekstom problema. Če je problem konceptualne narave oz. je mogoče vzpostaviti jasno povezavo med problemom in matematičnim pojmom, potem študenti izberejo strategijo posploševanje na osnovi posploševanja sklepanja, če pa je problem proceduralne narave, pri katerem prevladuje operiranje s števili oz. med problemom in matematičnim pojmom ni očitne povezave, študenti izberejo posploševanje na osnovi induktivnega sklepanja.

Ključne besede: reševanje problemov, proceduralni, konceptualni problemi, strategija reševanja problema, posploševanje, induktivno sklepanje, posploševanje na osnovi posploševanja sklepanja

Three- and four-dimensional regular 4-solids move in the computer 2-screen

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Abstract. In previous works (see [1], [2], [3]) the authors extended the method of central projection to higher dimensions, namely, for $\mathbf{E}^4 \to \mathbf{E}^2$ projection from a one dimensional centre figure, together with a natural visibility algorithm. All these are presented in the linear algebraic machinery of real projective sphere $\mathcal{P}S^4$ or space $\mathcal{P}^4(\mathbf{V}^5, \boldsymbol{V}_5, \sim)$ over a real vector space $\mathbf{V}^5$ for points and its dual $\boldsymbol{V}_5$ for hyperplanes up to the usual equivalence $\sim$ (expressed by multiplication by positive real numbers or non-zeros, respectively). In this presentation we attempt to further develop the exterior (Grassmann) algebra method (with scalar product) by computer to other effects of illumination: visibility and shading, e.g. for (regular) 4-polytopes on the base of the homepage: `http://www.math.bme.hu/~prok` (for free download).

Below we present the computer figures of the six regular 4-polytopes. We plan further develop the method in the above homepage.

This work is dedicated to the Memory of Ernő Molnár, the Father and first Mathematics Teacher of the presenter, on His 100th Birthday.

You see that the computer gives us phanthastic possibilities in visualizing higher-dimensional geometries as well.

Keywords: animation of regular figures for aesthetics of mathematics

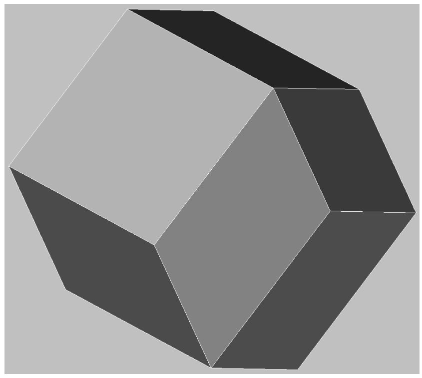


Figure 1. The 4-cube (8-cell) with Coxeter-Schläfli symbol $(4,3,3)$.

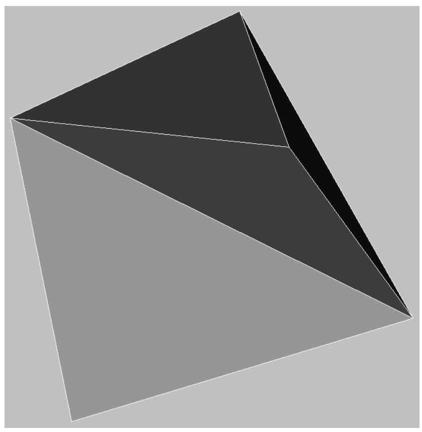


Figure 2. The 4-siplex (5-cell) with Coxeter-Schläfli symbol $(3,3,3)$.

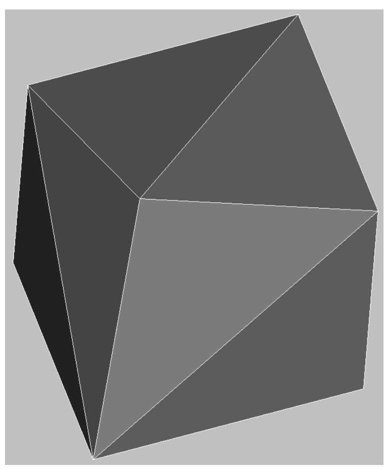


Figure 3. The Cross-polytope (16-cell) with Coxeter-Schläfli symbol $(3,3,4)$.

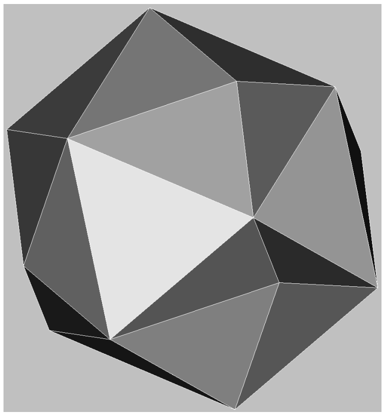


Figure 4. The 24-cell with Coxeter-Schläfli symbol $(3,4,3)$.

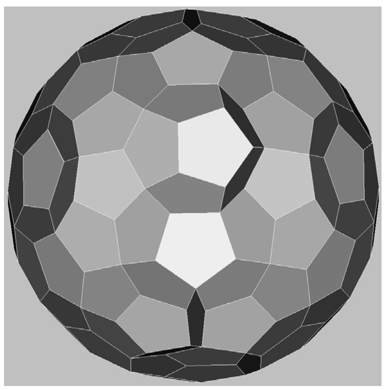


Figure 5. The 120-cell with Coxeter-Schläfli symbol $(5,3,3)$.

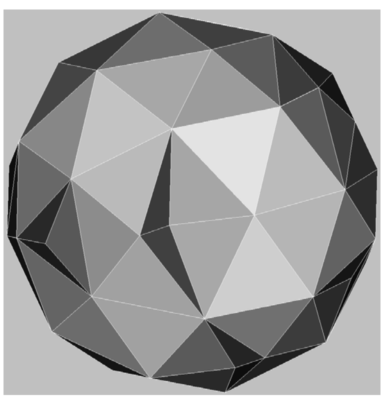
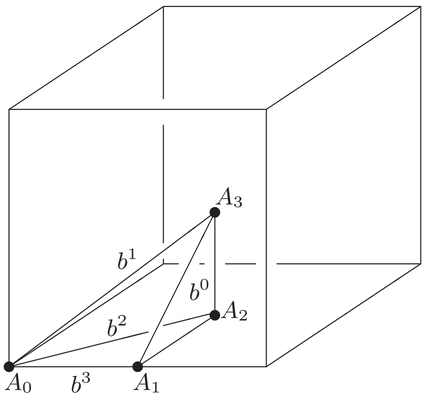
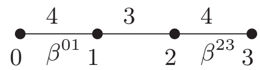
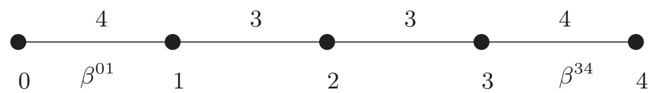


Figure 6. The 600-cell with Coxeter-Schläfli symbol $(3,3,5)$.





$$\begin{pmatrix} 1 & -\frac{\sqrt{2}}{2} & 0 & 0 \\ -\frac{\sqrt{2}}{2} & 1 & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 1 & -\frac{\sqrt{2}}{2} \\ 0 & 0 & -\frac{\sqrt{2}}{2} & 1 \end{pmatrix} = (b^{ij})$$



$$(b^{ij}) = \begin{pmatrix} 1 & -\frac{\sqrt{2}}{2} & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{2} & 1 & -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} & 1 & -\frac{\sqrt{2}}{2} \\ 0 & 0 & 0 & -\frac{\sqrt{2}}{2} & 1 \end{pmatrix}$$

Figure 7. Cube in E^3 and symbols for it. Diagrams and Coxeter-Schläfli matrices for the 3-cube and the 4-cube, respectevely.

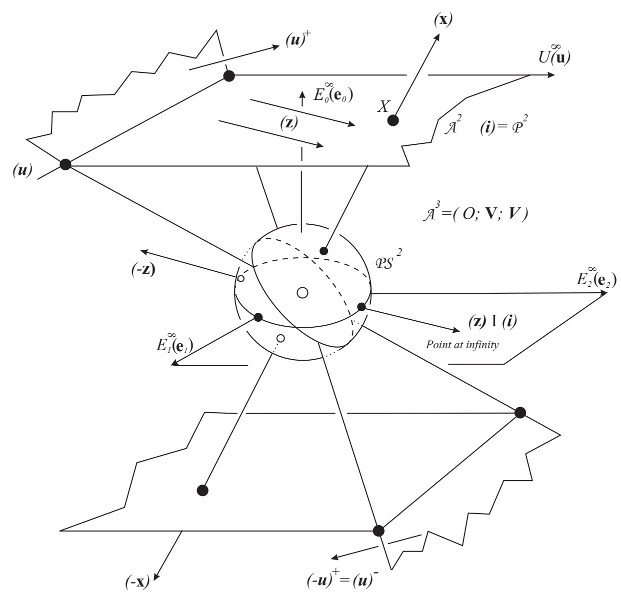


Figure 8. The projective sphere and plane in dimension 2.

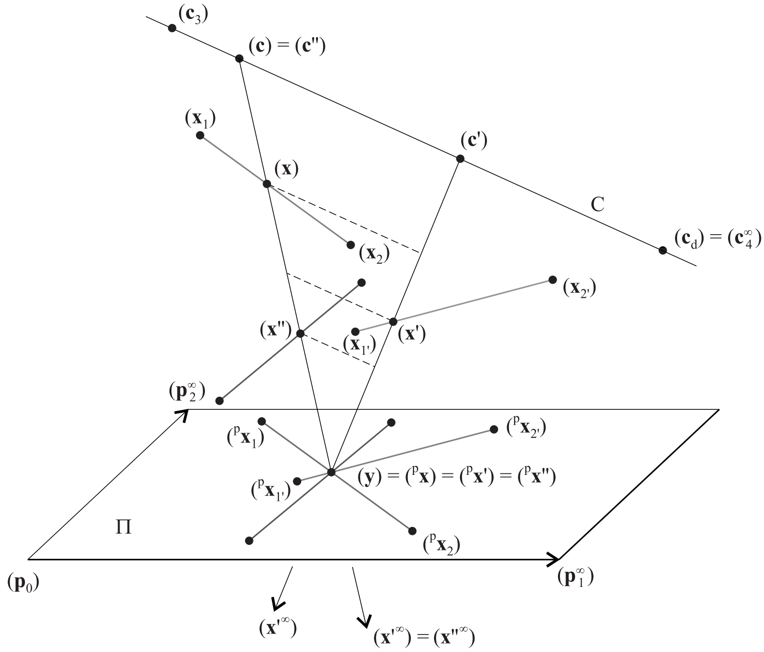


Figure 9. Projection of segments to extended local visibility.

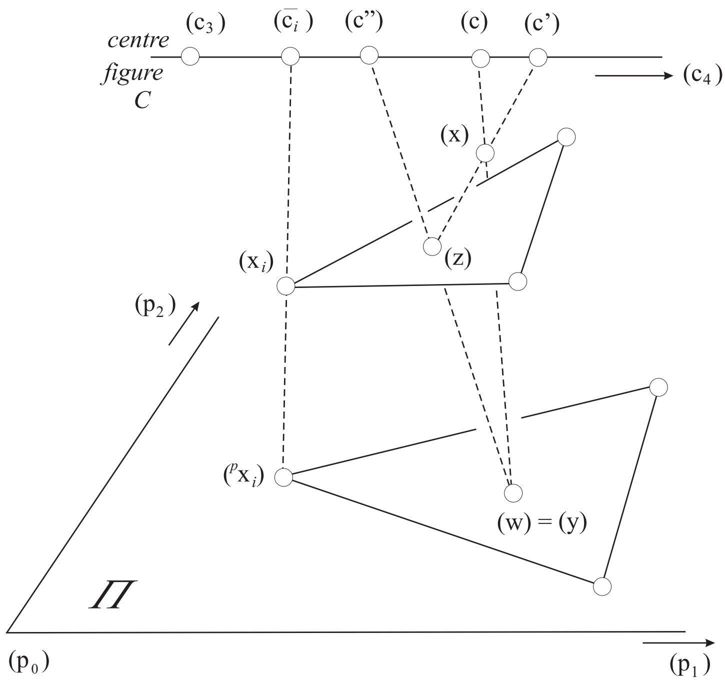


Figure 10. New initiative, illustrated in 4 → 2 projection.

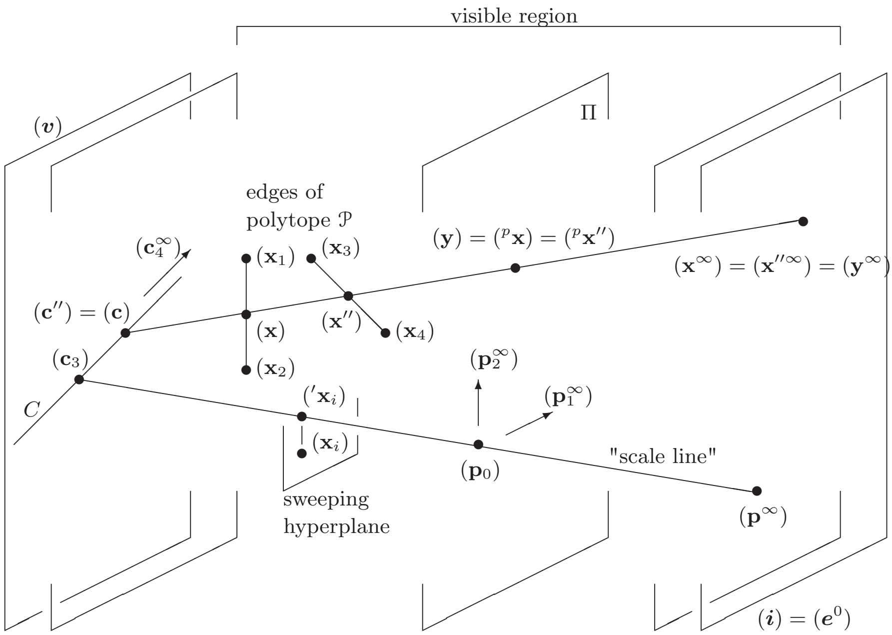


Figure 11. Ordering vertices to vanishing hyperplane $(\boldsymbol{v})$.

An affine-projective coordinate simplex represents the camera by

$$\begin{pmatrix} \mathbf{p}_0 \\ \vdots \\ \mathbf{p}_p^\infty \\ \mathbf{c}_{p+1} \\ \vdots \\ \mathbf{c}_d^\infty \end{pmatrix} \sim \begin{pmatrix} 1 & \dots & p_0^p & p_0^{p+1} & \dots & p_0^d \\ \vdots & & \vdots & \vdots & & \vdots \\ 0 & \dots & p_p^p & p_p^{p+1} & \dots & p_p^d \\ c_{p+1}^0 & \dots & c_{p+1}^p & c_{p+1}^{p+1} & \dots & c_{p+1}^d \\ \vdots & & \vdots & \vdots & & \vdots \\ 0 & \dots & c_d^p & c_d^{p+1} & \dots & c_d^d \end{pmatrix} \begin{pmatrix} \mathbf{e}_0 \\ \vdots \\ \mathbf{e}_p^\infty \\ \mathbf{e}_{p+1}^\infty \\ \vdots \\ \mathbf{e}_d^\infty \end{pmatrix}$$
$$\sim: (\mathrm{Cam}) \begin{pmatrix} \mathbf{e}_0 \\ \vdots \\ \mathbf{e}_d^\infty \end{pmatrix}.$$

Here any point $X(\mathbf{x})$ in the visible region can be expressed as

$$\mathbf{x} \sim (1, x^1, \dots, x^p, x^{p+1}, \dots, x^d) \begin{pmatrix} \mathbf{e}_0 \\ \vdots \\ \mathbf{e}_d^\infty \end{pmatrix}$$
$$\sim (y^0, y^1, \dots, y^p, c^{p+1}, \dots, c^d)(\mathrm{Cam}) \begin{pmatrix} \underline{\mathbf{e}}_0 \\ \vdots \\ \underline{\mathbf{e}}_d^\infty \end{pmatrix},$$

so that

$$(1, x^1, \ldots, x^p, x^{p+1}, \ldots, x^d)(\mathrm{Cam})^{-1} \sim (y^0, y^1, \ldots, y^p, c^{p+1}, \ldots, c^d)$$
$$\sim \left(1, \frac{y^1}{y^0}, \ldots, \frac{y^p}{y^0}, \frac{c^{p+1}}{y^0}, \ldots, \frac{c^d}{y^0}\right).$$

Relative visibility of $X(\mathbf{x})$ to $X'(\mathbf{x}')$ with $(')$ coordinates can be decided by Figures 9–11 and by an ordering prescription:

a) *the images* $({}^p\mathbf{x}) = (\mathbf{y})$ *and* $({}^p\mathbf{x}') = (\mathbf{y}')$ *are different (both are visible);*

b) *if the images are the same, i.e.* $\mathbf{y} \sim \mathbf{y}'$, *namely* $\frac{y^1}{y^0} = \frac{y^{1'}}{y^{0'}}, \ldots, \frac{y^p}{y^0} = \frac{y^{p'}}{y^{0'}}$, *then* $\frac{c^{p+1}}{y^0} > \frac{c^{(p+1)'}}{y^{0'}}$;

c) *if the above equalities hold, then* $\frac{c^d}{y^0} < \frac{c^{d'}}{y^{0'}}$ *(the reverse inequality holds for* $d = 4 = d'$*).*

Then $X(\mathbf{x})$ is nearer to the centre figure C than $X'(\mathbf{x}')$.

Figure 12. Projection calculus and visibility criteria.

The regular solids

After having constructed the regular 3-, 4-, 6-, 5-gons, respectively, we can meditate, why the regular 7-gon cannot be constructed by lineal and compass in the usual way. Then we can model and study the 5 regular (Platonic) solids: the cube, octahedron, tetrahedron, dodecahedron and icosahedron as follows.

The cube (hexahedron) has 6 regular square faces, its dual is the regular octahedron with 8 regular triangle faces.

Then or formerly we can study the regular tetrahedron (with 4 regular triangle faces) which is selfdual. That is the 4 midpoints of faces will be the vertices of a new regular tetrahedron, if we connect them, to get first the 6 edges; then we form the 4 dual faces by 3 edges to each face.

The regular dodecahedron has 12 regular pentagon faces 3 meeting at each vertex, there are 20 vertices at all, in a very attractive way. Its dual, named regular icosahedron, has 20 regular triangle faces. 5 edges in 5 faces meet in each vertex, there are 12 vertices at all.

In the home page [9] these move in the computer screen with visibility and shading. Moreover, you see there many other attractive combinations with various colors.

Let us consider the cube in Fig. 7 with its schematic parallel (axonometric) projection. Please, model also the 3-dimensional solid as follows. By A_3 we denote the cube centre (i.e. 3-dimensional or, in short, 3-centre); then A_2 is the 2-centre of a 2-dimensional face, bounding the solid; then A_1 is the 1-centre of a 1-dimensional edge of the former face; then A_0 is the 0-centre, i.e. a 0-dimensional vertex of the former edge.

Thus, we get a so-called characteristic or barycentric simplex $A_0A_1A_2A_3$ of the cube which describes a so-called flag-figure, i.e. consequetively incident vertex-edge-face-solid structure. A cube has $8 \cdot 6 = 48$ such simplices which are congruent. The neighbouring or adjacent simplices tile the cube.

The analogous procedure holds to the other Platonic solids. The following characterization will be valid for them as well, with corresponding parameters.

The side triangles of the characteristic simplex $b^0 = A_1A_2A_3$, $b^1 = A_0A_2A_3$ have the face angle $\beta^{01} = \pi/p = \pi/4$ because of the square face of the cube. This can be seen on the so-called Coxeter-Schläfli diagram besides the cube and on the matrix entry

$$b^{01} = \cos(\pi - \beta^{01}) = \cos\left(\pi - \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} \tag{1}$$

in the C-Sch matrix (b^{ij}) in Fig. 7.

With the face $b^2 = A_0A_1A_3$ we can form the face angles $\beta^{02} = \pi/2$ and $\beta^{12} = \pi/q = \pi/3$.

We see these on the C-Sch diagram and on the matrix as well. The nodes $0, 2$ are not connected with a branch, the nodes $1, 2$ are connected with a branch signed by $q = 3$. The matrix entries $b^{02} = \cos(\pi - \frac{\pi}{2}) = 0$ and $b^{12} = \cos(\pi - \frac{\pi}{3}) = -\frac{1}{2}$ also show these.

The side face $b^3 = A_0A_1A_2$ has the angles $\beta^{03} = \frac{\pi}{2} = \beta^{13}$ and $\beta^{23} = \frac{\pi}{r} = \frac{\pi}{4}$. The C-Sch diagram and matrix show these as above. Finally, we get the complete characterization of the cube by the C-Sch simplex, diagram $(p, q; r) = (4, 3; 4)$ and the symmetric matrix (b^{ij}) $(i, j = 0, 1, 2, 3)$ by

$$b^{ij} = \cos(\pi - \beta^{ij}) \quad \text{with} \quad \beta^{ii} = \pi, \; i.e. \quad b^{ii} = 1 \; . \tag{2}$$

Projective embedding

Behind this symbols we find the embedding of the Euclidean 3-space $\mathbf{E}^3$ into a 4-dimensional real vector space $\mathbf{V}^4$ and its dual (linear form) space $\boldsymbol{V}_4$, in the projective sense as Fig. 8. indicates this for $\mathbf{E}^2 \to \mathcal{P}^2(\mathbf{V}^3, \boldsymbol{V}_3, \sim) \subset \mathcal{P}\mathbf{S}^2$ embedding (by the usual multiplicative equivalence $\mathbf{x} \sim c\mathbf{x}$, $\mathbf{x} \in \mathbf{V}^3$ e.g.). This machinery is not easy, we advise the interested Reader to consult with the papers [1], [2], [3].

For Readers, familiar with projective geometry, we indicate the main steps by the previous figures 8–11.

To the origin Q and an orthonormed vector basis $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 \in \mathbf{E}^3$ we formally introduce a new origin O "out of" $\mathbf{E}^3$ and a new basis vector $\overrightarrow{OQ} = \mathbf{e}_0$. So we formally get a new vector space $\mathbf{V}^4$ with the new basis $\{\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$.

Any point X of $\mathbf{E}^3$ will have a new $x^0 = 1$ coordinate. Thus $X(x^0 = 1, x^1, x^2, x^3)$ will be a "proper" point $X(\mathbf{x})$; $\mathbf{x} \sim c\mathbf{x}$, $0 \neq c \in R$ for $\mathcal{P}^3$; $0 < c \in R$ for $\mathcal{PS}^3$. While $u^0 = 0$, so $(u^0 = 0, u^1, u^2, u^3)$ will describe a so-called ideal point (infinite, unproper point) $U(0, u^1, u^2, u^3) = U(\mathbf{u})$ with direction vector $\mathbf{u} \sim c\mathbf{u}$ (by the above convention).

Turning back to the 3-cube in Fig. 7, we can describe the vertices A_i $(i = 0, 1, 2, 3)$, of a characteristic simplex $A_0A_1A_2A_3$ by vectors $OA_i = \mathbf{a}_i$, and also by its side faces $b^i = A_jA_kA_l$, $\{i, j, k, l\} = \{0, 1, 2, 3\}$ as above. But now $b^i(\boldsymbol{b}^i)$ is described by linear form ("normal vector") $\boldsymbol{b}^i$ of the dual space V_4, so that

$$\mathbf{a}_i \boldsymbol{b}^j = \delta_i^j \quad \text{the Kronecker symbol,} \tag{3}$$

that expresses the incidences of the corresponding vertices and faces of the simplex $A_0A_1A_2A_3$. The normal vector $\boldsymbol{b}^i$ points into the half-space of A_i.

Then the scalar products of the forms $\boldsymbol{b}^i$ $(i = 0, 1, 2, 3$ normal vectors, say inward to the simplex) is just $\langle \boldsymbol{b}^i, \boldsymbol{b}^j \rangle = b^{ij}$ by formula (1.2).

As a general criterion, describing the Euclidean regular 3−solids and so the face angle $2\beta^{23}$, is that the C-Sch matrix (b^{ij}) shall be positive semi-definite, or, of signature $(+, +, +; 0)$. In other words, as the

Main theorem sais: *The sequence of minor determinants of* (b^{ij})

$$b^{00} = 1 > 0, \qquad \begin{vmatrix} b^{00} & b^{01} \\ b^{10} & b^{11} \end{vmatrix} = \begin{vmatrix} 1 & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & 1 \end{vmatrix} = 1 - \frac{1}{2} = \frac{1}{2} > 0,$$

$$\begin{vmatrix} b^{00} & b^{01} & b^{02} \\ b^{10} & b^{11} & b^{12} \\ b^{20} & b^{21} & b^{22} \end{vmatrix} = \frac{1}{4} > 0, \tag{4}$$

but $\det(b^{ij}) = 0$*, i.e.* $+, +, +; 0$ *should be valid, as now it is indeed.*

So we get an equation for $b^{23} = \cos(\pi - \beta^{23})$. This is illustrated in Fig. 7. for the 3-cube. So $\beta^{23} = \frac{\pi}{r} = \frac{\pi}{4}$, and the 3-cube with its congruent copies tile the space $\mathbf{E}^3$.

To the other Platonic solids this criterion provides β^{23} not of form $\frac{\pi}{r}$ with natural $r \geq 3$. Therefore, the other Platonic solids are not space fillers for $\mathbf{E}^3$! In [2], [3] we characterized the spherical space $\mathbf{S}^3$ and the hyperbolic space $\mathbf{H}^3$ as well.

Regular 4-polytopes

The previous analogy leads to the concept and definition of the characteristic 4-simplex $A_0A_1A_2A_3A_4$ for the 4-cube (i.e. 8-cell), then to the C-Sch diagram and symbol $(p,q,r;s) = (4,3,3;4)$ in Fig. 7. Then the C-Sch matrix b^{ij} $(i,j = 0,1,2,3,4)$ can be constructed by the former existence criterion with the signature $(+,+,+,+;0)$. This implies $b^{34} = \frac{\sqrt{2}}{2}$, so $\beta^{34} = \frac{\pi}{s} = \frac{\pi}{4}$, and again the space filler 4-cube (Fig. 1). The other 5 regular 4-polytopes (with congruent copies) do not fill $\mathbf{E}^4$, by the angle $2\beta^{34}$ of 3-hyperfaces, obtained from $\det(b^{ij}) = 0$ and by the corresponding criterion. All these are collected in Figures 1-6. with the short C-Sch symbols.

For the "nicest" 4-polytope, the regular 120-cell $(5,3,3)$ (Fig. 5) we have pentagonal 2-faces in front of us, projected into the computer 2-screen. We have 120 regular 3-dodecahedra (with C-Sch symbol $(5,3)$), 3 are joint each other on the 3-surface [5], [2]. The illumination, by the light from an ideal centre C^∞ from $C_3^\infty C_4^\infty$, also as view-point, is indicated by shading the 2-faces, depending on the area of their projection.

The projection procedure

Figures 8-12. indicate the projection very sketchily, since our papers [1], [2], [3] discuss these procedures for $d \to p$ projection from a complementary $s = d-p+1$ dimensional centre figure. Now we apply parallel (orthogonal) projection from an ideal line $C_3^\infty(\mathbf{c}_3)\ C_4^\infty(\mathbf{c}_4)$, $d = 4$, $p = 2$, $s = 1$.

Fig. 8 indicates the projective frame work for dimension $d = 2$ where $\mathbf{V}^3$, in the affine space $\mathbf{A}^3$, is the embedding vector space, as described in Section 2.

Fig. 9 shows the $4 \to 2$ projection and the visibility problem, Fig. 10 illustrates that a 2-face does not cover a point $(\mathbf{x})$ then an edge, and it can be vice-versa. The problem of visibility can be discussed by a unified method indicated in Fig. 12.

The general scheme of $4 \to 2$ projection are sketched in Fig. 11.

Fig. 12. describes by matrices and text the general vector calculus of our method to Fig. 11. In a projective "word coordinate system" we have the basis $\{\mathbf{e}_0;\mathbf{e}_1,\mathbf{e}_2,\dots,\mathbf{e}_d\}$ with the corresponding coordinate simplex E_0, E_1^∞, E_2^∞,..., E_d^∞, think of $d = 4$. We place there a polytope with a given coordinate representation (for $d = 4$ this is given by Coxeter [5], see also [2]). Any vertex $X(\mathbf{x})$ of the polytope can be expressed by $\mathbf{x} = x^i\mathbf{e}_i$ (by Einstein sum convention; $x^0 = 1$ can be assumed by the multiplicative equivalence $\mathbf{x} \sim c\mathbf{x}$ in the projective representation).

We define the camera by the picture plane $(\mathbf{p}_0,\mathbf{p}_1,\dots,\mathbf{p}_p, p = 2)$ and by the centre figure $(\mathbf{c}_{p+1},\dots,\mathbf{c}_d, d = 4)$ as by a basis in $\mathbf{V}^{d+1}$.

Then we get the camera matrix (Cam) in Fig. 12, may be moving by the time t with fixed polytope, or vice-versa: the polytope moves and the camera is fixed in the scene of Fig. 11.

The crucial inverse matrix $(Cam)^{-1}$ provides us the image coordinates $(y^0, y^1, \dots, y^p)$ and the centre components $(c^{p+1}, \dots, c^d)$ in Fig. 12.

Relative visibility preferring points $X(\mathbf{x})$ than $X'(\mathbf{x}')$ can be defined by the criteria in Fig. 12, that can be extended to global visibility. Of course, we did not mention many details which were applied in Figs. 1–6, and make the method to be general as possible.

Concluding remarks on aesthetics of mathematics

The novelty of our result seems to be obvious (?!), and we hope that you enjoy the animation (for free download). We think that this topic has some preferable aesthetical points of view:

- the regular polyhedra and polytopes are nice in themselves by their many symmetries (see also our References);
- the theory of their classification (by H. S. M. Coxeter in [5] and [2]) is mathematically very elegant;
- the theory uses a very elegant linear algebra;
- the projective geometry, developed to describe the visual phenomena, has also a nice history and further applications in optics and in visual arts;
- the application of computer makes the procedure much easier, this attraction is based on a new world record of programming (!?) by István Prok, the very modest colleague of the presenter.

Acknowledgement

The autors, thank Dr. Jenő Szirmai and Zsolt Molnár for preparing the presentation and the manuscript, respectevely; we thank also the Organizer of this already traditional Conference, and the Editors of this Volume for their enthusiastic work.

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Három- és Négy-dimenziós Szabályos testek mozognak a számítógép képernyőjén

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Összefoglaló. Korábbi munkáinkban [1], [2], [3] a centrális vetítés általános módszerét magasabb dimenzióra is kiterjesztettük, így speciálisan 4 dimenzióról a 2-dimenziós képernyőre történő vetítésre. Mindezt a 2-dimenziós lapok láthatósági algoritmusával és a szempontból történő megvilágítást érzékeltető árnyalással is kíbővítettük. Ez a magasabb dimenziós projektív geometria segítségével történik a lineáris algebra eszközeivel, a szabályos testek Coxeter-Schläfli jellemzésével. Úgyhogy a 2-dimenziós szabályos sokszögek és a 3-dimenziós Platon-féle testek is egységesen kezelhetők a

`http://www.math.bme.hu/~prok`

honlap képeivel (ezek szabadon letölthetők).

Ebben az előadásban a téma elméleti hátterét is érzékeltetjük a hat 4-dimenziós szabályos test látványos képei mellett.

Dolgozatunkat Molnár Ernő, a Győri Révai Miklós Gimnázium tanárának emlékére, születésének 100. évfordulójára ajánljuk, aki édesapaként az előadó első igazi matematika tanára volt.

A szabályos testek elméletét a 7. ábrához kapcsolva vázoljuk. A 8–11. ábrák a vetítés elméletét érzékeltetik. Láthatjuk, hogy a számítógép fantasztikus lehetőségeket nyjt a magasabb dimenziós geometriák szemléltetésében is. Az ilyen matematikai látványosságok a matematika szépségére, a lineáris algebra és a számítógép hasznosságára hívhatják fel a tanulók és a leendő tanárok, egyetemi hallgatók figyelmét.

Kulcsszavak: Szabályos alakzatok mozgása és a matematika esztétikája

Approaches to learning mathematics in engineering study program

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Abstract. The students' approaches to learning have been significantly researched in the last few decades, particularly since Marton and Säljö in 1979. elaborated about a deep/surface approach dichotomy. The above dichotomy appears to be very useful in the assessing of teaching, based on which one can obtain parameters according to which teaching can be improved. Precisely for these purposes the term approach to learning is more suitable than the term learning style. Learning style addresses ability-like dimensions while the term approach means that person can choose to learn in different ways depending on his/her motivation, the nature of the course taken and subject-matter, as well as the host of other variables. We use The Approaches and Study Skills Inventory for Students (ASSIST), developed by N.J. Entwistle, to investigate learning approaches of the first-year undergraduate engineering students in compulsory mathematics course. We emphasize that this inventory considers an additional type of approach, the strategic one. Inventory explored which of three approaches to learning (deep, strategic or surface) was chosen the most to cope with demands of the specific mathematics course, and how the chosen approach relates to the students' grades obtained in the course. The results showed that majority choose strategic approach, what might indicate that it was the nature of the mathematics course that resulted in this approach. Also, in this paper we discuss the potential main factors that could result in such a selection.

Keywords: approaches to learning, deep, surface, strategic

Introduction

One of the main goals of education is to build effective learners. There are several variables that affect students' learning. Many research point out that the approach to learning and study skills significantly influence on the quality of student's learning (e.g. Marton & Saljo, 1976; Entwistle & Ramsden, 1983; Biggs, 1993; Entwistle, 2000; Smith& Miller, 2005; Byrne, et al., 2009). According to Biggs (2001) a term *approach to learning* is more fitted than learning style because learning style addresses ability-like dimensions while approach means that students can choose to learn in different ways depending on their motivation, the nature of the course and subject-matter, and a host of other variables.

Approach to learning is a method or a way of dealing with learning material to facilitate understanding. Approaches have a relational nature and can vary according to learning context (Trigwell & Prosser, 1991, Entwistle et al., 2002; Leung et al., 2008). Biggs (2001) pointed out that students usually choose their approach after making a "cost/benefit analysis" for the course they are enrolled into. In the field of student learning in higher education, large number of empirical research has been conducted and as well as many theories have been developed in the last two decades (e.g. Biggs, 1999; Diseth, 2003).

Theoretical background

Approaches to learning emerged from work of Marton and Säljo (1976) and Ramsden (1979). Marton and Säljo identified deep and surface approaches to learning, whereas Ramsden identified a strategic one. *A deep approach* to learning implies that students have intention to understand the material, and to be actively engaged in their studies. Students use arguments and critically relate to new material using prior knowledge and other resources. According to Entwistle (2000) they also monitor the development of their own understanding and learning which presents an internal process to them. In mathematics, this means that learner is making connections between mathematical topics, concepts and procedures, and is aware of relationships between them. In contrast, students who adopt *a surface approach* tend to memorize the material without understanding. They rely on reproduction of the learning material and use different forms of rote learning. Mainly, they are limited by the specific learning task and do not go beyond it. In this approach, fear of failure and concern with the completion of a course is main guide through the learning process. Unlike surface approach, where usually a low level of understanding is achieved and learning is ineffective in a long run, deep approach is more likely to result in a high level of understanding and effective learning (Entwistle & Ramsden, 1983). Students who use the third approach, called a *strategic approach*, have no distinct learning strategy. In this approach, students use both deep and surface approaches (but not at the same time) when they found it appropriate and have a competitive motivation. The major intention is to achieve the highest grades managing study methods and time effectively. Strategic approach also involves monitoring one's study effectiveness and alertness (Entwistle et al., 2003).

Approaches to learning can be related to the assessment of student learning (e.g. grade), and this relation has been well established (Betoret & Artiga, 2011; Biggs et al., 2001; Byrne et al., 2004; Trigwell & Prosser, 1991). A deep approach is related to high quality of student learning, whereas surface learning is related to the poor learning outcomes (Biggs et. al., 2001; Trigwell & Prosser, 1991). Other findings are related to students' perceptions of the teaching–learning environment.

In a well-planned teaching–learning environment with clear goals, good quality teaching, appropriate assessment and workload, students tend to choose a deep approach to learning. In the teaching–learning environment with a poor quality of teaching together with assessment focused on memorizing and very high workload, students tend to choose a surface learning approach (e.g. Trigwell & Prosser, 1991)

When it comes to mathematics as a subject discipline, investigating how students view a nature of mathematics (fragmented vs. cohesive), Crawford et al. (1998) discovered that fragmented conceptions of mathematics were associated with surface learning approaches. Perceptions of workload and assessment were seen as inappropriate where the workload was too high and the assessment was focused on memorizing. On the other hand, the cohesive conceptions of mathematics, as well as perceptions of clear goals and good teaching were associated with deep learning approaches.

Methodology

The Approaches and Study Skills Inventory for Students (ASSIST) (Tait, et al., 1998) was used for collecting quantitative data relating to students' approaches to learning mathematics. The questionnaire contains 67 statements, where respondents indicate their agreement with each statement, using a five point Likert scale. ASSIST consists of four sections. The first section is a six-item measurement of the student's own conception of what the term "learning" means to them. The second section consists of 52 statements related to mainly three dimensions – deep, strategic, and surface. As mentioned above, every dimension has a subscale. Every approach has four or five subscales comprised of four items. The third section of ASSIST is an eight-item questionnaire measuring preferences for different types of teaching – lectures, courses, exams and books. In the fourth section, the students are asked what they think regarding their overall performance.

Participants were the first year engineering students enrolled in the compulsory course Mathematics 1. The sample comprised 2/3 of the cohort (69) and students were reached through direct contact in exercise lessons of Mathematics 1 where they were given paper copies of the questionnaire. Course Mathematics 1 is a one-semester course and students were surveyed almost at the end of the semester, before final colloquiums and exams. Participating in the survey was voluntary so no penalties were given for those who refused to take a part. Besides filling the questionnaire, the students were asked to leave the personal data to be able to track their scores in colloquiums and their final grades. All students who were present at the exercise lessons took part in questionnaire. The students who were not present at the time usually did not attend lessons so we were not in position to reach them.

Reliability and validity

ASSIST was translated into Croatian language with the great care, but we adapted some statements to fit better to mathematics environment. We performed confirmatory factor analysis to ensure that this translation of the instrument into

Croatian was successful and to examine the factor structure of the original inventory based on data obtained from the Croatian students. The goodness of fit of the confirmatory factor structure was assessed by the following fit indices: RMSEA (Root Mean Square Error of Approximation) GFI, and AGFI.

In Croatian version of the ASSIST, Cronbach's Alpha values ranged from 0.81 to 0.87 (whole questionnaire and deep, surface, strategic subscales) which could be considered as a high internal consistency. In the case of deep subscale, indices indicated very god fit of the data (RMSEA = 0.04, GFI = 0.98, AGFI = 0.97), and acceptable fit in the cases of the whole questionnaire (RMSEA = 0.08, GFI = 0.93, AGFI = 0.90), strategic subscale (RMSEA = 0.08, GFI = 0.88, AGFI = 0.86) and surface subscale (RMSEA = 0.11, GFI = 0.86, AGFI = 0.82).

Results

Analysis of the ASSIST revealed that a strategic approach to learning was the most commonly adopted by the participants, with 58% scoring most highly on this scale. This was followed by 29% scoring highest on the deep scale and 13% on surface. Average scores on each scale were 61.15 (SD = 14.59) on strategic, 56.39 (SD = 10.74) on deep and 49.55 (SD = 11.07) on surface approach to learning.

At the level of $\alpha = 0.05$ there were no statistically significant differences between mean scores of students using deep approach and strategic approach, while there were statistically significant differences between deep approach and surface approach, and between strategic approach and surface approach.

Grades obtained in the course range between 1 and 5, with 2 as minimum passing grade while 1 means that student did not pass the course. Looking at the correlations between grades and approaches to learning (Table 1), a significant negative correlations were found between surface approach and grades, positive correlations were found for strategic approach and negative or no correlation were found for deep approach.

Table 1. Correlation between grades and approaches to learning.

Approach	Grades in lecture colloquiums (theory)	Grades in exercise colloquiums (tasks)	Final grade
Deep	−0.249	−0.065	−0.046
Surface	−0.485	−0.448	−0.437
Strategic	0.142	0.197	0.275

Learning, conceptualised as a reproducing knowledge, is represented by three items in the questionnaire (see Table 2, bold). Surface approach to learning was negatively correlated to all of these items. Further, strategic and deep approach scores had positive correlations to some of these items. Learning, conceptualised as transformational, is represented by three items in the questionnaire (see Table 2, normal). Surface approach to learning had either no correlation or was negatively correlated to all of these items. Strategic and deep approach scores correlated significantly with the conceptualisation of learning as a transformational process that facilitates development as a person. Also, deep approach scores correlated significantly with the last item that describes learning as a new way of seeing things.

Table 2. Correlation between conceptions of learning and approaches to learning.

Items	Deep	Surface	Strategic
Making sure you remember things well	0.237	−0.074	0.228
Developing as a person	0.389*	0.065	0.611*
Building up knowledge by acquiring facts and information	0.238	−0.170	0.206
Being able to use information you've acquired	0.050	−0.118	−0.135
Understanding new material for yourself	0.059	−0.246	0.085
Seeing things in a different and meaningful way	0.366*	−0.033	0.001

*significant at 0.05

Third part of the questionnaire examined preferences for different types of course and teaching. There are four items that are related to a deep approach and support understanding (see Table 3, normal) and four items that are related to a surface approach and promote transmitting information (see Table 3, bold).

Table 3. Correlation between preferences for different types of courses and teaching and approaches to learning.

Items	Deep approach	Surface approach	Strategic approach
Lecturers who tell me exactly what to put down in our notes.	−0.042	0.144	0.101
Lecturers who encourage me to think for myself and show us how they themselves think.	0.367*	−0.179	0.103
Exams that allow me to show what I've thought about the course material for myself.	0.019	−0.365*	0.210
Exams or tests that need only the material provided in the lecture notes transmitting info.	−0.062	0.099	−0.007
Courses in which it's made very clear just what books we have to read.	−0.192	0.263	0.295
Courses where we are encouraged to read around the subject for ourselves.	0.364*	−0.016	0.516*
Books that challenge me and provide explanations that go beyond the lecture.	0.028	−0.097	0.135
Books that give me definitive facts and explanations that can easily be learned.	−0.199	0.536*	0.277

*significant at 0.05

Looking at the total number of students, results showed that 92% of them have preference for the surface approach and only 8% for the deep approach. We have found positive correlation with students' surface approach to learning scores and courses and teaching strategies based on the transmission of information, and moreover, a significant positive correlation with item "Books that give me definitive facts and explanations that can easily be learned." In the case of strategic approach scores, correlation was mostly positive but not significant. Students' deep approach to learning scores correlated negatively with those items.

Teaching and course types that support understanding correlated positively to deep and strategic approaches to learning, and in some cases are correlated significantly (Table 3). Furthermore, a significant negative correlation was found between the surface approach and item "Exams that allow me to show what I've thought about the course material for myself."

Discussion and conclusion

Students who enroll into a technical study programs at university usually have mathematics courses as compulsory ones. According to SEFI, these courses present a necessary requirement for the education of qualified engineering graduates and most of these courses are taken in the first years of studying. The role of these courses should be a service one, but in many cases such courses appear to be the eliminating ones, differentiating successful from non-successful students.

Students surveyed in this study were the first year engineering students, who were given a questionnaire almost at the end of the first semester. We believe that the questionnaire was given in a good time because students participated in colloquiums from other courses more related to their study program, and had opportunity to adapt to a new concept of mathematics that is different from the high school mathematics, and where mathematics theory was highly emphasized. Small number of studies has used the ASSIST to investigate approaches to learning in mathematics. Since students can employ different approaches for learning for different courses, depending on whether the course is more related to their future profession or not, our study enriches the corpus of research in mathematics education. Majority of students have chosen the strategic approach to learning. Similar results for mathematics can be found in Darlington (2011), who investigated first year mathematics students. Many studies (e.g. Speth et al., 2007) reported similar results for other subject disciplines.

Significant negative correlations that we have found between surface approach and grades (exercise and lecture colloquiums, and, consequently, final grade), have also been discovered in other studies (e.g. Entwistle & Ramsden, 1983). Similar holds for positive correlations between strategic approach and grades (Entwistle & Ramsden, 1983; Byrne et al., 2002), even though we have found positive correlation, but not the significant one. Absence of correlation with deep approach and grades is a concern, and this has also been reported by other researchers (Byrne, et al., 2004). The possible explanation for these results could lie in the examination style that does not asses what the examiner believes it should asses. It is possible that examination of knowledge is structured in a way where strategic approach is encouraged, meaning that students using lecturers and teaching assistants hints and remarks can improve their performance, but not their understanding. This is also in line with Entwistle's comment that in fact many study programs in higher education are promoting a strategic approach when they are using summative assessments. That means that students are combining deep and surface approach in order to achieve the best possible grades, organizing their learning time in the effective way.

Combining these results with the first and the third part of the questionnaire gives better overview of students' approaches to learning. The results concerning conceptions of learning and approaches to learning might suggest that engineering students who choose surface approach do not consider learning of mathematics as a transformational process nor as a pure reproduction of learned facts. Also, findings suggest that some students who adopt deep or strategic approach for learning mathematics might conceptualize learning as a reproduction of facts, at least in some parts. This may be connected with the rote-learning of a definition in order to be able to understand the meaning or application of a theorem or procedure.

It is interesting to discover that many students have preference for courses that promote surface approach to learning. This can be related to the structure of the mathematics course but also with other courses that students were taking in the first semester. Such preference can indicate overloaded mathematics course syllabi and overburden students who are coping with many different courses at the same time.

Given these results, it seems that in our case engineering students estimated that deep approach is unrewarded, but also that surface approach is not the best way to achieve the success in mathematics course. Biggs (1991) and many others report similar findings in other subject disciplines. Although we would prefer that engineering students choose the deep approach toward learning mathematics, we believe that good strategic approach should be developed, that has a potential to later outgrow into the deep approach. This seems more realistic and usable in the context of such study program, where mathematics is service course but is highly important as a base for further studying.

Cano & Berben (2009) discovered a pattern between achievement goals and approaches to learning in mathematics what suggests that they can be intertwined. In order to fully understand engineering students' motives for choosing certain approach to learning in mathematics courses, we should certainly broaden our research in this direction.

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Pristupi učenju matematike na tehničkom studiju

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Sažetak. Pristupi učenju značajno su istraživani u posljednjih nekoliko desetljeća, osobito nakon što su 1976. Marton i Säljö razradili dihotomiju pristupa učenju podijelivši ga na dubinski i površinski pristup. Navedena dihotomija se pokazuje vrlo korisnom pri procjenjivanju nastave, na osnovu čega se dobivaju parametri prema kojima se nastava može unaprijediti. Upravo za te potrebe je pojam pristupa učenju pogodniji od pojma stila učenja. Stil učenja se odnosi na nečije sposobnosti, dok pristup učenju označava da studenti mogu izabrati različite načine učenja, ovisno o njihovoj motivaciji, sadržaju i prirodi upisanog kolegija te mnogim drugim varijablama. Pri istraživanju pristupa učenja studenata prve godine jednog tehničkog studija na obaveznom kolegiju iz matematike, koristili smo Upitnik o pristupu i vještinama učenja (ASSIST), koji koristi još jednu dodatnu vrstu pristupa (strateški pristup) te kojeg je izradio N. J. Entwistle. Navedenim upitnikom smo istražili koji je od tri pristupa učenju (dubinski, površinski ili strateški) najviše odabiran kako bi se studenti nosili sa zahtjevima kolegija, te su proučeni odnos pristupa učenju i ocjene postignute na kraju kolegija. Rezultati su pokazali da je većina studenata odabrala strateški pristup, što bi moglo ukazivati da je i priroda proučavanog kolegija rezultirala ovim odabirom. Također, u radu diskutiramo i potencijalne glavne čimbenike koji su mogli za rezultat imati takav odabir.

Ključne riječi: pristup učenju, dubinski, površinski, strateški

University students' understanding of graphs in mathematics, physics and other contexts

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Abstract. Scientific data are very often presented through graphs which allow the skilled user to quickly extract information on the data presented. In physics especially, the analysis of data, such as dependence of variables appearing in interpretation of physical phenomena presented by a graph, relies heavily on interpretation of graph slopes and areas under the graphs. Students meet these concepts as purely mathematical concept in mathematics education but also within their application in different contexts and disciplines (e.g. physics). The physics curriculum often expects that understanding of these concepts is well developed in mathematical courses, whereas physics teachers report on students' lack of mathematical knowledge and the ability to apply their knowledge in physics ([3]). This is the problem of a transfer of learning which is concerned with the student's ability to apply what has been learned in one context to a new context. It is one of the biggest challenges in education ([2]), especially valid for mathematical knowledge which is considered as a prerequisite for science education. It is not only the problem of recognizing mathematical concepts in different contexts, but also of recognizing problems common to mathematics and other disciplines and problems specific to the specific discipline. In physics, for example, problems stripped off their physical context "often involve representations that deviate from those typically used in the mathematics domain" ("physicsless physics questions", [1]).

In the study presented, our aim was to investigate whether and how students recognize mathematics to be used in contexts problems which involve analysis of graph slopes and areas under the graphs. We summarize our findings on understanding of these concepts across three different domains, mathematics, physics (kinematics) and context other than physics (economy, biology, everyday life). The last domain did not require any additional context-dependent knowledge. We have collected data on 385 first year students of mathematics and physics on the test developed by authors consisting of eight set of parallel questions. Questions were parallel in the sense that the (final) required mathematical procedure was the same in each set of three items. However, whereas the problem of areas under the graphs in mathematics domain was concerned with numeric calculation of an area, for the context other than physics it involved the interpretation of an area under the graph as a cumulative growth of a quantity. Such an interpretation lies in the fundamentals of concept of integration ([4], [5]). The analysis of results of our study suggests that the concept of a graph slope is equally difficult in all three areas, whereas the difficulty of the concept of an area under graph differs across domains. Furthermore, the findings suggest that students' mathematical knowledge is not the only important factor for student's success in solving graph problems in physics or other contexts.

Keywords: graphs, mathematics, physics, slope, area

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Studentsko razumijevanje grafova u matematici, fizici i drugim kontekstima

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Sažetak. Znanstveni se podaci vrlo često prikazuju grafovima koji omogućuju vještom korisniku brzo očitavanje informacija o prikazanim veličinama. Posebno u fizici, analiza ovisnosti veličina koje se pojavljuju u interpretaciji fizikalnih pojava i prikazane su grafom, često se oslanja na interpretaciju nagiba grafa i površine ispod grafa. S tim se pojmovima, kao sasvim matematičkim pojmovima, učenici i studenti sreću u matematičkom obrazovanju, no sreću se i unutar njihove primjene u različitim kontekstima i disciplinama (npr. u fizici). Kroz nastavni plan i program fizike često se očekuje da je razumijevanje tih pojmova dobro usvojeno u matematičkom obrazovanju, dok nastavnici fizike svjedoče o nedostatnom matematičkom znanju svojih učenika i studenata, posebno kod primjene tih znanja u fizikalnom kontekstu ([3]). Problem prijenosa znanja upravo se bavi proučavanjem sposobnosti učenika kako primijeniti ono što su naučili u jednom kontekstu u neki novi kontekst. To je jedan od najvećih izazova u obrazovanju ([2]), a osobito se tiče matematičkog znanja za koje se smatra da je preduvjet za obrazovanje u prirodoslovlju. Problem nije samo u tome kako prepoznati matematičke pojmove i koncepte u različitim kontekstima, nego i utvrditi probleme koji su zajednički matematici i drugim disciplinama, odnosno probleme koji su specifični za pojedinu disciplinu. Na primjer, u fizici problemi koji su "oslobođeni" svog fizikalnog konteksta često uključuju prikaze koji nisu tipični za matematičku domenu ("fizikalna pitanja bez fizike", [1]).

U provedenom istraživanju, naš je cilj bio proučiti da li i kako studenti prepoznaju matematičke pojmove kao što su nagib grafa i površina ispod grafa u kontekstualnim problemima. Prezentirat ćemo sažetak rezultata o razumijevanju tih pojmova u tri različite domene, u matematici, fizici (kinematici) i kontekstima različitim od fizike (ekonomija, biologija i stvarni život). Zahtjevi u posljednjoj domeni nisu pretpostavljali nikakva dodatna znanja o kontekstu. Dobiveni su odgovori 385 studenata matematike i fizike prve godine studija na testu kojeg su sastavile autorice, a koji se sastojao od 8 grupa paralelnih pitanja. Pitanja su bila paralelna u smislu da je (konačni) matematički postupak potreban za rješavanje problema bio isti u svakoj grupi od tri pitanja. Međutim, na primjer, dok se problem površine ispod grafa u matematičkoj domeni sastojao u numeričkom izračunu zadane površine, u domeni konteksta različitog od fizike uključivao je interpretaciju površine ispod grafa kao kumulativnog rasta neke veličine. Takva interpretacija leži u samim temeljima koncepta integriranja ([4], [5]). Analiza rezultata sugerira da je pojam nagiba grafa jednake težine u sve tri domene, dok težina pojma površina ispod grafa varira po domenama. Također, rezultati sugeriraju da matematičko znanje nije jedini važni faktor uspjeha studenata u rješavanju problema s grafovima u fizici i drugim kontekstima.

Ključne riječi: grafovi, matematika, fizika, nagib, površina

The sine and tangent function – concept images of pre-service mathematics teachers

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Abstract. According to the actual Croatian high school mathematics curriculum, trigonometric functions of a real argument are introduced and taught in the 3rd grade of grammar and of four-year technical high schools and assessed at the national level at the higher level of State Matura after the completion of fourth grade. Results show that students' performance on tasks involving this mathematical content is not satisfactory although a significant number of lesson periods has been allocated for its comprehension. Considering that teacher quality is an important school-based factor affecting students' achievements, we have conducted a survey investigating the basic concepts and procedures related to trigonometric sine and tangent functions among the university mathematics education students in Croatia. We have acquired data over entire populations of the 3rd year and of the 5th year students of mathematics education programmes at the largest Croatian university on a questionnaire with open-ended questions and on related semi-structure interviews. Pre-service teachers' responses were then analyzed against the theoretical framework of concept image vs. concept definition (Tall & Vinner, 1981). The obtained results suggest that the right triangle trigonometry and degree measure make significant part of their concept images of sine function, whereas the tangent function is seen dominantly only algebraically, that is, as a ratio of sine and cosine, without referring to its geometric interpretation in circle trigonometry. This finding is most evidently seen in pre-service teachers' approaches to solving simple trigonometrice quations and inequalities, such as $\sin x = -0.5$, $\tan x > -1$ or $\sin x < \cos x$. Moreover, the evidence gained shows that such tasks are prevalently solved by employing procedural knowledge rather than conceptual. Some of these findings on Croatian data set confirm and further extend earlier results in Fi, 2006, and Topçu, Akkoç, Yılmaz & Önder, 2006.

Keywords: Mathematics education, trigonometric functions, concept image

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Funkcije sinus i tangens – slike koncepata kod studenata nastavničkih smjerova matematike

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Sažetak. U skladu s važećim nastavnim planom i programom matematike za srednje škole u Hrvatskoj, trigonometrijske funkcije realnog argumenta uvode se i poučavaju u trećem razredu gimnazija i srednjih tehničkih škola, a ispituju na nacionalnoj razini po završetku četvrtog razreda, na višoj razini Državne mature. Učenički rezultati na zadacima vezanima uz ovaj nastavni sadržaj nisu zadovoljavajući iako je njegovom svladavanju posvećen veliki broj nastavnih sati. Imajući u vidu da je kvaliteta nastavnika važan školski faktor utjecaja na učenička postignuća, provele smo istraživanje među studentima nastavničkih studija matematike u Hrvatskoj o osnovnim konceptima i procedurama vezanima uz trigonometrijske funkcije sinus i tangens. Podaci o cijelim populacijama studenata treće i pete godine nastavničkih studija matematike na najvećem hrvatskom sveučilištu prikupljeni su putem upitnika s pitanjima otvorenog tipa te na njima zasnovanima polustrukturiranim intervjuima. Odgovori budućih nastavnika potom su analizirani unutar teorije odnosa slike i definicije koncepta (Tall & Vinner, 1981). Dobiveni rezultati upućuju na značajnu zastupljenost trigonometrije pravokutnog trokuta i stupanjske mjere u studentskoj slici koncepta funkcije sinus, dok funkciju tangens studenti doživljavaju dominantno algebarski, kao omjer sinusa i kosinusa, bez povezivanja s njenom geometrijskom interpretacijom na trigonometrijskoj kružnici. Ovaj je nalaz najočitiji u pristupima budućih nastavnika rješavanju jednostavnih trigonometrijskih jednadžbi i nejednadžbi, kao npr. $\sin x = -0.5$, $\tan x > -1$ ili $\sin x < \cos x$. Prikupljeni dokazi također pokazuju da se ovakvi zadaci pretežno rješavaju primjenom proceduralnog znanja, a ne konceptualnoga. Neki od ovih nalaza na hrvatskim podacima potvrđuju i proširuju ranije rezultate iz Fi, 2006, te iz Topçu, Akkoç, Yılmaz & Önder, 2006.

Ključne riječi: matematičko obrazovanje, trigonometrijske funkcije, slika koncepta

An approach to Operations Research course in the curriculum for computer science students

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Abstract. Operations Research is a compulsory course taught in the winter semester at the Department of Informatics, University of Rijeka. The profile of students that are enrolled into this course varies. It includes single major computer science students (a teacher training module, a module Information and Communication Systems and a module Business Informatics), and double major students of computer science in combination with History, English Language, German Language and other combinations. The problem which we often face with is different level of pre-knowledge of Mathematics for different groups of students. Based on the results of the past surveies and the student's attitude, both single major and double major students of Informatics lack in motivation to study Mathematics courses which are offered at undergraduate and graduate levels of studies. These are the courses in which students get lower marks.

According to the research results presented in the literature, it has been concluded that there are factors on which professors can influence in order to improve the students' performance in the course. One of such is a level of complexity of the presented materials. Guided by the fact that holding lectures in teaching Mathematics courses is not sufficient for students to gain knowledge, we decided to develop a set of activities by which we tried to overcome the lack of pre-knowledge and explain complex and abstract concepts which are the basis of Operations Research in a more approachable manner. While teaching this course we used the ICT which is common to the computer science students.

In this article we will present our approach of designing and teaching the course Operations Research with the help of the ICT and compare results of a student pass rate for the previous three academic years.

Keywords: ICT, computer science students, Operations Research, instruction design, online activities

Introduction

ICT has become an important part of daily life, and it has also found its use in the education. In teaching Mathematics, technology brought many changes. According to the researches (Rahman, Ghazali, Ismail, 2003), ICT supports and enhances the problem solving ability and changes the approach to mathematical problems. The use of computer programs in performing iterative procedures shortens the time of exhausting arithmetic calculations which are normally done by hand (Ittigson, Zeve, 2003) and enables the students to focus on the analysis of results, construction of a mathematical model and other activities that belong to the higher cognitive levels. Moreover, ICT enhances understanding of basic mathematical concepts (Keong, Horani, Daniel, 2005), for example by drawing and displaying a graphical representation of abstract concepts. Along with available software packages, there are various online learning management systems (LMS) that allow creation of digital educational materials, solving online quizzes for self-evaluation, conducting online tests with multimedia elements, watching video lectures, etc. With the appearance of the Web 2.0 tools and new Internet technologies, such as HTML5, more and more easy-to-use widgets become available for inclusion in the learning system.

In this article we will present an approach of designing and teaching the course Operations Research with the help of the ICT and present the results of the survey in which we examined in which measure the designed activities helped students to acquire knowledge.

Operations Research at the informatics studies

Operations Research is an interdisciplinary field whose task is to solve scientific and economic problems with quantitative methods (Müller-Merbach, 1973). Most of the practical problems are optimization problems. Therefore, it is necessary to familiarize students with the mathematical background in optimization (Ramirez i sur., 2004). At the University of Rijeka, operations research is included in study programs of the Department of Informatics University of Rijeka, Faculty of Civil Engineering, Faculty of Economics, Faculty of Maritime Studies, Faculty of Engineering and Faculty of Tourism and Hospitality Management.

At the Department of Informatics, University of Rijeka, operations research problems are taught in the course Operations Research 1 (equivalent to the course Operations Research) and Operations Research 2, both being taught on the first year of graduate studies in the winter and summer semester, respectively.

After completing the course Operations Research 1, students will be able to:

- Differ and explain basic concepts of the operations research
- Define problems from the aspect of the operations research

- Construct a mathematical model for the linear programming problems
- Solve linear programming problems with a graphic metod and a simplex method
- Analyse the optimal solution
- Solve linear programming problems in the GNU Octave programming language
- Solve sensitivity analysis problems in the GNU Octave programming language and interprete the given solution
- Differ and explain basic concepts of transportation problem
- Construct mathematical models and solve closed and open transportation problems.

Table 1. Operations Research in the Informatics studies.

Faculty/Department	**Studies**	**Course title**	**ECTS**	**Course topics**
Department of Informatics, University of Rijeka	*SM (Business Informatics module)	Operations Research 1	6	Construction of a mathematical model for LP** problems, graphic method for solving LP problems, simplex method of minimum and maximum, duality, degeneracy, postoptimal analysis (sensitivity analysis), transportation problems.
	SM (Information and Communication Systems module)		6	
	SM (Teacher Training)		5	
Faculty of Humanities and Social Sciences	*DM	Operations Research	5	
Department of Informatics, University of Rijeka	SM (Business Informatics module)	Operations Research 2	6	Scheduling problems, fractional programming, decision trees, integer programming, graph theory, maximal flow problems, shortest path problem, queuing theory.
	SM (Information and Communication Systems module)		6	
	SM (Teacher Training)		5	

*Abbreviations SM and DM are used for single major Informatics studies, and double major Informatics studies in combination with history, philosophy, pedagogy, history of arts, English language, German language, Croatian language and Italian language and literature respectively.

**Abbreviation LP is used for linear programming.

The results of the course pass rate for the previous three academic years indicated that the average GPA of a student is slightly decreasing (Figure 1). Although the number of students that got a non-satisfactory final mark for the course did not change in the previous three years, the general GPA of the students did. The weakest results were achieved in the computer lab classes, and in the academic year 2011/2012 in the theoretical exam.

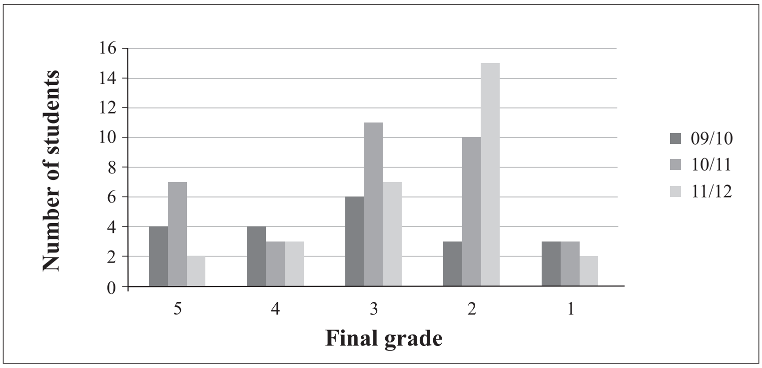

Figure 1. Comparison of the students' final grades for the course.

1. *A new approach to designing a course and teaching experiences*

For the academic year 2012/2013, some changes were introduced in the course Operations Research 1.

According to the researches (Mahmud, Saliman, 2012), there are factors by which teachers can influence on the students' performances. In the instruction design approach for the course Operations Research 1 for the academic year 2012/2013, the following factors were taken into consideration:

A problem of the limited time of lectures. According to Moazeni (2012), in teaching mathematics courses, lectures by its self are not sufficient for the students to acquire knowledge. Bearing this in mind, we designed a set of online activities for the students which were performed at home and whose aim was to help students to successfully prepare for the written exams. A high number of 63,8% of enrolled students accessed and completed these activities for extra points.

Competition among students. Individual work and competition for higher achievement is one of the student habits (Blumenfeld et al., 1996). We wanted to use this habit in order to achieve a more frequent student activity and continuous coursework. For this purpose, we designed web quest activities for extra points. In order to encourage the students to approach the task in the shortest time possible, only the first student who found the correct answer was awarded with an extra point. Apart from encouraging competition, interaction among students and teacher and students was encouraged during the lectures. Students were also encouraged to create a learning community and help each other during studying and preparation for the test. Apart from the interaction in the classroom, students communicated with teachers by sending them e-mail and private messages in the learning management system Instructure Canvas, as well as meeting with the teachers in person during the office hours.

The level of knowledge and skills which students have to achieve in order to pass the final test. As a prerequisite for the test, students had to solve two homework assignments and an online self-assessment quiz. One week before the test, an optional class was held for those who wanted to repeat the taught algorithms and procedures in the presence of a teacher and solve mathematical exercises similar to those in the forthcoming test. Students of the Department of Informatics, University of Rijeka, are used to the mathematical theoretical tests in which they are asked to reproduce mathematical definitions and proofs. In order to avoid learning of definitions by heart and without understanding, from the academic year 2012/2013 the format of the test was changed. Students no longer had to reproduce definitions, but make short and precise conclusions based on the mathematical proof. Due to the disadvantages of the Bloom's taxonomy for the mathematics courses (Divjak, Ostroški, 2009; Rizvi, 2007), we decided to use Smith's MATH taxonomy (Smith et al., 1996) in order to analyze the amount of questions for each level of knowledge (Table 2).

Table 2. Number of questions for each level of knowledge according to the MATH taxonomy.

A test type	**Level of knowledge according to the MATH taxonomy**	**Number of questions**	**A percentage of points**
Written test (theoretical)	A1	3	10%
	A2	2	25%
	A3	0	0%
	B1	0	0%
	B2	3	30%
	C1	1	10%
	C2	0	0%
	C3	3	25%

Level of complexity and quantity of presented materials. The materials for the course were uploaded to the system Instructure Canvas, an online learning management system based on the cloud technology. The course units were divided into shorter lessons and organized into separate topic blocks in the system. Apart from the textual content presented in the theoretical and practical classes, each block contained at least one activity for extra points, a quiz for self-assessment, a PDF lesson and a PowerPoint presentation that was presented in a f2f class, as shown in Figure 2.

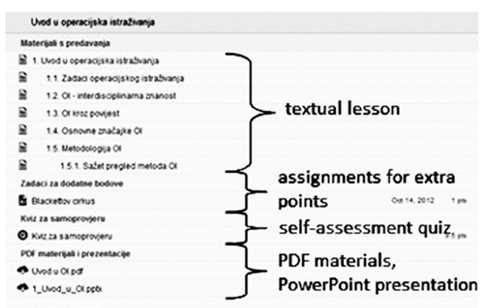

Figure 2. A topic block for the lesson "Introduction to Operations Research" in the system Instructure Canvas.

In order to bring the operations research topics closer to the students, they were given everyday examples. Finding such examples was relatively easy due to the interdisciplinary nature of operations research. In their researches, Cochran (2004), Thanasis, Konstantinos and Sifaleras (2005) list practical examples which helped raise the level of students' motivation and improve understanding of abstract mathematical concepts. Since students of different studies were enrolled in the course, a set of motivational examples was designed in order to promote interaction with each student group. For example, for the students of double major studies in History and Informatics, a picture of the stock market crash in the time of the Great Depression was shown. The target group of students recognized the history event and explained to the whole class the economic situation of the period. The class was continued with an introduction to a Stigler's diet problem in which he tackled the question of the most cost-effective choice of the nutrients. The flow of the activities for this class is presented in the Figure 3.

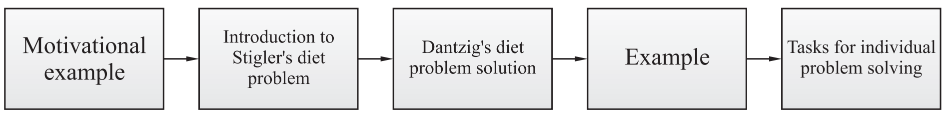

Figure 3. The flow of the activities used while teaching the minimization problem.

In addition to real life examples, students were also motivated with examples from the popular culture, such as an excerpt from a movie "Good Will Hunting".

For the students of the modules Business informatics and Information and Communication Systems, computer lab classes were organized in which students had an opportunity to get familiar with computer linear problem solving in the high level programming language GNU Octave which is used for mathematical calculations. In the past two years a significant decrease in computer lab assignment results was noticed which pointed to a necessity of changes. The attempt to improve these results was achieved by redefining the purpose and the outcomes of the laboratory classes (Table 3).

Table 3. A comparison of the learning outcomes for the lab assignment for the academic years 2011/2012 and 2012/2013.

Outcomes for the academic year 2011/2012.	**Outcomes for the academic year 2012/2013.**
Student will be able tosolve a postoptimal analysis problem using the software LPSolve.	Student will be able to solve LP problems using a programming language GNU Octave, solve a postoptimal analysis problem using a programming language GNU Octave and interprete the given solution.

Research Methods

During the semester, we conducted a survey with a goal to examine in which measure the designed activities helped students learn the content presented in the course. The survey was consisted of 22 questions and was conducted on the sample of the students enrolled in the Operations Research course in the academic year 2012/2013. Out of 41 enrolled students, 30 students have completed the survey (73%). The survey was organized into three parts. In the first part, the examinees were asked to express their attitude towards the mathematics courses taught in the Informatics studies (likert scale). The second part of the survey referred to the student's experience in the course Operations Research 1, which included partly a likert scale and partly multiple choice questions. The last part of the survey allowed students to enter additional comments about the course Operations Research and its activities.

Results

The results of the survey revealed that 58% of the examinees recognized the purpose of mathematics courses in the Informatics studies. Out of the same group, 47% consider that their mathematical pre-knowledge has an influence on their final course grade, 41% consider that one's effort has the most influence on the final course grade, while 12% think that pre-knowledge does not have any influence on the final grade.

In total 64% of the examinees said that they completely or mostly recognize the use of operations research in everyday life. In accordance with defined learning outcomes, students learn simplex method which is consisted of a set of iterative steps that are easy to master. On the other hand, abstract mathematical concepts require a higher cognitive effort from students (Mahmud, Salimian, 2012). According to the survey results, 30% of the students had difficulties in mastering exactly these concepts (Figure 4).

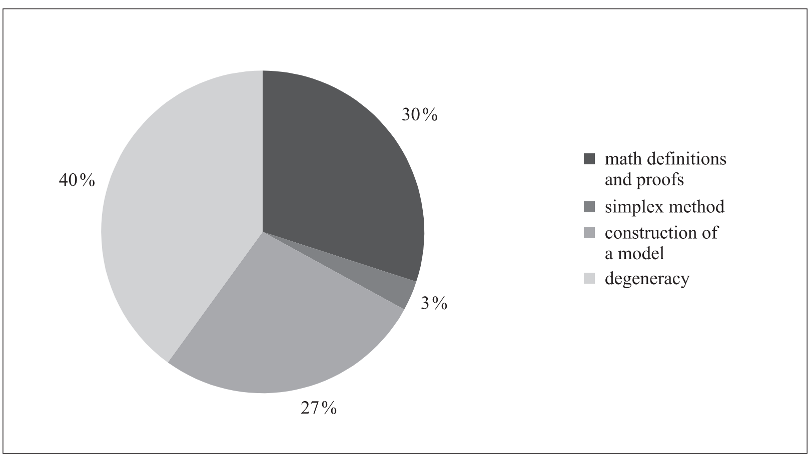

Figure 4. Course content that student found hardest to learn.

Although the use of technology in teaching brings many advantages, in addition to online course materials in the system Instructure Canvas, students found solving the problems on the blackboard and the homework assignments as the most helpful course elements. Examinees could also additionally add another course element, which was done by 23% of the students. Each of the additional answers was: “teachers’ methods of teaching” (Figure 5).

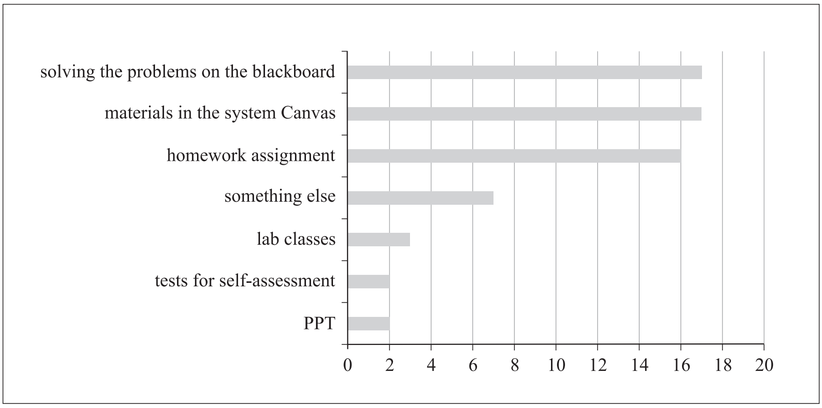

Figure 5. A histogram of the elements of the course that students found most helpful in learning.

In total, 50% of the examinees liked the new format of the test, with only 10% of the students expressing the negative attitude. Computer lab classes were successful, with an average of 6,48 achieved points (out of 10 possible). The results of the survey have indicated that 50% of the students wants to get to know additional software for solving linear programming problems, while 63,3% would like to have more computer lab classes.

Conclusion

Although the ICT brings many advantages, the quality of mathematics courses do not depend entirely upon the technology. Taking into consideration the factors on which the teachers can influence to improve students’ performance and in order to achieve the learning outcomes, we designed a set of activities to explain complex and abstract mathematical concepts of the operations research in a more understandable manner.

The new approach showed its positive sides. Students were satisfied with the new form of a written test. The results of the computer lab class assignments were significantly better than the ones in the previous two academic years. In the academic year 2012/2013, the learning management system used for the course became more than just a repository of the course materials. The designed online activities for extra points encouraged the students to work continuously throughout the semester and helped them in achieving better test results.

The survey results revealed new guidelines for a further improvement of the course Operations Research. There is a greater need for practical computer problem solving tasks, more computer lab classes and in general more computer work at the graduate level in the Informatics studies, along with further changes in the Operations Research teaching plan and program.

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Jedan pristup kolegiju Operacijska istraživanja u programima informatičara

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Sažetak. Operacijska istraživanja obavezan je kolegij koji se održava u zimskom semestru na Odjelu za informatiku, Sveučilište u Rijeci. Profil studenata koji upisuje ovaj kolegij je raznolik, a uključuje studente jednopredmetne informatike (nastavnički smjer, modul informacijski i komunikacijski sustavi, modul poslovna informatika), te studente na dvopredmetnom studiju informatike u kombinaciji s povijesti, engleskim jezikom, njemačkim jezikom i ostalim studijskim kombinacijama. Problem s kojim se često suočavamo je različita razina predznanja iz matematike za različite grupe studenata. Prema rezultatima dosadašnjih anketa, ali i stavu studenata, studentima jednopredmetne i dvopredmetne informatike općenito nedostaje motivacije za učenje matematičkih predmeta koji se nude na preddiplomskoj i diplomskoj razini studija. Sukladno tome, upravo iz ovih predmeta studenti imaju slabije ocjene.

Prema provedenim istraživanjima koja su opisana u literaturi utvrđeno je da postoje faktori na koje profesori mogu utjecati kako bi poboljšali rad studenta na kolegiju, a jedan od njih je razina složenosti prezentiranih materijala. Vodeći se činjenicom da u podučavanju matematičkih predmeta predavanje samo za sebe nije dovoljno kako bi studenti usvojili gradivo, akademske godine 2012./13. smo osmislili niz aktivnosti kojima smo pokušali prevladati nedostatak predznanja i objasniti složene i apstraktne matematičke pojmove koji su temelj operacijskih istraživanja. U provođenju ovog kolegija koristili smo se ICT-om koja je bliska studentima informatike.

U ovom radu predstavit ćemo pristup dizajnu i provođenju kolegija Operacijska istraživanja uz pomoć ICT-a te usporediti rezultate prolaznosti studenata postignute na kolegiju u protekle tri akademske godine.

Ključne riječi: ICT, studenti informatike, operacijska istraživanja, instrukcijski dizajn, online aktivnosti

Fundamental prospective teachers' algebraic knowledge

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Abstract. There is no question that elementary school teachers are the ones who lay the foundations of mathematical knowledge. The choice of future career of their students is largely dependent of the strength of given mathematical foundations. But, what is the quality of the mathematical foundations of prospective elementary school teachers? Are the fundamental mathematical knowledge firm and deep enough to allow prospective elementary school teachers to practice methodical processing of these knowledge?

In order to provide insight on these questions, the research on basic algebraic skills of prospective elementary school teachers was conducted. Sixty 3rd year students at Pedagogical faculty in Sarajevo participated in the research. Special attention was given to diagnose difficulties that prospective elementary school teachers show in the case of different representations of basic algebraic concepts and their transformations. For this purpose, prospective elementary school teachers solved the tasks of recognition, representation, transformation and interpretation of basic arithmetic and algebraic expressions and properties.

Qualitative analysis of the data indicates the difficulties and misconceptions rooted in primary education, and points to the necessity of carrying out modifications in the content of pre-service education of prospective teachers.

Keywords: algebraic knowledge for teaching, concept representation, concept transformation

Introduction

Knowledge of algebra is essential for learning and mastering mathematics and other disciplines. One of the main strengths of algebra is that algebra is a tool for generalizing and solving variety of problems. Algebra is a symbolic language that enables users to describe and analyze relationships between quantities. By simplifying and representing problem situations in condensed form, algebra allows students to see problem structure and become better problem solvers (Krutetskii, 1976; Schoenfeld, 1992).

Teachers have important role in supporting and developing arithmetic and early algebraic reasoning in the lower grades of primary school. Contemporary trends in the teaching of mathematics in primary school imply slight involvement of algebra in arithmetic contents. Therefore, it is reasonable to ask whether the teachers, educated at our universities, able to promote and develop early algebraic reasoning.

Studies conducted worldwide (primarily in the U.S.) show that the level of teachers' basic early algebraic knowledge is quite modest, and their skills of generalization of arithmetic early algebraic laws insufficiently high (Ma, 1999; Doerr, 2004; Dobrynina, Tsankova 2005; Kieran, 2006). There are many reasons to pay more attention to the development of algebraic thinking in elementary schools. This applies primarily to the reasons for the ease transitions from arithmetic to the understanding of complex and abstract concepts of variables (Kieran, 1992; Kaput, 2000). Taking into account given reasons, the survey was conducted among pre-service teachers -students at the University of Sarajevo, in order to provide insight into the level of their basic early algebraic knowledge and typical mistakes that they conduct.

Theoretical framework

A number of different characterizations of algebra can be found in the mathematics education literature. For example, Usiskin (1988) described four conceptions of algebra: generalized arithmetic, the set of procedures used for solving certain problems, the study of relationships among quantities, and the study of structures. Kaput (1995) identified five aspects of algebra: generalization and formalization; syntactically guided manipulations; the study of structure; the study of functions, relations, and joint variation; and a modeling language. Kieran (2004) categorized school algebra according to the activities typically engaged in by students: generational activities, transformational activities, and global meta-level activities

Algebra is a subject with a complex structure. Algebra is dealing with expressions with symbols and the extended numbers in order to solve equations, to analyze functional relations and to determine the structure of the representational system which consists of expressions and relations. However, activities such as solving equations, analyzing functional relations and determining the structure are not the purpose of algebra. Those activities are tools for modeling of real world phenomena and problem solving related to the various situations.

Furthermore, algebra is more than a set of facts and techniques. It is a way of thinking. Lew (2004) states that success in algebra depends on a minimum of six kind of mathematical thinking abilities: generalization, abstraction, analytical thinking, dynamic thinking, modeling and organizing. Skills of representing algebraic concepts in different forms (verbal, symbolic, graphic), as well as transforming different representations from one form to another are inherent to these six mathematical thinking abilities (Pjanić, 2011). Complex algebra structure is presented the Figure 1.

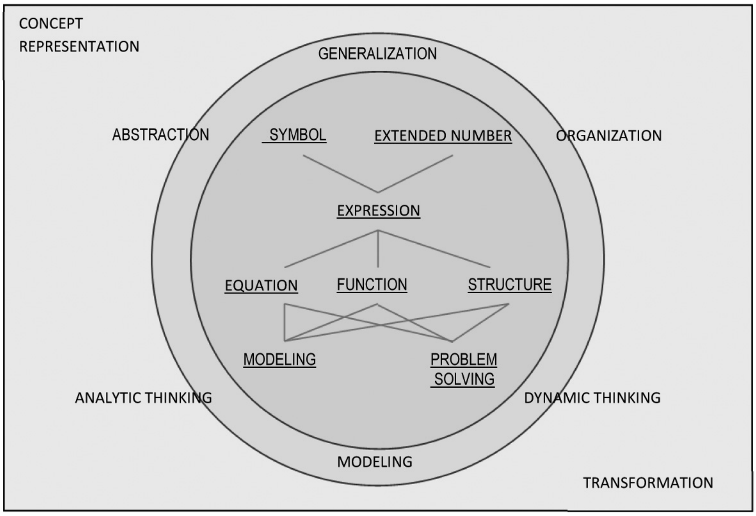

Figure 1.

Necessary mathematical thinking skills that influence success in algebra are determined as follows:

Generalization. Most of all algebraic objects or concepts are results of the generalization process, namely a process to find a pattern or a form. Algebra begins with the patterns identified in the given set of objects and then the observed patterns are generalized and expressed in symbolic form. For example, the commutative property of addition is a pattern observed in many numerical expressions such as $3+2=2+3$ and $10+20=20+10$, etc.

Abstraction. Unlike arithmetic, algebra deals with symbols. Symbol is an abstract object in the sense that it is de-contextualized. Seldom does algebraic language have concrete meaning. For example, variable x has no concrete meaning accompanied with the image. In geometry, for example, a rectangle as an object abstracted from various rectangular shapes, has image to describe the shape of rectangle. In algebra the symbol x does not evoke such an image. This characteristic is a serious stumbling block for students whose developmental ability has not reached the formal level for learning algebra.

Analytic thinking. An important activity in algebra is the process of solving equations. Solving an equation is the process of finding some unknown value requested in the expression written in terms of the unknown value. It will be solved by a series of necessary conditions. Considering that the unknown value is determined, by the analytical method, we find relationships in the equation and conditions necessary to find a solution. Working backward, by the synthetic method, we confirm the accuracy of solutions. The significance of the analysis is the fact that it reveals inverse operation to those given in the expression, which enables finding the necessary conditions for solving equations.

Dynamic thinking. One of the most important concepts in algebra is the concept of variable. The variable is an object to encapsulate changing objects. Variable is essential for understanding a function. Dynamic thinking could be developed by hypothetical deduction and the trial and error strategy for monitoring and controlling the dependent action for each of changing variables.

Modeling is a process to represent the complex phenomena and situations using mathematical expressions, to investigate the situation with a model, and to draw some conclusions from the activities. This is the essence algebra and the reason to learn algebra. So, when we teach an equation, it is important for students to represent some situation with an equation and to solve the equation to get the solution of the original situation.

Organization. One purpose for learning algebra is to develop a tool for organizing the complex situations by using a table and a diagram. Very important to many problem-solving activities is finding of the independent variables. By sorting and organizing data by making a table, a whole picture about the problem situation and the relation between conditions of the problem can be grasped and the relation between an independent variable and the corresponding dependent variable can be controlled more easily.

Concept representation and transformation. Ability to represent a concept in a variety of forms (symbolic, graphical, verbal) is closely related to the degree of understanding of the concept. Although the algebraic concepts are expressed primarily in the symbolic representation, it does not imply the non-use of other representations and transformations among each other. The mere transformation of concept representations is fundamental process in learning and comprehending mathematical (and thus algebraic) concepts (Pjanić, 2011).

On the basis of given characteristics of algebra, algebraic thinking can be defined as thinking that involves the formation of generalizations from experience with numbers and calculations with numbers, formalizing obtained generalizations using coherent system of symbols, and the studying of forms and functions. Algebraic thinking is about generalising arithmetic operations and operating on unknown quantities. It involves recognising and analysing patterns and developing generalisations about these patterns. While the language of arithmetic focuses on answers, the language of algebra focuses on relationships (MacGregor, Stacey, 1999).

Given implies five segments of algebraic thinking:

- generalizations arising from arithmetic, but also from the patterns in other mathematical disciplines,
- meaningful use of symbols,
- studying the structure of the number system,

- studying patterns and functions,
- the process of modeling, which integrates the first four segments.

These five segments of algebraic thinking are followed by two aspects of understanding mathematical, and hence algebraic, concepts and relations: conceptual and procedural understanding, i.e. knowledge and understanding the essence of concepts and relationships among them, and knowledge of the rules, symbols and algorithms required to solve mathematical problems.

Research shows that students can easily understand algebra when they have a solid knowledge of the general properties of numbers (for example, commutation rule, constancy of the sum, etc.), the relationship between numbers and the impact of the basic operations of numbers, rather when they are only focused on finding answers. This concept is best taught at early age, because in this way the development of misconceptions can be avoided. Misconceptions that are created in early age often prevent student to develop the ability to work with symbols and generalizing. In order to make contents that are taught in lower grades slightly algebraic it is essential that teachers understand basic algebraic concepts and that they are willing to promote and develop algebraic thinking of their pupils. Studies indicate that the level of knowledge and skills in algebraic generalization of arithmetic and algebraic principles that teachers have in the Far East is very good, as opposed to the lack of knowledge and skills of teachers in the West (Ma, 1999).

Research methodology

The main goal of the research, which the first results are presented in this paper, is to provide insight into the fundamental (early) algebraic knowledge of future teachers, that is, consideration of segments of algebraic thinking of pre-service teachers. Segments algebraic thinking which are the focus of interest are analyzing of the relationships between quantities, identifying the structure, identifying changes, verbal and symbolic representations of concepts and transformation of one representation to another.

The research goal is concretized by following research tasks:

- To consider segments of pre-service teachers' algebraic thinking, mainly the ability to analyze the relationships between quantities, observe structures, transform expressions.
- To determine the rate of success in expressing relationships between quantities in verbal and symbolic representation.
- To determine the rate of success in the transformation of verbal to symbolic representation.
- To examine which of the forms of understanding (conceptual or procedural) in solving elementary equations is prevalent among students.
- To establish a characteristic errors.

The study sample was consisted of 60 3rd year students (pre-service teachers) of the Faculty of Education at the University of Sarajevo. The study was conducted during the summer semester of 2011/2012 academic year. Survey instrument was a test that compiled set of six objective type exercises. The tasks were designed to check early algebraic knowledge. The tasks were related to the expression of mutual relations of the components of mathematical operations in verbal and symbolic representation, the process of solving equations and discussion of elementary equations:

1. Solve on the basis of given instructions $(x, y, \ldots, v \in \mathbf{N})$:
a) $x + b = c \implies b = c - x$
b) $v : y = t \implies v =$ __________ $(y \neq 0)$
c) $a \cdot b = c \implies a =$ __________ $(b \neq 0)$
d) $m : x = g \implies x =$ __________ $(x \neq 0, g \neq 0)$
e) $s + t = 2 \implies t =$ __________
f) $x - y = a \implies x =$ __________
g) $x - b = c \implies b =$ __________

2. Write how to determine:
– factor unknown __________
– addend unknown __________
– dividend unknown __________
– subtractor unknown __________
– divisor unknown __________
– subtrahend unknown __________

3. Write using symbols:
a) number for 2 greater than x __________
b) number 2 times greater than x __________
c) number 2 times less than x __________
d) number for 2 less than x __________
e) half of number a __________
f) fifth of number y __________
g) doubled the number t __________

4. Solve equation
a) $100 : (2x) = 10$
b) $18 + (x - 17) = 100$

5. How many solutions have each of equations
a) $0 \cdot x = 0$ __________
b) $0 \cdot y = 3$ __________
c) $0 : a = 4 - 4,\ a \neq 0$ __________

6. Solve equation $(((x : 2) : 2) : 2) : 2 = 1$.

Results and discusion

After analyzing students' answers to the given tasks, the segments of (early) algebraic thinking of prospective teachers could be perceived, especially the ability to analyze the relationships between quantities, to identify structures and transform expressions. The general impression is that respondents indicate amazingly low level of achievement related to the aforementioned segments of algebraic thinking.

Analysis of the responses in tasks 1 and 2 gave an insight into students' rate of success to express relationships between quantities in verbal and symbolic representation. Task 1, which referred to the symbolic representation of the relationship of components of mathematical operations, was completely solved by 52.55% of the respondents. Most respondents indicated difficulties in expressing the unknown divisor (52.55% correct answers), and subtrahend (58% correct answers). The 96.6% of respondents were able to express addend, factor and dividend. In task 2, where verbal representations of relations between components of mathematical operations was sought, respondents showed poorer results. Only 42% of respondents correctly expressed each term. As in the case of symbolic representation, majority of mistakes in verbal representation were dealing with concepts and divisor and subtrahend. By the analysis of students' answers in task 2, it was observed that students have difficulties in using the terms the sum, difference, product and quotient, which in their formulations appointed term result. Such formulations are not satisfactory, hence the low percentage of correct answers.

Analysis of the responses of students in task 3 obtained insight into students' rate of success to transform verbal representation of relationships between quantities to symbolic representation. All components of this task were successfully solved by only 48% of respondents.

After analyzing responses in tasks 4, 5 and 6 it was found that procedural form of solving equations is prevalent among respondents. Half of the respondents apply the parentheses rule in tasks 4 and 6, but only 26.6% of them gave correct answers in task 5. Typical errors detected by analyzing respondents' answers can be classified into two categories: procedural errors and conceptual errors. Procedural errors are reflected in the lack of knowledge of performing procedures in solving equations. Conceptual errors are reflected in the inability to express the components of mathematical operations in both representations, as reflected on the impossibility of solving equations, identification phrases "for much larger (smaller)" and "so many times larger (smaller)" and a misunderstanding of the concept of solution of equation as well as concept of numbers of solutions of equation.

Conclusions

Analysis of the survey results indicates worryingly low level of understanding early algebraic concepts that originated in the primary education. Program and mathematics textbooks developers in B&H are choosing the recommended path of transition from arithmetic to algebra. Namely, pupils in lower grades of elementary school become acquainted with the symbolic representations of arithmetic generalizations, symbolically describe the relationships of components of mathematical operations. Also, they usually come to the concept of equations by modeling simple everyday life problems. When solving equations, it is insisted on recognizing the relationship between the component calculations, which occurs in the equation.

However, test results indicate that these concepts are not adopted at the level of understanding, at least in the case of students who have opted for the teaching profession. Neither teaching of algebra in middle school as well as in high school, that is focused on solving equations, factorization of polynomials and drawing graphics of functions, does not contribute to the developing algebraic thinking. Besides the necessary modifications to the process of realization of algebraic content in elementary and high school, modifications of realization of these contents are required also in the initial teacher education, in the sense to put focus to relational components of operations, modeling, representing and solving problems, noticing changes and structures, predicting, justifying and proving.

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Temeljna algebarska znanja studenata razredne nastave

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Sažetak. Neupitno je da su učitelji oni koji postavljaju temelje matematičkih znanja. Od čvrstoće tih temelja uveliko zavisi i izbor budućeg zanimanja njihovih učenika. No kakvi su sami matematički temelji studenata razredne nastave, budućih učitelja? Jesu li dovoljno čvrsti i duboki da omoguće studentima da se bave njihovom metodičkom obradom?

S ciljem da se pruži uvid na postavljena pitanja obavljeno je istraživanje o temeljnim algebarskim znanjima budućih učitelja. U istraživanju je učestvovalo 60 studenata treće godine Pedagoškog fakulteta u Sarajevu. Posebna pažnja je bila usmjerena da se dijagnosticiraju poteškoće koje studenti pokazuju u slučaju različitih reprezentacija osnovnih algebarskih pojmova i njihovih transformacija. U tu svrhu, studenti su rješavali zadatke prepoznavanja, reprezentovanja, transformiranja i interpretiranja elementarnih algebarskih izraza i aritmetičkih zakonitosti.

Kvalitativna analiza dobijenih podataka ukazuje na poteškoće čiji je korijen u osnovnoškolskom obrazovanju, te upućuje na neophodnost modifikacije realizovanja ovih sadržaja u okviru inicijalnog obrazovanja budućih učitelja.

Ključne riječi: algebarska znanja za podučavanje, reprezentacija pojma, transformacija pojma

Correlation between pupils' managing of graphic data and their level of geometric thinking

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Abstract. Since the year 2010 the National Curriculum Framework[1] in Croatia has, among other, defined the general goals for the educational area of mathematics, as well as the expected achievements of pupils at the end of each educational cycle with regards to two segments: mathematical processes and mathematical concepts. In primary school the first educational cycle is completed at the age of ten. Accordingly, in the aforementioned document it is anticipated that at the end of the first cycle within the mathematical concept labelled *Data* ten-year-old pupils will be able to:

• *collect, classify, and organise data culled from their everyday lives, and present them using simple tables, pictograms (i.e. graphic charts), and bar charts,*

• *read and interpret data presented in simple tables, pictograms and bar charts,...*

Due to the fact that the mathematics curriculum in Croatia does not anticipate the development of the aforementioned knowledge and skills until the seventh grade of primary school, the students of the fifth year of teacher studies attempted to investigate:

1. how to encourage lower primary school pupils to successfully manage graphic data,

2. whether the level of geometric reasoning of ten-year-olds is correlated to their data reading skills, as well as the skills of interpreting and presenting data.

We sought out these answers in collaboration with the students by working with fourth grade pupils within a pilot research.

Keywords: levels of geometric thinking/reasoning according to van Hiele, graphic presentation of data, reading and interpreting graphic data, student competences

[1] National Curriculum for Preschool Education, General Compulsory and Secondary School Education, `http://public.mzos.hr/Default.spx?sec=2685(March, 2012)`

Introduction

The key to a successful mathematics instruction and its satisfactory outcomes is a high quality communication between teachers and pupils. It is, therefore, important for students of teacher studies to learn to adjust their communication practices to the pupils' levels of thinking. On the other hand, it is important for children already at a preschool age to be made aware of mathematics in a sensory and experiential manner during their everyday activities. For this reason data interpretation and presentation has become a part of required mathematics curriculum from the first grade of primary school in several Croatia's neighbouring countries (1999, Slovenija). Accordingly, pupils are expected to express quantitative data by means of pictures, in other words in a graphic manner, and vice versa. However, the question arises whether the managing of data interpretation and presentation correlates to the level of pupils' geometric reasoning. Studies conducted worldwide point to the fact that lower primary school pupils experience numerous difficulties when solving various geometric tasks (TIMSS, 2011). Based on the assessment of the respondents' process of reaching a solution to the task and their subsequent argumentation, the Dutch mathematician Pierre M. van Hiele (1986, Burger and Shaughness) defined 5 levels of geometric reasoning (Visualization, Analysis, Informal Deduction, Deduction and Rigor). A number of researchers confirm van Hiele's findings which show that the level of geometric reasoning does not depend solely on the age of the pupil, in other words the grade the he/she is attending (Burger and Shaughnessy, 1986; Wu and Ma, 2006). Additionally, van Hiele asserted that pupils do not necessarily need to transgress to the next level of geometric reasoning if they have never had the opportunity to perform at a higher level. In 2006 Wu and Ma conducted research determining which levels of geometric reasoning are exhibited by 5581 pupils from the first to the sixth grade. The results of their research showed that all pupils of the first, second, and predominantly third grade are mostly at the level of visualization. The pupils of the fourth, fifth and sixth grade were predominantly at the level of analysis, and only 20% of the fifth and sixth grade pupils performed at the level of informal deduction.

Research goal and hypothesis

While working with the fourth grade pupils (ten-year-olds), the students of the fifth year of teacher studies expressed their desire to master the teaching approaches to interpreting and presenting data and to learn how to recognise the level of geometric reasoning of pupils according to van Hiele. The direct instruction of the pupils was conducted at the Faculty at the *Little school of mathematics* which was founded by M. Pavleković as part of the teacher studies curriculum. The participants of the *Little school of mathematics* thus engaged in extracurricular mathematical activities. Accordingly, 25 of them comprised the experimental group of the research. The goal of the research was to find out whether the pupils' level of geometric reasoning is correlated to their managing of data from their sorroundings. The students hypothesized that there would be a correlation between the level of the pupils' geometric reasoning and their successful managing of graphic data. The control group consisted of 27 randomly selected pupils of the same age as those in the experimental group who attended additional mathematics classes in their schools.

Research methodology and procedure

The students taught the experimental group for two periods per week during one semester. During October and November 2012 the students used one period per week to gather, categorise, present, read and interpret graphic data, and the other to solve geometric tasks. While solving the geometric tasks, the students, aided by university instructors, learned how to recognise the level of geometric thought in each pupil. In order to determine the level of geometric reasoning, as well as the pupils' managing of reading, displaying and interpretation of data, we devised the examination material composed of seven tasks in the duration of 45 minutes. The preparation of the exam tasks and the examination procedure of the pupils in the experimental and control group was implemented by the students of teacher studies during January 2013. The final analyses were done in StatSoft Statistica. The comparison of the success in solving each task in two separate (experimental and control) groups was conducted in a non-parametric manner by means of the Mann-Whitney U test. The correlation within groups across two variables (level of geometric reasoning and the level of success in managing graphic data) was determined by means of the Spearman rank correlation coefficient.

Results and interpretation

The examination material intended for the assessment of the level of geometric reasoning consisted of 4 groups of tasks, with each group comprised of further subtasks, making this the total of 15 assignments which each pupil was required to complete. At the zero level we categorised the pupils' answers which were based on various descriptive qualities of geometric shapes, as well as the answers which were nonsensical from the mathematical point of view. The first level of geometric reasoning contained the pupils' answers from which it was obvious that the pupils understand the functions of geometric shapes and the connection between them, and are capable of argumenting their actions soundly. Based on the available research, it was expected that a great number of pupils will be on the threshold between levels 0 and 1 (they do not perceive shape as a whole, describe characteristics of shapes which are not in accord with their true function, arguments are not entirely clear, etc.), which is why we introduced level 0,5. The skills of reading, argumenting and graphic presentation of data were tested by means of three tasks. For each subtask the pupils were awarded points in all seven tasks.

In the first task each pupil needed to recognise and name five shapes, more specifically: two rectangles, a triangle, a square and a hexagon. The side length ratio of one of the rectangles was 1 : 2, and of the other 1 : 5. For the purpose of more easily distinguishing the two, we named the second rectangle the “narrow” rectangle.The students monitored their pupils’ task solving procedure and discussed the levels of geometric reasoning in the following manner:

The recognition and naming of the rectangles in the control group was completed at a slightly lower level of geometric reasoning. Out of the total of 27 pupils, 13 of them recognised and named both rectangles (level 1), and two pupils did not recognise either of the rectangles (level 0). The remaining 12 respondents recognised only the “narrow” rectangle, and two pupils called the “narrow rectangle” a quadrangle, which constitutes level 0,5. Six of the pupils provided incorrect explanations for the “narrow rectangle” (ruler, quadrarectangle, cylinder), and two failed to assign name to the shape. These are the features of the lowest level of geometric reasoning. As opposed to that, 22 pupils out of 25 in the experimental group recognised and named the rectangle in bothe cases. One pupil called both rectangles a square and the remaining two pupils did not name the “narrow rectangle”.

Both groups were equally successful in recognising and naming a square. Accordingly, 21 out of 27 pupils in the control group recognised and correctly named the square. Two pupils named it a quadrangle which suggests that they still have not reached the level of analysis in geometric reasoning. Four pupils named the square a cube and a parallelepiped, which suggests the lowest level of geometric reasoning. Similarly, 23 out of 25 pupils in the experimental group recognised and correctly named the square. Consequently, one of them believed that the drawing represented a cube and one pupil did not provide an answer.

Recognising and naming the right-angled triangle with a side length ratio of 1 : 5 was a very difficult task for the pupils. 17 out of 27 pupils in the control group recognised and correctly named the triangle. Three pupils failed to name the shape, and 7 pupils provided an incorrect answer (five named the shape a cone, one called it a tip, one noted that it was an angle). Judging from the pupils’ answers, the students concluded that ten pupils in the control group were at the lowest level of geometric reasoning. In the experimental group 24 out of 25 pupils correctly named the triangle, while one pupil did not provide the answer.

Finally, recognising and naming the regular hexagon in the experimental group was achieved by all the respondents at the highest level. On the other hand, 18 out 27 pupils in the control group recognised and correctly named the hexagon. Seven of them failed to name the shape, and two provided the incorrect term (heptagon, hexigon).

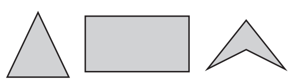
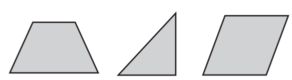
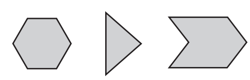
Table 1. Achieved levels of geometric reasoning of ten-year-olds in recognising and naming of geometric shapes.

	Rectangle		**Square**		**Triangle**		**Hexagon**	
Level	Contr.	Exp.	Contr.	Exp.	Contr.	Exp.	Contr.	Exp.
0	8	3	4	1	10	1	9	–
0.5	6	–	2	1	–			–
1	13	22	21	23	17	24	18	25

From table 1 it is visible that more than a quarter of the pupils in the control group experienced difficulties recognising and naming the shapes. These pupils do not perceive geometric shapes via their specific features, and are, therefore, still at the visual level of geometric reasoning.

The second task required the pupils to find the odd man out in the sequence of three and argument their choice. What was tested in this instance was the pupils' ability to single out an object from a group of objects according to a specific property which makes that object different from the others, and rationally explain their choice. Regardless of the chosen "odd man out" in the sequence, if the pupil sensibly explained their criteria (highlighted features) according to which the selected geometric shape differed from the remaining two shapes, we assigned the answer to level 1. However, if the pupil explained their choice of the "odd man out" by means of the properties possessed by the other two shapes and omitted the lack of properties of the "odd man out", their answer was categorised at level 0,5. A nonsensical explanation or a failure to name the "odd man out" placed the respondent's answer at level 0. The pupils were presented with three sequences. The biggest challenge for the pupils was providing a sensible explanation in each individual instance.

Table 2. Achieved levels of geometric reasoning of ten-year-olds while argumenting the selection of the "odd man out" in a given sequence.

						
Level	Contr.	Exp.	Contr.	Exp.	Contr.	Exp.
0	16	10	20	17	11	5
0,5	7	8	3	9	10	8
1	4	7	4	9	6	12

In the third task the pupils were offered a set of six quadrangles, two of which had two pairs of parallel sides, two only one pair of parallel sides, and the remaining two no pairs of parallel sides. The pupils were required to produce three assignments. Firstly, within the offered set of quadrangles each pupil needed to colour the quadrangles with no parallel sides using the same colour. Secondly, they needed to use a different colour to designate all the quadrangles with two pairs of opposite parallel sides. In properly completed assignments the final picture needed to contain all coloured quadrangles except for those with one pair of parallel sides. The next step required the pupils to independently construct a quandrangle with only one pair of parallel sides. If the pupil in the first and the second subtask correctly coloured two and only two of the required shapes, their assignment was considered completed at level 1 of geometric reasoning. If the pupil correctly coloured at least one of the quadrangles according to the proper solution and did not additionally colour any of the quadrangles which do not have the required property, the level of geometric reasoning was considered on the threshold between 0 and 1. If in the subtask the pupil coloured one correct and one incorrect shape, the answer was categorised at level 0. The pupil who completed each step in all three subtasks correctly was considered to be at level 1 of geometric reasoning in this particular task. Those pupils who partially completed the tasks were included in the level 0,5 group. The pupils with incorrectly completed assignments or those who have not attempted to solve the task were placed in the level 0 group.

Table 3. Achieved levels of geometric reasoning of ten-year-olds in recognising and construction of a quadrangle with assigned properties.

	Isolating objects according to specific properties		Constructing a shape with assigned properties	
Level of geom. reasoning	Contr.	Exp.	Contr.	Exp.
0	6	1	10	11
0,5	14	10	–	–
1	7	14	17	14

In the fourth task the pupils were expected to construct an arbitrary triangle. Following this activity they were asked to construct a triangle which was different from the previous one and explain the difference. Finally, they were asked how many different triangles they could draw.

Table 4. Achieved levels of geometric reasoning in constructing an arbitrary triangle, a triangle differing from the previous one, arguing the difference, and assessing the possible number of different triangles.

Level of geom. reasoning	Control group	Experimental group
0	7	5
0,5	16	14
1	2	6

Only 15% of the pupils solved this task at level 1. These pupils, for example, first constructed an equilateral triangle and subsequently a right-angled triangle providing an explanation that in the latter triangle the angles were of different sizes, while in the equilateral triangle all angles were of equal sizes. Moreover, only a few of the pupils answered that they could draw very many triangles. The largest number of pupils thought that they had constructed a different triangle as soon as they had changed the position of the initial triangle, which constitutes geometric reasoning at the level of analysis. Furthermore, the pupils mostly did not manage to reasonably explain in which way the second triangle differed from the first.

In the fifth and the sixth task the pupils needed to demonstrate the skill of reading a pie chart and a bar chart. Accordingly, the pie chart displayed the participation of pupils of a certain school in athletics, table tennis, basketball, football and handball practice. The football players designated a quarter of a circle. The pupils were asked how many pupils practised football if the school housed a total of 240 athletes and each of them had opted for only one sport. While solving this task the pupils in the experimental group were frequently more accurate in their responses. We believe that the reason for such an outcome might be that the students had practised dividing a circle into 6, 3, 12, and 2, 4 and 8 equal parts with the pupils, as well as reading and presentation of data in a pie chart. The bar chart featured the number of sunny days in each month of the year. The pupils were required to name the month with the least number of sunny days and read the total of how many days. Additionally, they were expected to name the month with the highest number of sunny days and read how many sunny days there were in that particular month. A high percentage of pupils in both groups successfully solved this task.

In the seventh task the pupils were given three maps of their homeland which highlighted the temperatures in all the major cities recorded on the last day of three different years. The task was to create a bar chart in order to present the temperature of the assigned city in all three years. All the pupils were very successful at solving this task.

Table 5. Level of success in reading and graphic presentation of data by ten-year-olds.

Points	Reading and interpreting of data from a pie chart		Reading and interpreting of data presented in the bar chart		Organising and presenting data by means of a bar chart	
	Contr.	Exp.	Contr.	Exp.	Contr.	Exp.
0	17	3	3	–	2	2
0,5	3	–	2	3		
1	7	22	22	22	25	23

Due to the fact that in all seven tasks we assigned points to each of the subtasks for each pupil from bith groups (control and experimental), we conducted the analysis in StatSoft STATISTICA with regards to two variables (level of geometric reasoning and level of success in managing graphic data). These variables model the correlation of the pupils' level of geometric reasoning (GEOM_Z) and the level of success in managing graphic data (GP_95Z). The values of the variables are expressed by means of the ordinal measurement scale. The pupils belonged to two independent groups. The control group was comprised of 27 pupils, whereas the experimental group consisted of 25 pupils.

The mode values in rating success for the solving of each individual task are presented in the following table:

Table 6. Mode values of success.

Task	Control group	Experimental group
Task 1. GEOM	4	4
Task 2. GEOM	0	1,5
Task 3. GEOM	2	3
Task 4. GEOM	3	3
Task 5. GRAPH. DATA	0	1
Task 6. GRAPH. DATA	1	1
Task 7. GRAPH. DATA	0	1

The median values in rating success for the solving of each individual task are provided in the following table:

Table 7. Median values of success.

Task	Control group	Experiemental group
Task 1. GEOM	2,75	4
Task 2. GEOM	0,5	1,5
Task 3. GEOM	2	3
Task 4. GEOM	3	3
Task 5. GRAPH. DATA	0	1
Task 6. GRAPH. DATA	1	1
Task 7. GRAPH. DATA	0	1

For the purpose of comparing the level of success in solving each individual task in these two independent groups of pupils, we applied the non-parametric Mann-Whitney U test. The results of the test showed that on the level of significance 5% there is no statistically significant difference only in the achieved level of geometric reasoning in solving the fourth task and the reading of the bar chart in task six ($p4 = 0,18728$; $p6 = 0,63394$). In the majority of the remaining cases there is a statistically significant difference between the control and experimental group ($p1 = 0,00075$; $p2 = 0,0093$; $p3 = 0,00349$; $p5 = 0,00018$; $p7 = 0$).

The correlation within groups between the achieved level of geometric reasoning and the managing of graphic data was determined by means of the Spearman rank correlation coefficient. In this manner we found that there was a positive, yet weak correlation in the control group only between the achieved level of geometric reasoning in solving the first task and the success in reading the data from the pie chart in task five ($rS = 0,40$; $p = 0,039$). In the experiemental group there was a positive but weak correlation only between the achieved level of geometric reasoning in solving the second task and the reading of the bar chart ($rS = 0,41$; $p = 0,041$).

Conclusion

The students of the fifth year of teacher studies have mastered the teaching approaches to reading, presenting and interpreting data in lower primary school by means of direct instruction with the pupils, using appropriate materials form mathematical journals as well as other available research. Furthermore, while observing the paths to the pupils' solving of geometric tasks and with the support of university instructors, they learned how to assess pupils' level of geometric reasoning according to van Hiele, as well as how to, by means of the appropriate type of communication, encourage a pupil to transgress to a higher level of reasoning. The analysis of the answers provided by all the fourth grade pupils included in this research shows that the pupils are in approximately 70% of the cases situated on the threshold between the level of visualization and the level of analysis of geometric thinking. The half of the remaining part of the pupils is at the lowest level, whereas the rest are at the level of visualization, which is in accord with the previously conducted research. When observing the two groups separately, the pupils in the experimental group displayed a slightly higher level of geometric reasoning and a slightly more successful managing of graphic data than the pupils in the control group. We believe that such results were obtained partly because of the extracurricular activities which we provide for the pupils. However, regarding the correlation between the level of geometric reasoning and the level of success in reading, interpreting and recording of data within each of the two independent groups of pupils, we found no significant relationship. We recommend for these conclusions to be tested on a larger sample of pupils.

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Suodnos učenikova snalaženja u grafičkim podatcima i njegove razine geometrijskoga mišljenja

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Sažetak. Od 2010. godine Nacionalnim okvirnim kurikulumom[1] u Hrvatskoj su, između ostaloga, definirani opći ciljevi za matematičko područje te očekivana postignuća učenika na kraju svakoga obrazovnoga razdoblja i to u dvije dimenzije: matematički procesi i matematički koncepti. Prvo obrazovno razdoblje završavaju desetogodišnjaci. U spomenutom dokumentu zapisano je da će desetogodišnjaci u okviru koncepta nazvanoga *Podatci*, između ostaloga, znati:

- *prikupiti, razvrstati i organizirati podatke koji proizlaze iz svakodnevnih situacija, te ih prikazati jednostavnim tablicama, piktogramima (slikovnim dijagramima) te stupčastim dijagramima,*

- *pročitati i protumačiti podatke prikazane jednostavnim tablicama, piktogramima i stupčastim dijagramima,...*

Kako se u nastavnom programu matematike u Hrvatskoj tek u sedmom razredu predviđaju teme za razvoj ovih znanja i vještina, studente pete godine učiteljskih studija zanimalo je:

1. kako poticati snalaženje učenika razredne nastave u grafičkim podatcima,

2. je li razina geometrijskoga mišljenja desetogodišnjaka u suodnosu s njihovom vještinom čitanja, tumačenja i prikazivanja podataka.

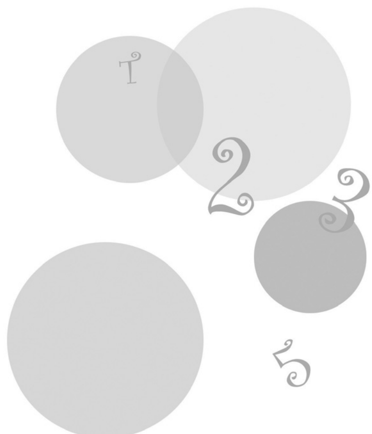
U suradnji sa studentima, odgovore smo potražili radeći s učenicima četvrtoga razreda osnovne škole i provedbom pilot istraživanja.

Ključne riječi: razine geometrijskoga mišljenja po van Hiele-u, grafičko prikazivanje podataka, čitanje i tumačenje grafičkih podataka, kompetencije studenata

[1] Nacionalni okvirni kurikulum za predškolski odgoj i obrazovanje te opće obvezno i srednjoškolsko obrazovanje, `http://public.mzos.hr/Default.spx?sec=2685(1.3.2012.)`

4

The influence of convictions, views, norms, emotions and attitudes of mathematics teachers on the efficiency of mathematics instruction



In this chapter the authors emphasize the importance of convictions, views, norms, emotions and attitudes which teachers as problem-solvers and instruction designers project on instruction as a problem-based situation. Moreover, during task solving in mathematics instruction different attitudes of teachers, as well as those of pupils (joy, motivation, interest) can cause the participants of the teaching process to experience a variety of emotions (frustration, anxiety, pleasure, joy, impatience, rage). The authors suggest that we take into consideration teachers' attitudes, as they influence the efficiency of different segments of mathematics instruction. The essence of teaching is the teacher's capability to engage a pupil into thinking independently and critically, and not to imitate their skilful instructor.

Future teachers' attitudes toward the use of concept maps

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Abstract. This article discusses the experiences with the use of concept maps in teaching mathematics in the first semester at the Faculty of Teacher Education of the University of Zagreb. We present and analyze the results of a survey conducted among students that reflects their attitudes toward the use of concept maps. The results are compared with the results of previously conducted survey among future teachers in the United Arab Emirates. Finally, we give directions for our future work and research.

Keywords: concept maps, mathematics teaching, teacher education, students' attitudes

Introduction

Data, information and knowledge visualization as a way of representation is used for creation, but also for transfer, communication and use of knowledge, and is very important in domain of education. There are many different graphical and other representations, and also strict formal methods and languages used for knowledge representation (Juričić Devčić, Mrkonjić, Topolovec, 2012).

Figure 1. Italian primary school students are constructing concept maps.

Source: Berioni, A., Baldoni, M. O. (2004), *The words of science: The construction of science knowledge using concept maps in Italian Primary School*, Proc. of the First Int. Conference on Concept Mapping, Pamplona, Spain.

Concept maps are graphical tools for organizing and representing knowledge that are very successful in the classroom, as evidenced by numerous studies (Juričić Devčić, Topolovec, Mrkonjić, 2011). They are well accepted by the students and teachers at different levels of education (Mrkonjić, Topolovec, Marinović, 2011).

In this paper, we will deal with the future teachers' attitudes toward the use of concept maps.

Mehmet and Nihal Buldu (Buldu, M., Buldu, N., 2010) describe the way in which they introduced the concept map as a tool for formative assessment in work with students of teachers' study in the United Arab Emirates. Two teachers worked with students in two teacher education institutions. Students have used concept maps in 6 courses, and demonstrated their satisfaction with concept maps by filling out a questionnaire. The result of the survey is shown in Table 1. The study included 166 students.

Table 1. Student teachers' attitudes on the use of concept maps in the United Arab Emirates.

Perceptions		1	2	3	4
1.	**Concept mapping helped me to understand the key concepts of the subject I studied in the class.**	2.4	9.0	33.7	**54.8**
2.	**Concept mapping helped me to improve my learning of the course content.**	2.4	7.8	**48.8**	41.0
3.	**Concept mapping increased my motivation in learning the course content.**	3.6	7.8	**47.6**	41.0
4.	**Concept mapping increased my involvement in the class.**	0.6	2.4	30.7	**66.3**
5.	**Concept mapping helped me to communicate my learning to others in the class.**	0	4.8	**54.8**	40.4
6.	**Concept mapping stimulated me to think independently.**	0	5.4	**55.4**	39.2
7.	**Concept mapping helped me to learn cooperatively with my class colleagues.**	0.6	10.8	**84.9**	3.6
8.	**Making connections among concepts in concept mapping tasks challenged my thinking.**	0	1.2	**59.6**	39.2
9.	**Concept mapping helped me to clarify the interrelationships among course content.**	1.2	6.6	**48.2**	44.0
10.	**Concept mapping helped me to see the missing components in my learning of the course content.**	0	5.4	36.1	**58.4**
11.	**Visualizing my learning through concept mapping tasks reduced the ambiguity of the concepts expressed verbally by the instructor.**	1.2	7.8	**69.3**	21.7
12.	**Concept mapping helped me to gain a better understanding of my learning processes in the class.**	2.4	6.6	33.7	**57.2**

Note: 1 = Not at all, 2 = Very little, 3 = Somewhat, 4 = To a great extend

Source: Buldu, M., Buldu, N. (2010), Concept mapping as a formative assessment in college classrooms: Measuring usefulness and student satisfaction, Procedia Social and Behavioral Sciences, 2, pp. 2099–2104.

Table 2. Postgraduate student teachers' attitudes on the use of concept maps at the University of Braga.

N°	Likert Scale Questionnaire Items	% 1	2	3	4	5	Mean
2	To build maps allowed me to relate complex subjects in a visual representation.	0	0	0	50	50	**4.5**
4	Relating concepts helped to a deeper understanding of contents.	0	0	7.7	26.9	**65.4**	**4.58**
5	I don't believe that to build concept maps enhances learning.	**57.7**	34.6	3.8	0	3.8	**1.58**
7	To construct concept maps makes one organize topics in a logical format.	0	0	0	30.8	**69.2**	**4.69**
8	It is compulsory to focus on key concepts.	0	0	0	19.2	**80.8**	**4.81**
9	To build maps was very useful for my learning.	0	0	4	48	48	**4.44**
10	Instead of simplifying it only confused me more.	**61.5**	30.8	3.8	3.8	0	**1.5**
11	It encourages non-linear thinking.	7.7	11.5	7.7	**61.5**	11.5	**3.58**
12	It develops skills for organizing information distinguishing what is essential and secondary.	0	0	0	30.8	**69.2**	**4.69**
13	To build concept maps helped me to organize better RME topics.	0	0	0	50	50	**4.5**
1	It helped to understand the complexity of RME core.	0	0	3.8	38.5	**57.7**	**4.54**
14	I don't think it was important to build maps to learn RME core.	**61.5**	30.8	3.8	3.8	0	**1.5**
15	To build concept maps helped to reflect on my learning process.	0	0	15.4	**53.8**	30.8	**4.15**
16	While doing concept maps for a RME topic I was making a balance of what was already learned.	0	8	8	**44**	40	**4.16**
17	To build maps was useful to my learning because I was aware of what I needed to study.	0	0	11.5	**46.2**	42.3	**4.31**
18	To build concept maps helped learning as it forced me to discipline myself.	0	0	15.4	**50**	34.6	**4.19**
19	To construct, modify and maintain online concept maps was very motivating.	0	0	11.5	**46.2**	42.3	**4.31**
20	To visualize the maps of other colleagues was useful to my learning.	0	7.7	11.5	**42.3**	38.5	**4.12**
21	The feedback of the instructor helped me to increase performance on building maps.	0	0	0	19.2	**80.8**	**4.81**
3	I will certainly use concept maps in my professional life.	0	3.8	3.8	34.6	**57.7**	**4.46**
6	I think I will use this tool class with my pupils.	0	4	8	**52**	36	**4.2**

1 = Strongly Disagree, 2 = Disagree, 3 = Neither Agree or Disagree, 4 = Agree, 5 = Strongly Agree

Source: Coutinho, C. P. and Bottentuit Junior, J. B. (2008), Using concept maps with postgraduate teachers in a web-based environment: an exploratory study, Proceedings of the Workshop on Cognition and the Web: Information Processing, Comprehension and Learning. Granada: Universidade de Granada, 4–26 de Abril de 2008, pp. 139–145. Available at: `http://hdl.handle.net/1822/7811`

C. P. Coutinho at the University of Braga (Portugal) in the academic year 2007/2008 conducted a study (Coutinho, Bottentuit Junior, 2008) with students of postgraduate studies for teachers, in the course 'Research Methods in Education' (RME). During one semester, students were introduced to and learned to use concept maps. They were making their concept maps based on reading comprehension, with the help of the concept maps they were planning projects, and comparing and opposing opinions during the discussion in the classroom. Within 15 weeks, as the semester lasts, they were taught to use software CmapTools to create concept maps. The study included 26 postgraduate students. At the end of the semester, a survey on the attitudes of students toward concept maps was conducted (see Table 2).

Research methodology

Attitudes of student teachers in the United Arab Emirates and postgraduate student teachers in Portugal encouraged us to introduce concept maps in teaching mathematics at the Faculty of Teacher Education in Zagreb.

In the first semester of the academic year 2012/2013, in the course Math 1, students were taught of the use of mental maps in mathematics, and also were introduced to the concept maps and their characteristics that distinguish them from the mental maps, and examples of the use of concept maps in mathematics. Two tasks related to the making of mental and concept maps were given to students as a part of homework (see Appendix A), and those tasks were relatively successfully done.

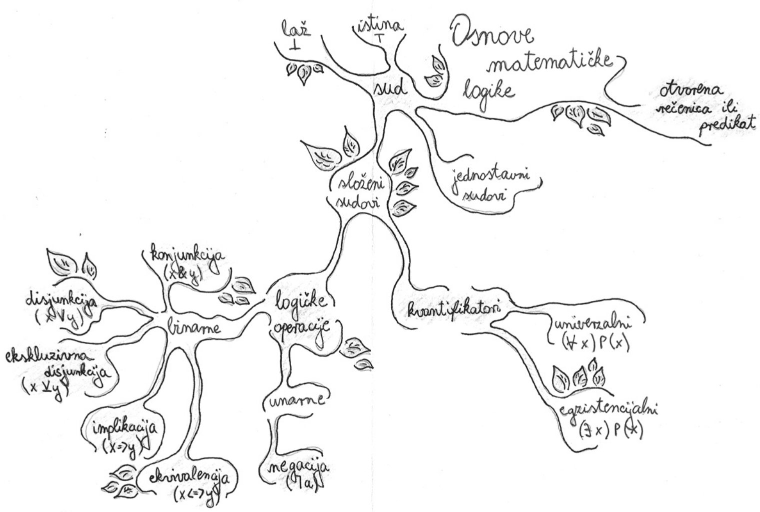

Figure 2. Mental map on topic 'Mathematical logic'
(work of student Lucija Pečk attending the module 'Visual Arts').

Students were taught of making mental maps by Buzan's model (Buzan, 1995), but, as shown in Table 3, most of their work was made by their own model, which shows that they were faced with making mental maps at primary and secondary school and already have their own style of mind maps.

Table 3. Types of mental maps.

mental maps	N	%
Buzan's model	**10**	**12.66**
others	**69**	**87.34**
Total	79	100

When creating concept maps, all the students followed the instructions that were provided by the instructor, which indicates that this is a new model of knowledge visualization for them and that they tried to manage them properly. Concept maps they made were different in many ways, some of them were less successful, and some were great. In some cases, we noticed the lack of connecting words on the linking lines, so the content of the map was insufficient. Although the students are taught how to access free software for concept maps, only few of them constructed a concept map on the computer, using CmapTools package (see Table 4).

Table 4. Concept maps according to the way they are designed.

concept maps	N	%
made by hand	**76**	**96.20**
supported by computer	**3**	**3.80**
Total	79	100

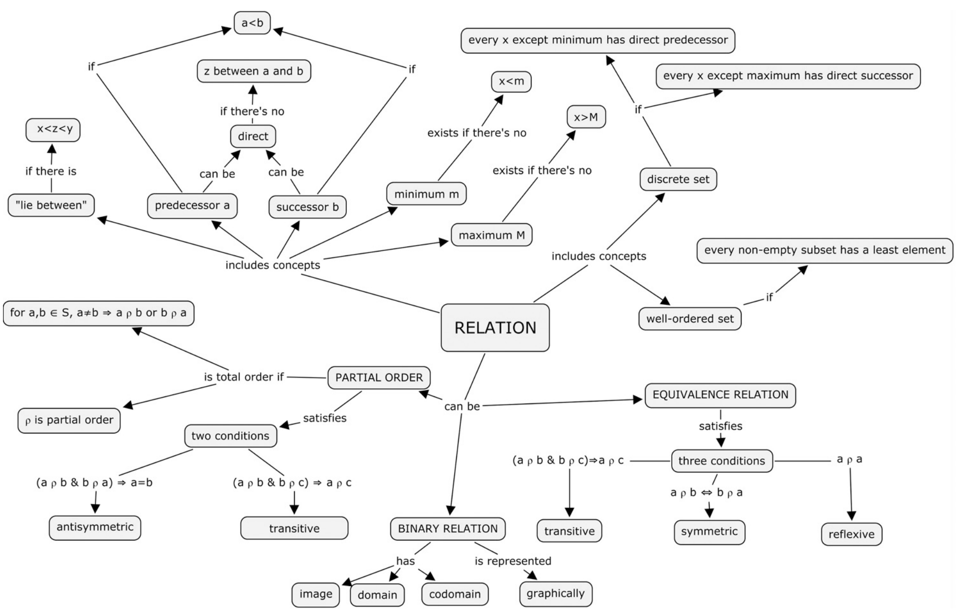

Figure 3. Concept map on topic 'Relations'
(work of student Petra Kuntin attending the module 'Educational Sciences')

Students had the option to choose the form of a concept map according to the way that the concepts in the map are hierarchically organized (vertical, horizontal and radial hierarchy). Concept maps with vertical hierarchy are commonly used (Novak, Gowin, 1984), as the students were warned. However, a large number of students have chosen a radial map, probably because they were already familiar with radial form of mental maps (Table 5).

Table 5. Concept maps according to the format.

format	N	%
vertical	**53**	**67.09**
horizontal	**1**	**1.27**
radial	**25**	**31.64**
Total	79	100

After the students' first experience with building a concept map, we have requested the feedback by a survey on students' attitudes. We used the same questionnaire (Appendix B) that Buldu, M. and Buldu, N. (2010) used in their research. In our research, 79 students participated in building maps. The survey covered a total of 70 students, 69 of them were females and one male, aged about 19 years. All students attended classes of course Math 1 in the first year of Teacher Studies' program with modules at the Faculty of Teacher Education (these are modules *Croatian Language*, *Visual Arts*, *Educational Sciences* and *Computer Science*).

Research results

Table 6. The results of our survey.

Perceptions	**1**	**2**	**3**	**4**
1. Concept mapping helped me to understand the key concepts of the subject I studied in the class.	30.0	**35.7**	27.1	7.1
2. Concept mapping helped me to improve my learning of the course content.	25.7	**50.0**	22.9	1.4
3. Concept mapping increased my motivation in learning the course content.	**44.3**	32.9	17.1	5.7
4. Concept mapping increased my involvement in the class.	**48.6**	31.4	15.7	4.3
5. Concept mapping helped me to communicate my learning to others in the class.	**42.9**	40.0	17.1	0.0
6. Concept mapping stimulated me to think independently.	37.1	**42.9**	14.3	5.7
7. Concept mapping helped me to learn cooperatively with my class colleagues.	**37.1**	32.9	24.3	5.7
8. Making connections among concepts in concept mapping tasks challenged my thinking.	28.6	31.4	**34.3**	5.7
9. Concept mapping helped me to clarify the interrelationships among course content.	24.3	**37.1**	32.9	5.7
10. Concept mapping helped me to see the missing components in my learning of the course content.	37.1	**42.9**	17.1	2.9
11. Visualizing my learning through concept mapping tasks reduced the ambiguity of the concepts expressed verbally by the instructor.	**32.9**	30.0	28.6	8.6
12. Concept mapping helped me to gain a better understanding of my learning processes in the class.	37.1	**41.4**	17.1	4.3

Note: 1 = Not at all, 2 = Very little, 3 = Somewhat, 4 = To a great extend

Since we have requested the feedback from students after their first contact with concept maps, we did not expect high ratings. Our expectations have been vindicated. Students' attitudes toward the use of concept maps in course Math 1 are shown in Table 6. Shares of the students' responses to numerated perceptions are given in percentages.

Analysis of the results

Results of the survey conducted by Buldu, M. and Buldu, N. are compared with our results in Table 7 and on the graph shown at Figure 4. In Table 8, we show the difference in the average ratings of two surveys, sorted from minimum to maximum.

Table 7. Comparison of the average ratings given by students in both studies.

Perceptions		Average scores		
		Buldu, Buldu	Our research	Difference
1.	**Concept mapping helped me to understand the key concepts of the subject I studied in the class.**	**3.41**	**2.11**	**1.30**
2.	**Concept mapping helped me to improve my learning of the course content.**	**3.28**	**2.00**	**1.28**
3.	**Concept mapping increased my motivation in learning the course content.**	**3.26**	**1.84**	**1.42**
4.	**Concept mapping increased my involvement in the class.**	**3.63**	**1.76**	**1.87**
5.	**Concept mapping helped me to communicate my learning to others in the class.**	**3.36**	**1.74**	**1.62**
6.	**Concept mapping stimulated me to think independently.**	**3.34**	**1.89**	**1.45**
7.	**Concept mapping helped me to learn cooperatively with my class colleagues.**	**2.91**	**1.99**	**0.92**
8.	**Making connections among concepts in concept mapping tasks challenged my thinking.**	**3.38**	**2.17**	**1.21**
9.	**Concept mapping helped me to clarify the interrelationships among course content.**	**3.35**	**2.20**	**1.15**
10.	**Concept mapping helped me to see the missing components in my learning of the course content.**	**3.53**	**1.86**	**1.67**
11.	**Visualizing my learning through concept mapping tasks reduced the ambiguity of the concepts expressed verbally by the instructor.**	**3.12**	**2.13**	**0.99**
12.	**Concept mapping helped me to gain a better understanding of my learning processes in the class.**	**3.46**	**1.89**	**1.57**

As it is evident from Table 8, the smallest difference in the attitudes of students is achieved at the perception 'Concept mapping helped me to learn cooperatively with my class colleagues' (No. 7). We believe that here the use of concept maps had the strongest effect among our students, and we justify it by the fact that the tasks were given for the homework, and most of students resolved them in collaboration with colleagues. Also, our ratings shows that students mostly agree with perceptions ‘Visualizing my learning through concept mapping tasks reduced the ambiguity of the concepts expressed verbally by the instructor’ (No. 11) and ‘Concept mapping helped me to clarify the interrelationships among course content’ (No. 9).

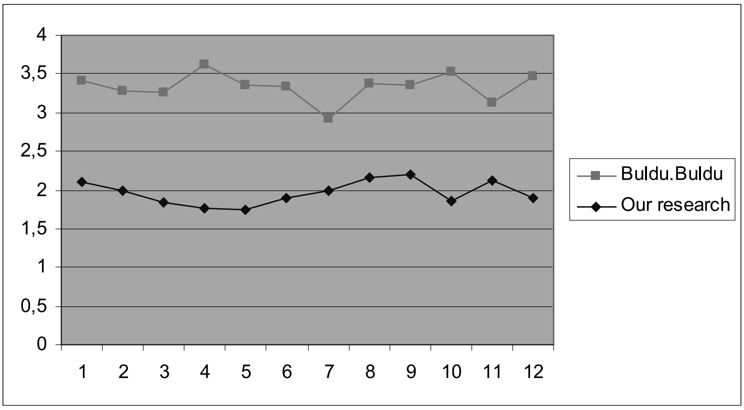

Figure 4. Graphic comparison of research results pf Buldu, M., Buldu, N. (2010) with our research results.

The largest difference in the ratings is achieved at the perception ’Concept mapping increased my involvement in the class’ (No. 4). The fact that students did not make concept maps in the class, but at home, had an impact on this low rating.

Table 8. Differences in the ratings sorted from minimum to maximum.

Perception	Difference
7.	**0.92**
11.	**0.99**
9.	**1.15**
8.	**1.21**
2.	**1.28**
1.	**1.30**
3.	**1.42**
6.	**1.45**
12.	**1.57**
5.	**1.62**
10.	**1.67**
4.	**1.87**

When we were processing the survey, we noticed some questionnaires with answers that contradict to each other, so it is possible that some of the students did not understand some questions or do not know enough about their learning processes. We noticed several flaws in the implementation of the survey:

1. We could have requested from students to express their attitudes toward mathematics in order to establish how attitudes toward mathematics influence their attitudes toward the use of concept maps in mathematics.
2. We could have supplemented the survey with a series of questions that would have allowed us to assess metacognitive skills of each student surveyed.
3. We didn't ask for feedback on whether students have already had experience with concept maps, although it is evident that they have not.
4. We didn't sort the survey by modules that students attend. It would be better to have the data collected and analyzed within a single module, since the earlier experience has shown that, depending on the module, there are different students' attitudes toward mathematics, as well as there are different cognitive and metacognitive abilities of students.

Conclusion

Positive experiences of scientists and teachers in the world literature that highlight the use of concept maps in mathematics teaching, but also in teaching in general, have prompted us to try this new approach to learning at the Faculty of Teacher Education.

First results and feedback are satisfying, but they also show some disadvantages. It has been shown that students can easily adopt this knowledge visualization tool, even though at the outset they are not aware of its overall usefulness.

Poorer results of our survey, compared to the other studies carried out in the world, should not have discouraged us, given that the survey was conducted at the very beginning of the introduction of concept maps into work with students of the Faculty of Teacher Education.

With this paper we wanted to highlight the need for introducing a concept map in the school, as a useful auxiliary tool for successful learning.

Directions for future work and research

We plan to continue with the use of concept maps in teaching mathematical courses at the Faculty of Teacher Education. We intend to introduce the other forms of knowledge visualization, such as v-diagram, into teaching. We shall also continue with surveying students on efficacy of the implementation of these forms of knowledge visualization.

We think it would be good to follow this generation of students till higher study years and to continue to use concept maps in different courses, and then to re-conduct the same survey in order to compare the results with the initial survey and identify potential improvement.

In the near future, we plan to experimentally test the use of concept maps in work with children, and then conduct a survey that will reflect the children's opinion on the use of concept maps.

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Appendix A

Task 1

Draw a mind map for one of the topics: MATHEMATICAL LOGIC, STRUCTURE OF MATHEMATICS, following these instructions:

- Draw on separate clean sheet of A4 paper with no lines;
- Set the paper horizontally;
- Use markers or crayons;
- Sign your name on the back of the paper.

Task 2

Draw a concept map for one of the topics: SETS, RELATIONS, following these instructions:

- Draw on separate clean sheet of A4 paper with no lines;
- Set the paper horizontally or vertically (as you wish);
- You can draw by hand or by using a computer;
- Sign your name on the back of the paper.

Appendix B

QUESTIONNAIRE

For each perception please circle a number from 1 to 4 for which you believe that corresponds to your attitude.

1.	**Concept mapping helped me to understand the key concepts of the subject I studied in the class.**	**1**	**2**	**3**	**4**
2.	**Concept mapping helped me to improve my learning of the course content.**	1	2	3	4
3.	**Concept mapping increased my motivation in learning the course content.**	1	2	3	4
4.	**Concept mapping increased my involvement in the class.**	1	2	3	4
5.	**Concept mapping helped me to communicate my learning to others in the class.**	1	2	3	4
6.	**Concept mapping stimulated me to think independently.**	1	2	3	4
7.	**Concept mapping helped me to learn cooperatively with my class colleagues.**	1	2	3	4
8.	**Making connections among concepts in concept mapping tasks challenged my thinking.**	1	2	3	4
9.	**Concept mapping helped me to clarify the interrelationships among course content.**	1	2	3	4
10.	**Concept mapping helped me to see the missing components in my learning of the course content.**	1	2	3	4
11.	**Visualizing my learning through concept mapping tasks reduced the ambiguity of the concepts expressed verbally by the instructor.**	1	2	3	4
12.	**Concept mapping helped me to gain a better understanding of my learning processes in the class.**	1	2	3	4

Note: 1 = Not at all, 2 = Very little, 3 = Somewhat, 4 = To a great extend

Stavovi budućih učitelja o korištenju konceptualnih mapa

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Sažetak. U ovom radu izložena su iskustva vezana uz korištenje konceptualnih mapa u nastavi kolegija Matematika 1 na prvoj godini učiteljskih studija na Učiteljskom fakultetu Sveučilišta u Zagrebu. Prikazani su i analizirani rezultati ankete provedene među studentima koja odražava njihove stavove prema korištenju konceptualnih mapa. Rezultati su uspoređeni s rezultatima ranije provedenog istraživanja među studentima učiteljskih studija u Ujedinjenim Arapskim Emiratima. Na kraju dajemo smjernice našeg budućeg rada i istraživanja.

Ključne riječi: konceptualne mape, nastava matematike, obrazovanje učitelja, stavovi studenata

The interrelations of the cognitive and metacognitive factors with the affective factors during problem solving

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Abstract. Problem solving is an extremely complex human endeavor that involves a complex interplay between cognition and metacognition. Students' rich store of mathematical knowledge and facts, ability to access and organize knowledge already possessed, plan strategies for implementing what is known, and monitor the effectiveness of these strategies are one of many factors adversely affecting problem-solving performance. Nevertheless, affective attributes, such as motivation, interest, pleasure, impatience, anxiety, beliefs, and persistence in problem solving are of high significance and may influence students' problem-solving performance (Goldin, 2000). Moreover, effective management of negative affective behaviors, such as anxiety and frustration, is instrumental for participants' perseverance during problem solving (Carlson & Bloom, 2005). Veenman, Van Hout-Woulters, and Afflerbach (2006) contend that we as researchers need to focus on understanding how individual differences and contextual factors interact with learning, and problem-solving processes.

With these considerations in mind, in this paper I focused on identifying affective behaviors, and describing situations where a particular affective behavior occurred in the context of problem solving with technology. Moreover, special attention was given to elucidate mechanisms participants used to cope with both productive, and counterproductive affective behaviors, and its effect on subsequent problem-solving endeavors. The results of the study showed that effectiveness of solution approaches was dependent on the presence of managerial decisions. Cognitive problem-solving actions not accompanied by appropriate metacognitive monitoring actions of affective behaviors appeared to lead to unproductive efforts. Redirection and reorganizing of thinking in productive and positive directions occurred when metacognitive actions guided the thinking and when affective behaviors were controlled.

Keywords: affective behaviors, mathematical behavior, metacognition, problem solving, regulation

Introduction

A deep and comprehensive view of problem solving as an art form emerged from works of Pólya (1945/1973). Since the 1980s the inclusion of problem solving in school mathematics was strongly endorsed by various professional organizations such as, NCTM, PISA, reform curricula, educators, and researchers making problem solving now more prominent in school mathematics than ever. Problem solving is an extremely complex human endeavor involving much more than the simple recall of facts, recall of concepts, or the application of well-learned procedures. Rather, it requires numerous cognitive and non-cognitive activities as well as many types of knowledge (Schoenfled, 1985, 1992). Nevertheless, some studies (Carlson & Bloom, 2005; Schoenfeld, 1988) point into an important direction of the influence of various affective dimensions (e.g., beliefs, attitudes, and emotions) on the problem-solving process. Hence, individual factors one brings to a problem-solving situation can greatly impact the problem solving process.

The interrelationship between metacognition, and cognition with non-cognitive factors, such as affective behaviors (e.g., attitudes, beliefs, emotions, values) during problem solving is of high significance and may influence students' problem-solving performance (Carlson & Bloom, 2005). Goldin (2000), for instance, argues that affective domain can be both productive and counterproductive; it can foster mathematical ability and creativity, but it can also inhibit the problem-solving process and have consequences in future problem-solving situations. The role of affective attributes, such as motivation, interest, pleasure, impatience, anxiety, and persistence in problem solving has a long history, but it is not stressed often (Mayer, 1998). With these considerations in mind, in this paper I focused on identifying affective behaviors participant exhibit during problem solving endeavors. Moreover, special attention was given to elucidate mechanisms participants used to cope with both productive, and counterproductive affective behaviors, and its effect on subsequent problem-solving endeavors. The research questions for the study are as follows: What are and how the affective behaviors students exhibit develop during problem-solving endeavor in a technological context? What situations and interactions during problem solving in a technological context promote both productive, and counterproductive affective behaviors? What coping mechanisms students use to engage in a productive, and successful problem-solving endeavor?

Theoretical background on the role of cognition, metacognition and affective domain

The literature on problem solving describes cognition as a set of mental processes that may be described as an experience of doing or knowing. It includes resources (factual and procedural knowledge) and heuristics (strategies) used to explore, analyze, and probe aspects of nonroutine problems in an attempt to formulate pathways to a solution (Schoenfeld, 1985). Research (Garofalo & Lester, 1985; Schoenfeld, 1985) shows that students' low problem-solving performance is not due to the inadequacy of mathematical content knowledge and facts, but rather is associated with students' inability to analyze the problem, to fully understand it, to evaluate the adequacy of given information, to organize knowledge and facts they possess with the goal of devising a plan, to evaluate the feasibility of the devised plan before its implementation, and to evaluate the reasonableness of the results. In other words, the utility of a problem solver's resources and strategies depend on the factor of metacognition. Metacognitive behaviors (regulation of cognitive activities) during an problem solving act include processes such as decisions (decision-making) that problem solvers make regarding if (assessing), when (planning), and how (monitoring) they will use their factual knowledge, procedural knowledge (resources), and heuristics while attempting to deal with nonroutine problems. Hence, cognition is implicit in any metacognitive activity, but metacognition might or might not be present during a cognitive act (Garofalo & Lester, 1985) or cognition is necessary to perform a task, whereas metacognition is of great importance to understand how a task was preformed. That is, metacognition is necessary to understand how and why is task performed whereas cognition in only necessary to merely perform the task.

Recent research (Hannula, 1999, 2002; Meyer, 1998; Veenman et al., 2006) call for giving attention to individual factors, such as beliefs, attitudes and emotions any problem solver brings in the problem solving situation. *Belief systems* refer to the student's view about self, about the environment, about the topic, and about mathematics. Consequently one beliefs about self and/or mathematics shape cognition and determine the perspective with which one approaches mathematics and mathematical tasks, and should therefore be included in any investigation of why individuals succeed or fail in their attempts to solve mathematics problems. Often during problem solving different attitudes (enjoyment, motivation, interest) and emotions (frustration, anxiety, pleasure, joy, impatience, anger,) become evident (Carlson & Bloom, 2005).

Mayer (1998) borrows from interest theory, self-efficacy theory, and attribution theory when arguing that "the will to learn depends partly on how the problem solver interprets the problem solving situation" (p. 56). *Interest* refers to natural curiosity or willing participation in activity. According to interest theory, students engage in deeper thinking when they are interested rather than uninterested in a particular activity. *Self-efficacy* refers to an individual's judgment of his or her own capabilities. According to self-efficacy theory, students who have high self-efficacy understand the material better and are more successful in problem solving. *Attributions* refer to the explanations one uses to make for success (e.g., effort) or failure (e.g., difficulty). One's feelings and beliefs about one's interest and ability to solve problems can help or hinder the problem-solving process (Mayer, 1998). Similarly, Goldin (2000) refers to the affective domain more broadly as a tetrahedral model including beliefs and belief structures, attitudes, emotional states, values, ethics and morals. Affective states are described as "local changing states of feeling that the solver experiences and can utilize during problem solving – to store and provide useful information, facilitate monitoring, and evoke heuristic processes" (p. 209). Affective states interact productively or counterproductively with problem solving and include curiosity, puzzlement, bewilderment, frustration, anxiety, fear and despair, encouragement and pleasure, elation and satisfaction.

Affect can foster mathematical ability and creativity, but it can also inhibit the problem-solving process and have consequences in future problem-solving situations. Moreover, Carlson and Bloom (2005) showed that effective management of negative affective behaviors, such as anxiety and frustration, was instrumental for participants' perseverance during problem solving. These studies point in an important direction focusing on understanding the influence of complex construct of affect during problem solving and extend it to characterizing these affective states and their use during problem solving. The above discussed theoretical and empirical literature served as lens for coding the data and to help describe and understand critical situations where non-cognitive factors productively or counterproductively interacted with (meta)cognitive processes.

Methodology

Methods

For this study, a qualitative research design was chosen. A purposeful sampling strategy (Patton, 2002) was utilized as a way of collecting rich and in-depth data from the research participants. The participants in this study were two preservice teachers, Wes and Aurora (pseudonym), from the mathematics education program at a large southeastern university in the United States with whom I had previous experience working. These two participants already had experience working in a technological environment, were comfortable working with me, and could easily verbalize their thoughts and feelings. Data sources for this study consisted of different verbal reports (think aloud protocol, concurrent verbalization methods, such as prompts and probing), individual interviews after each problem-solving session, students' written solutions to given three problems, researcher's observation notes, video files of problem solving sessions and a final interview. Each participant solved individually one nonroutine geometry problem per problem solving session using unlimited amount of time. The mathematical problem solving tasks included three nonroutine geometry problems selected and modified from a variety of sources, including mathematical journals, textbooks and mathematical web sites. The problems were chosen such that they demanded strategy flexibility, thinking flexibility, provided participants with opportunities to engage in metacognitive activity, and covered mathematical content area in geometry. Three types of problems were used for this study: construction, applied, and exploration problem where the nature of the problems allowed exhibiting different metacognitive processes, multiple solution paths, and different uses of the Geometer's Sketchpad (GSP) that consequently enhanced understanding the nature of metacognition when problem solving in a DGE.

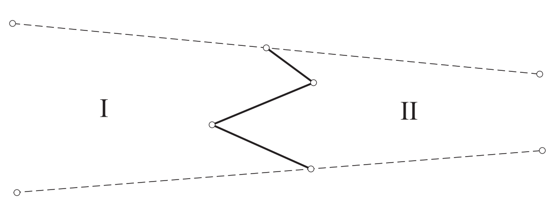
Data Analysis

For the purpose of this study, multiple stages of analysis, as suggested by Patton (2002) were conducted using inductive analysis. First, all data from the think-aloud sessions, the interview sessions, hard copies and GSP sketches of participant's solutions, and researcher's field notes were transcribed after each problem solving session. I then analyzed the data for convergence or determining which pieces of data were similar using inductive analysis that allows construction of themes or theories that are "grounded" in the data. When using inductive analysis, I focused on creating codes and categories from the data, developing or enhancing theory during the act of analysis and the use of constant comparative method during analysis of the data. Using the constant comparative method, I categorized all data that consisted of comparing and generating categories, integrating categories, and delimiting the theory to help illuminate common themes across cases and within cases. During this process it was important to keep the voice of participant present to help develop a deep and sound understanding of a phenomenon under investigation. Data analysis was a continuous task that started at the beginning of data collection.

Results and discussion of findings

In this section I describe the affective behaviors and the coping mechanisms by the two participants when completing the "Land Boundary Problem" (see Figure 1), that was taken from the book *Euclidean and Transformational Geometry: A Deductive Inquiry* (Libeskind, 2008), but it was also available from the TIMSS video study. I chose this exploration problem because, of the problem used in this study, it provided the richest exhibition or personal factors and its relation to (meta)cognitive behaviors. The participants received this problem after the first part of the problem, where the border was just once bent, was solved. In the following sections I describe shortly Wes's and Aurora's solution with respect to the research questions. A more detailed description of the problem solving paths of both participants can be found in Kuzle (2011).

The boundary between two farmers' land is bent, and they would both like to straighten it out, but each wants to keep the same amount of land. Solve their problem for them.



Justify your answers as best as you can.

Figure 1. The Land Boundary Problem.

A description of Wes's solution

When having received the extension of the problem, Wes was shocked and expressed his concern that his confidence might be shattered now because he assessed the problem harder than the original one. Past first signs of anxiety, he read the problem before considering what needed to be done. He immediately decided on a choice of perspective – use the strategy from the first part of the problem. That is, drawing a parallel line through the "peak" point to the segment opposite to that point. In his first attempt he tried to use the strategy at once and was confident it would work out; that is, applying the strategy on both bents simultaneously (see Figure 2, left). He promptly jumped into implementation of it without carefully examining his plan. However, since that attempt was not successful, he was surprised by the outcome and could not comprehend what he did wrong. However, he then revised his problem solving approach deciding to use the strategy this time is a systematic way, that is, applying the strategy on a bent one at a time (see Figure 2, right). For both plans, although he had strategy ready at hand, selection of steps was not assessed prior to their implementation though confident it would work.

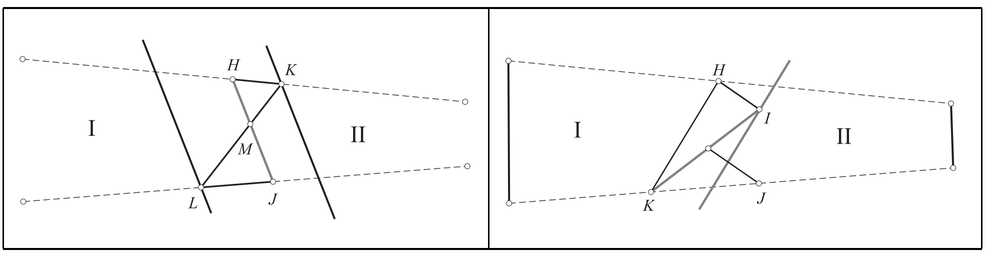


Figure 2. Wes's attempts while solving the problem.

Before devising his third and final solution plan which involved reducing the number of constraints, he took a step back and reflected on the process and solution before directing his thinking to a new plan as he did not want to be wrong again showing signs of pride and ego. Taking a step back was a copying mechanism that allowed orchestration of both metacognitive and non-cognitive behaviors. He made two false attempts thus far and though different negative affective behaviors were exhibited, he persevered in his attempt to solve the problem. Hence, personal motivation and beliefs about mathematics "overwrote" the negative affects exhibited during the problem solving process. Though his plan as a whole was not coherent, careful evaluation of undertaken activities and regulation of his thinking and knowledge, led him to solving the problem. Hence, when a result was refuted, he assessed the approach, revised the plan prior to moving to a new problem solving cycle. The use of technology did not cause any negative affective behaviors, bur rather supported his (meta)cognitive actions not hesitating when to use technology to execute or check his thinking. He exhibited signs of joy and excitement when he realized he solved the problem correctly and having learnt a new problem solving strategy.

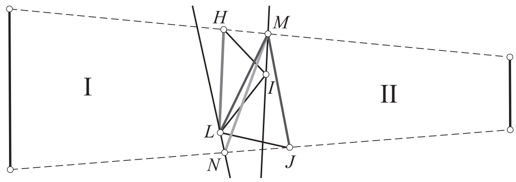


Figure 3. Wes's solution of the Land Boundary Problem.

A description of Aurora's solution

Aurora started the session by reading the problem in silence, and immediately tried to consider knowledge and strategies relevant to the problem. Her motivation to make sense of the problem was influenced by curiosity and pride. Even though she contemplated between using the strategy from solving the first part of the problem (drawing a parallel line through a "peak" point to the segment opposite to that point) or approaching the problem by measuring the area and working backwards, she decided to go with the latter as she did not know how to use the strategy from the original problem (see Figure 4).

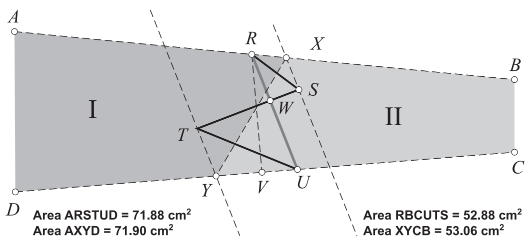


Figure 4. One of Aurora's attempts while solving the problem.

Her beliefs about mathematics, hence, guided her to use technology to guide her understanding and solving of the problem. The rest of the session was spent on searching for a solution where lack of assessment of potential utility of her planned actions and frustration made the following endeavors unproductive. She persevered throughout, however, and by taking a step back from the problem, evaluating what was done and what needed to be done, directed her thinking to how to use the strategy from the original problem, which ultimately increased her confidence. Similarly to Wes, she focused first on one bent and straightening it out. Then she obtained one bent instead of two using the strategy again to obtain one straight bent. This process was sequential and monitored but made the overall endeavor a success. Thus, fruitful problem solving efforts relied upon regulation of affective behaviors as well. She evaluated her solution by using the measurement function of the technological tool. Hence, another dimension of affect, namely her Aurora's mathematical integrity was exhibited. Here the measurement provided by the software was used as a standard to validate if her solution was correct and if problem was satisfactory solved. Both positive affective behaviors, such as satisfaction, interest and confidence, and negative behaviors, such as frustration, anxiety and embarrassment, change frequently during the problem solving process; puzzlement and success lead to motivation and interest. Failure, after high confidence, and struggle, on the other hand, lead to frustration and anxiety. This session clearly demonstrates how lack of monitoring and evaluation skills may prompt negative affective behaviors that consequently influence the problem solving process; cognitive processes take domination over metacognitive processes until regulation of unhealthy affective behaviors and reflection on undertaken activities occurs.

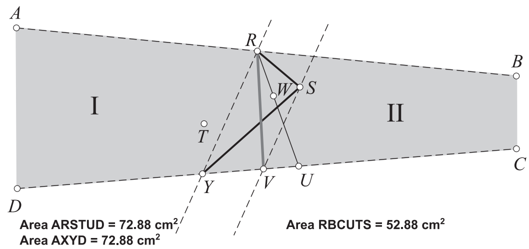


Figure 5. Aurora's solution of the Land Boundary Problem.

Conclusions

The findings of the study showed that affective behaviors, such as perseverance, persistence, confidence, interest, and frustration occurred frequently during the problem solving activity. These affective behaviors changed during the process of solving a problem, and were related to participants' success when problem solving. More specifically, these affective behaviors influenced variety of metacognitive processes, such as planning of cognitive activities, monitoring of cognitive activities, and evaluating the outcomes of former activities. For instance, both participants had a natural curiosity, confidence and motivation to solve the problem, and at the beginning of problem solving they engaged in deep thinking. They were confident in their understanding of the problem, and an approach to solve it. The affective states of interest and confidence interacted positively with problem solving. During the planning phase (Pólya, 1945/1973), participant's beliefs about mathematics and problem solving influenced what and how the decisions were made. However, once their problem solving approach did not result in an expected way, lack of evaluation of the plan and monitoring of quality thinking during the execution of a plan causing negative affective behaviors (anxiety, frustration, joy) to arise similar to results of Hannula (1999, 2002). Within the last phase of problem solving (Pólya, 1945/1973), many affective behaviors were displayed, whereas frustration at this point sometimes overwhelmed the solver and influenced cognitive behaviors to take domination over metacognitive processes, which was mostly observed in the case of Aurora. In these problem-solving situations as a result of an incorrect path, she stopped monitoring and evaluating her progress, and engaged in lengthy pursuits characterized by weak structure and impetuous jumps from one particular direction to another. She persevered in her problem-solving path, but never persisted too long on a chosen problem-solving path. However, she most commonly persisted on a choice of strategy (trial-and-error – as a result of her beliefs. Moreover, she did not get discouraged by false attempts using this problem solving strategy, because she believed that false moves are part of doing mathematics helping her learn mathematics, and attain a correct solution. Once she engaged in a coping mechanism (taking a step back), she was able to effectively manage the affect of frustration. During the act of problem solving different affective states acted both productively and counterproductively with metacognitive processes as was similarly shown in other studies (Carlson & Bloom, 2005). Redirection and reorganizing of thinking in productive and positive directions occurred when metacognitive actions guided the thinking and when affective behaviors were controlled. Effective management of different affective behaviors such as, anxiety and frustration, allowed both participants to persevere in their problem solving activity and experience joy, pride, and satisfaction.

The observations made in this study support the arguments from other researchers (Carlson & Bloom, 2005; Veenman et al., 2006); affective responses are extremely complex entailing structures of intimacy, integrity and meta-affect. As Goldin (2000) pointed out, affect is fundamental during problem solving – it can both foster ability, but it can also inhibit the current and future problem-solving process. Therefore, research on problem solving should not be studied in isolation, but take into consideration complex construct of affect during problem solving and extend it to characterizing these affective states and their use during problem solving. Last but not least, mathematics teacher education programs should allow preservice teachers with opportunities to learn about a variety of pedagogical and learning issues, and means for implementing problem solving within their lessons, as well as to also experience them with respect to (meta)cognitive and noncognitive aspects of problem solving

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Međusobni odnosi kognitivnih i metakognitivnih čimbenika s afektivnim čimbenicima tijekom rješavanja matematičkih zadaća

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Sažetak. Rješavanje problema je izuzetno složen ljudski poduhvat koji uključuje kompleksnu interakciju između spoznaje (kognicije) i metakognicije. Učenikovo matematičko znanje potrebno prilikom rješavanja matematičkih zadaća, pristup i mogućnost organizacije znanja koje posjeduje, različitost i bogatstvo strategija potrebnih za provedbu zadataka i njihov nadzor neki su od mnogih čimbenika koji utječu na uspješno rješavanje matematičkih zadaća. Ipak, afektivni atributi, poput motivacije, interesa, zadovoljstva, nestrpljivosti, anksioznosti, stanovišta i upornosti su od velikog značaja prilikom rješavanja zadaća i mogu utjecati na učenikov čin rješavanja zadaća (Goldin, 2005). Štoviše, učinkovito upravljanje negativnim afektivnim ponašanjima, kao što su tjeskoba i frustracija, je instrumentalno za učenikovu upornost tijekom rješavanja matematičkih zadaća (Carlson i Bloom, 2005). Veenman, Van Hout-Woulters i Afflerbach (2006) tvrde da se znanstvenici trebaju usredotočiti na razumijevanje kako se individualne razlike i kontekstualni čimbenici odnose s učenjem i procesima prilikom rješavanja matematičkih zadaća.

S tim razmatranjima na umu, u ovom članku sam se usredotočila na prepoznavanje afektivnih ponašanja, te na opisivanje situacija u kojima se konkretno afektivno ponašanje dogodilo prilikom rješavanja matematičkih zadaća u geometrijskom dinamičkom okruženju. Štoviše, posebna pažnja posvećena je rasvjetljavanju mehanizama koje su sudionici koristili kako bi se nosili s produktivnim i kontraproduktivnim afektivnim ponašanjima i njihov utjecaj na kasnije rješavanje problema. Rezultati studije su pokazali da je djelotvornost pristupa bila ovisna o prisutnosti rukovoditeljskih odluka. Kognitivni procesi koji nisu popraćeni odgovarajućim metakognitivnim nadzorom nad afektivnim ponašanjem rezultirali su neproduktivnim naporima. Preusmjeravanje i reorganizacija razmišljanja u korisnim i pozitivnim smjerovima dogodila se kada su metakognitivne akcije upravljale razmišljanjem i kada su afektivna ponašanja bila pod kontrolom.

Ključne riječi: afektivna ponašanja, matematičko ponašanje, metakognicija, rješavanje matematičkih zadaća, regulacija

The influence of formal education and personal initiatives on the willingness to use ICT in teaching mathematics

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Abstract. Application of computer technology in the classroom is a challenge for every teacher who wants to follow the development of the information society. Teacher's competence to work with computer technology certainly depends on formal education but also on personal initiatives.

This paper presents the results of study on the willingness of mathematics teachers to use ICT in teaching mathematics in the different educational cycles with respect to their competences. Teachers are aware of the need to use ICT, but point out the shortcomings of their formal education, that shows progress, and emphasize a great need for personal initiatives and further education.

Keywords: mathematics, ICT, IT competences, formal education, personal initiative

Introduction

Observing the educational system, we are witnessing of an active integration of information and communication technologies (abbreviation: ICT) in teaching process at all levels. The young generations, especially generations of today pupils, develop a specific type of coexistence with ICT, it is easy to assume that we can expect from young generations of teachers a high level of ICT integration in teaching mathematics. This statement was discussed in a population of students, future math teachers (Dobi Barišić et al., 2011). Young generations are those that will not be considered ICT as a stumbling block in the future work, because it is a part of their everyday life and a component part of a formal education as well. However, what about the elder generations of teachers who are not in coexistence with ICT? Is there, among them, any interest for integration of ICT into teaching, and whether, is interest the most important or perhaps some other components are more important for the successful integration of ICT in teaching mathematics?

In addition, we wonder, whether the application of ICT in teaching process depends on the curriculum that teachers perform, that is designated by the educational cycle in which they teach. Ministry of Science, Education and Sports of the Republic of Croatia (2011) defined, by the National Curriculum Framework, four educational and developmental cycles based on the developmental stages of pupils that comprise: (1) the first four grades of elementary school, (2) fifth and sixth grades of elementary school, (3) seventh and eighth grades of elementary school and (4) first and second grades of vocational and art high school and all four grades of gymnasium. In vocational education, cycles continue depending on the level and standards of qualification.

Under the application of ICT in teaching, we imply possession of certain competencies so that integration and lessons would be successfully completed. Competencies include knowledge, skills and attitudes needed to perform the job, and as a special type of competencies we observed digital competencies. They include reliable and critical use of ICT for learning, self-development and participation in society (Marcetić et al., 2010). Learning and development through self-initiative, as elements of digital competencies, perhaps are the most important factors that could be considered if we compare mathematics teachers.

This paper presents the results of research conducted on the population of mathematics teachers. We examined their views on several issues related to the integration of ICT into teaching: the perception of the need of possessing digital competence, IT qualifications gained by university education, the personal initiatives in the area of digital competences and IT infrastructure. Mathematics teachers we have observed through two criteria: (k1) year of enrolment in the study compared to 2000, (k2) educational cycle in which teacher teaches (first, second or third, fourth).

Considering that Croatian resources required for the implementation of ICT in teaching experienced a significant growth after 2000, formal education of teachers who enrolled University before 2000 is largely lag in terms of education by using ICT and about ICT. Teachers of these generations, in this paper, we considered teachers of elder generations. The teachers, whom in this paper we considered the younger, were university educated at the time when the infrastructure was developed enough to represent ICT in a formal university education. Namely, Croatian national centre for exchange Internet traffic was established in 2000, making the Internet and exchange of the data accessible to the general public, so we have defined this year as a breaking for study enrolment. Also, in 2001 pan-European Academic and Research Network GEANT was put into, on which CARNet is connected to and it was a big step for the development and application of ICT in Croatian schools and academic community[1].

Literature review

In 2006 research was conducted in 22 countries which has shown that, across educational system, a proportion of the math and science teachers were using ICT extensively and that the traditional and lifelong curriculum goals are considered

[1] http://www.carnet.hr/o_carnetu/o_nama/kronologija, 5.1.2013

equally important. Higher use of ICT in education was associated with higher level of digital competencies and self-initiative for their development, and with a greater affinity to collaborate with teachers from other schools (Voogt, 2010).

A modern mathematics curriculum in Ghana emphasizes the need for integration of ICT in teaching mathematics at all levels of education, in order to improve student achievement in mathematics. Within the curriculum is not defined whether teachers will be provided an aid or assistance for the integration of ICT in teaching in the future, and this is what is considered to be a major problem. Research has included 180 math teachers, of which 60 in-service and 120 pre-service teachers. The results showed that the biggest barriers to the integration of ICT in teaching mathematics are lack of knowledge about how to integrate ICT in teaching and lack of training on this issue. Barriers that are still evidenced are lack of infrastructure and lack of interest in school for integration of ICT. Although even 98% of the respondents answered positively to the question whether their school has a computer room, only 15% of them have a computer in the mathematics classroom and 47% access to the Internet. Of all the teaching strategies, teaching with the use of ICT was the least represented, although the percentage of teachers who expressed enthusiasm for the integration of ICT in teaching was 98% of in-service and 94% of pre-service teachers. Only a few teachers attended some form of education related to the integration of ICT in teaching mathematics, and the author considers development of such programs in the future a step in the right direction (Agyei & Voogt, 2011).

In Pakistan, in Lahore city, research on the utilization of ICT in teaching mathematics was conducted. Research included 20 teachers and 100 math students from all educational institutions of Lahore city. The results showed that teachers use ICT in teaching but that the need for its use is far greater. There is a lack of people who know how to apply ICT in teaching in the observed institutions, although it is closely associated with time saving, workload reduction and creation of interest among students and teachers. The use of ICT in teaching has shown greater efficacy than some other aids (Azeem & Ashfaq, 2010).

Results of research conducted in Greece at 118 primary school teachers, have shown that majority is not pleased with the current training for application of ICT in teaching, that they consider the cause of inappropriate use of ICT. However, they have a positive attitude about the idea of training and believe that it should be a continuous, systematic and appropriate for the subject they teach, and focused on new pedagogical approaches to teaching, learning and classroom management (Panagiotis et al., 2011).

Research conducted in the Pakistani high schools about the effectiveness of ICT in teaching mathematics showed that teaching with ICT gives better results at a lower level mathematics than traditional teaching. In the sample were 120 students, 60 of whom attended public schools and 60 private schools. The teaching units in the both the groups were Sets, Algebraic Expressions and Logarithms. Each sub-sample was divided into experimental and control groups of 30, with the attempt to ensure equivalence of groups by knowledge. The experimental group was learned with the help of ICT, while the control group learned with traditional methods (Safdar et al., 2011).

Sample and data

Competence of teachers to work with computer technology depends on the formal education as well as on the personal initiatives. In the previous paper (Dobi Barišić et al., 2011) we examined willingness of final year students enrolled in Master of Arts in Teaching Primary Education program and Master of Arts in Teaching Mathematics and Computer Science program to use ICT in teaching mathematics. There we established a conceptual model of the situation which shows the impact of formal education initiatives and mathematics teachers to integrate ICT in teaching mathematics. Using that model we have created a survey where we have collected data on the population of interest, such as mathematics teachers.

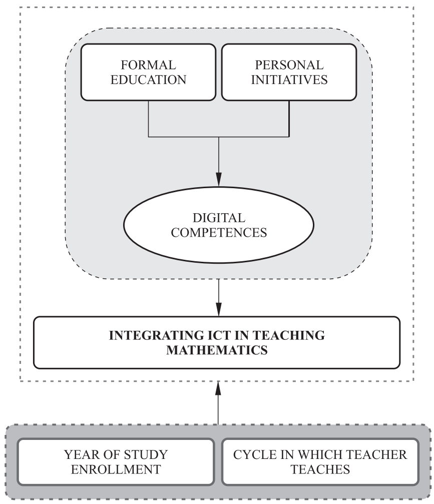

Figure 1. Conceptual model which shows the influences on the integration of computer technology in teaching mathematics in practice.

We made initial hypothesis that there exists a correlation between the time of study enrolment and educational cycle in which the teacher teaches and the following concepts: (a) the perception of the need of possessing digital competence, (b) IT qualifications gained by university education, (c) the personal initiatives in the area of digital competences and IT infrastructure, (d) the integration of ICT in teaching mathematics. Therefore, we divided the mathematics teachers with respect to two different criteria: (k1) years of enrolment in the study compared to 2000, (k2) educational cycle in which teacher teaches (first, second or third, fourth). The teachers that were categorized as younger ones were enrolled in the university at a time when computer science was developed enough to be represented in the formal education. Our initial hypothesis is represented with the model in Figure 1.

Within core digital competences we analysed the possession of knowledge about the following: using electronic mail, creating digital textual and presentational materials, creating spreadsheets, drawing and image processing, and using Internet as additional source of information necessary for creating teaching materials. Special digital competences were analysed through the possession of knowledge and skills to work in some ICT tools appropriate for integrating in mathematics lessons, such as mathematical educational software.

We have examined to what extent mathematics teachers used ICT in the classroom and how much of IT training they experienced during their formal education. We examined if they used the Learning Management System, the digital presentation materials and the dynamic geometry tools. Attention was given to the student-teacher communication via email and to the existence of Web pages for enrolled courses. Within the IT training, we had investigated if mathematics teachers gained the skills to work in a dynamic geometry tool, used the Internet as a source of additional information, and if they used ICT for the development and presentation of a seminar or lecture.

We examined personal initiatives of mathematics teachers through the constant update of their digital competences and the possession of IT infrastructure. We focused on the need for continuous update of digital competence through various forms of informing such as reading IT magazines, browsing relevant web portals, attending computer courses and self-studying of ICT tools. IT infrastructure is investigated through possession of personal computer with Internet access at home.

We also examined the attitude of mathematics teachers about the application of ICT in teaching mathematics and their willingness to integrate ICT within their teaching.

In this study, we had used an online questionnaire as a method of collecting data. Data are obtained from the samples of individuals from the population of teachers (n =146) who teach mathematics in different educational cycles. Data were collected in January 2013, and include mathematics teachers employed in the eastern region of the Croatia. Mathematics teachers are divided according to two different criteria (k1) years of enrolment in the study compared to 2000, and (k2) educational cycle in which teacher teaches (first, second or third, fourth). The questionnaire was completed by 100 (68,49%) teachers enrolled in study before 2000, while the number of those enrolled in the study in year 2000 or after is 46 (33,51%). Given the educational cycle in which the respondents teach, the following data were collected: 49 (33,56%) respondents teaches in the first cycle, 49 (33,56%) in the second and third, 47 (32,19 %) in the fourth the educational cycle, and one respondent did not plead in which cycle he teaches. The structure of the sample is described in the table below (Table 1).

Table 1. Sample structure.

Teachers in educational cycles	**Total in cycle**	**Enrolled in study before 2000**	%	**Enrolled in study in 2000 and after**	%
first	49	43	87,75	6	12,25
second & third	49	27	55,10	22	44,90
fourth	47	29	61,70	18	38,30

Responds collected from the sample of individuals are discrete quantitative data that were recorded on meaningful integer numerical scale from 1 to 5. They label a grade given to certain statement, where 1 stands for “I totally disagree” and 5 stands for “I completely agree”.

Methodology

After collecting data on variables of interest from introduced model (Figure 1), we conducted statistical analysis using Soft STATISTICA 7.0. We estimated population parameters and tested statistical hypotheses that derived from the research hypothesis in order to make conclusion about the population of interest among every categorization criterion.

Testing statistical hypothesis is conducted on data collected from the sample of individuals in order to make inferences about differences in means for variables of interest considering every categorization criterion. Hypothesis testing is conducted by following proper steps (McClave et al., 2001). Level of significance of tests applied in this paper is α =0,05. For comparing means of the population of teachers considering the year of study enrolment, we used z-statistic for comparing two populations' means for large-sample. For comparing means of the population of teachers considering the educational cycle in which teacher is teaching, and for comparing means of the population of young teachers considering the year of study enrolment, we used Kruskal-Wallis test for comparing more than two probability distributions and post hoc multi criteria comparing in case of rejecting null hypothesis.

Results

At the beginning we investigated the math teacher's perception of the application of ICT in teaching mathematics. Results obtained from the statistical tests reveal that at the 5% level of significance neither one of categorization criteria has influence on described perception (p_1 =0,912219; p_2 =0,2371). This variable mean is estimated with high average rate (from 3,880752 to 4,577509) for all groups of teachers what indicates teachers' awareness about the necessity of ICT use in teaching mathematics despite teachers' characteristics

Onwards as part of introductory research we explored the math teacher's perception of the readiness for ICT use in teaching mathematics. Results obtained from the statistical hypothesis testing reveal that at the 5% level of significance there is statistically significant difference in described perception considering both categorization criteria (p_1 =0,044154; p_2 =0). Teachers, who enrolled in the study in the 2000th or later, are more ready than the others (p =0,036751). Teachers who teach in second or third (p =0,000053) and in fourth (p =0,000002) educational cycle are more ready than those teaching in first educational cycle. Furthermore, we declared teachers, who evaluates their readiness with grade 4 or higher, ready to use ICT in teaching mathematics. Proportion of ready math teachers is estimated with 95% confidence interval [0,53564; 0,695124].

Since using computer technology in teaching requires core and special digital competences possessing, in addition we explored math teachers' attitudes about need of possessing those competences. Considering first criterion (year of study enrolment) we infer that at the 5% level of significance there is statistically significant difference only in the perception of need of possessing knowledge about using Internet as additional source of information in creating teaching materials (p =0,002456). Younger teachers consider this competence more valuable than the elders, while there is no significant difference in the evaluation of other digital competences between younger and elder teachers (p >0,05). This finding acquaints the awareness of math teachers about possessing digital competences. The absence of statistically significant difference between older and younger teachers reveals that teachers despite the shortcomings of their formal education, which are justified by the development of computing, are perceptive about the need of following the development of knowledge in the field of computer science. Considering second criterion (educational cycle) we infer that at the 5% level of significance there are statistically significant differences in perceptions of the need of possessing knowledge about using Internet as additional source of information in creating teaching materials (p =0) and working in some specialized computer tool for mathematics education (p =0,0115). Math teachers, who teach in first educational cycle, consider these knowledge less important than the others. Since the agreement about the values of the most digital competence (5 of 7) among math teachers' groups is achieved, we believe that math teachers are perceptive about the need of possessing digital competence for using computer technology in their teaching practice.

Additionally we explored IT knowledge acquiring at university education and the need for personal initiatives on this subject.

First we analysed the math teacher's perception of IT knowledge acquiring at university education. Results obtained from the statistical hypothesis testing reveal that at the 5% level of significance there is statistically significant difference in explored aspects of university education considering the year of enrolment criterion (p =0 for all variables of interest). Younger teachers evaluated all those aspects statistically significant higher than the others (p =0 for all variables of interest). We believe that reason for this is the development of computer science. For this finding we investigated the perception of IT knowledge acquiring at university education only for younger math teachers. Results obtained from the statistical hypothesis testing reveal that at the 5% level of significance there is no statistically significant difference in explored aspects of university education considering the educational cycle criterion (p >0,05 for all variables of interest). Younger teachers showed homogeneity in evaluation of their IT knowledge acquired at university education considering the educational cycle criterion. In order to identify progress in IT knowledge acquiring at university education, we grouped younger math teachers considering the enrolment year: 2000th and 2001st (n =11), from 2002nd to 2004th (n =18), from 2005th to 2007th (n =16). Results obtained from the statistical hypothesis testing and based on described grouping reveal that at the 5% level of significance there is statistically significant difference in following aspects of university education of younger math teachers: teaching supported by learning management system (p =0), using dynamic geometry tools by university teachers for presentation and demonstration in their teaching (p =0,0451), communication with teachers using electronic mail (p =0,0006), existence of course web site with updated data (p =0), acquiring competence of working in some dynamic geometry tool (p =0,006). All these differences indicate that university education ensured better IT training for those who were enrolled later. This finding points to improvements in the educational system that is trying to trace the development of computer technology, and to ensure proper IT training for students preparing for teaching mathematics.

We discussed the perception of the necessity of the math teacher for taking self-initiatives in direction of acquiring and updating digital competences and realisation of IT infrastructure. Results obtained from the statistical tests reveal that at the 5% level of significance neither one of categorization criteria has influence on described perception (p >0,05 for all variables of interest). This finding indicates that math teachers know that the update of digital competence is necessary for adequate integration of computer tools in teaching. Besides, they know how their competences can be updated: following IT journals ($\bar{x}$ =3,527397; σ =1,090367) and web portals ($\bar{x}$ =3,938356; σ =0,926414), attending IT seminars ($\bar{x}$ =4,171233; σ =1,012744) and studying computer tools independently ($\bar{x}$ =4,397260; σ =0,817730). Obtained results indicate that math teachers, regardless of their specific features described in introduced criteria, believe that the possession of the IT infrastructure at home is a prerequisite for high-quality preparation of teaching mathematics. IT infrastructure includes a personal computer ($\bar{x}$ =4,575342; σ =0,869686) with internet access ($\bar{x}$ =4,547945; σ =0,895061). This finding indicates that teachers depend on their own resources if they want to prepare quality lessons, what includes the use of computer technology.

Discussion

In this paper we found that math teachers are aware of the need for using ICT in teaching regardless of their features. Younger teachers and those who are teaching in second, third and fourth educational cycles are more ready to use technology in their teaching.

Math teachers showed a high perceptiveness of the need for possessing digital competence what is positive inference. Besides, they know that the actualization of the digital competence is necessary and how to update their knowledge. In the following researches we plan to examine whether they really work at it.

Results indicate that the age of teachers, i.e. study enrolment year, does not affect the attitudes about using computer technology in teaching. Despite the shortcomings of their formal education, which are justified by the development of computing, math teachers are perceptive about the need of following the development of knowledge in the field of computer science.

University education ensured better IT training for those who were enrolled later. This finding points to improvements in the educational system that is trying to trace the development of computer technology, and to ensure proper IT training for students preparing for teaching mathematics.

Finding of this study that should be pointed out is that teachers depend on their own IT resources if they want to prepare quality lessons, what includes the use of computer technology.

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Utjecaj formalnoga obrazovanja i osobnih inicijativa na spremnost za primjenu ICT-a u poučavanju matematike

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Sažetak. Pravilna primjena računalne tehnologije u nastavi predstavlja izazov za svakog predavača u praksi koji nastoji pratiti razvoj informacijskog društva. Osposobljenost predavača za rad s računalnim tehnologijama dakako ovisi o formalnom obrazovanju kao i o osobnim inicijativama.

U ovom radu su prikazani rezultati istraživanja o spremnosti predavača matematike na primjenu ICT-a u nastavi matematike u različitim obrazovnim ciklusima s obzirom na njihovu osposobljenost. Predavači su svjesni potrebe za primjenom ICT-a, ali naglašavaju nedostatke svoga formalnog obrazovanja, koje pokazuje napredak, te veliku potrebu za osobnim inicijativama i kontinuiranom edukacijom.

Ključne riječi: matematika, ICT, informatička osposobljenost, formalno obrazovanje, osobne inicijative

Humour in teaching mathematics and computer science courses – yes or no?

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Abstract. The aim of this study was to examine the role of humor in teaching mathematics and computer science courses at Faculty of Teacher Education in Osijek. The paper explores whether the humor is used and in which intensity it is used as an effective mean of teaching and communication at mathematics and computer science classes, and it also identifies which acceptable form of humor is most often used. Furthermore, the paper investigates whether the use of humor in teaching computer science and mathematics courses has a stimulating effect on the environment for learning in the sense of creating a nice "classroom" climate without stress. During this research two questionnaires were created and used. One of the designed questionnaires was used to test students' attitude about the use of humor in teaching. In fact we wanted to see whether or not students participated in the teaching process which included humor in some of its acceptable forms, was the effect of that humor stimulating for their learning and did it have impact on overall impression of the course where the humor was used and what effect did the use of humor had on communication with the teacher who has using it. The second questionnaire was used to collect data from teachers themselves, and it was based analogous to the first questionnaire. Answers obtained by the means of used questionnaires were analyzed and the results showed that although teachers of mathematics and computer science courses at Faculty of Teacher Education in Osijek recognize the benefits of using humor in the classroom, they use it rarely and occasionally in the practice.

Keywords: humor in the classroom, improving the learning environment, attitude of students, the impact of humor in communication

Literature review

There are many studies about effects of humour on stress and anxiety reduction, increasing mood and self-respect so as on health condition in general. So called “laugh therapies” are becoming popular and they yet have to show their therapeutical action. Humour researcher describe laugh as response to pleasant and amusing physical, emotional and/or intellectual savour which impacts on brain in an interesting and very complex way (Lovorn, 2008). It is obvious that persons who approach with smile on their face will make pleasant beginning of communication in contrast to serious, business approach. It is becoming clear that humour can have significant impact on daily communication and working environment. Polimeni & Reiss (2006) emphasize that humour is strictly set to be cognitive, intellectual process which "success" in majority depends on the moment in a sense of the social environment and emotional state of the individual. For using humour in communication it is not necessary to be a comic or a clown, transfer of idea is important as well as the communication which in appropriate situation can result with humour (Weaver & Cotrell, 2001). Even though there is no culture which does not know humour, understanding humour and humour assessment are not equally recognized in all social areas (Shiyab, 2008). It is important to emphasize intercultural level of humour because of the subjects that are intended for precise group of people (political, business and ethnographic). At this sort of humour it is important that presenter and listener belong to the same group or the one that understands and accepts the meaning of illustrated humour (Shiyab, 2008). We can say that both area of our interest, mathematics and computer science courses, develop specific type of humour.

In term "humour in teaching" it is necessary to differ two significantly various aspects. Teacher can be humorous person by nature and popular among students by expressing through classroom communication which can serve for successful obtaining and retaining students attention during teaching. On the other hand systematic and rational implementation of humour in teaching, tests and course materials is relatively new didactical idea which often comes to skepticism (Lovorn, 2008). In traditional classes and for traditionally oriented teachers using humour in teaching is considered as a waste of time within a context of time provided for teaching and the way of communication which results by reducing the authority of teacher and overall decreasing of the teaching quality (Lovorn, 2008; Shiyab, 2008). Humour can be powerful tool for relaxing students by fostering sense of openness, respect and security which improves process of learning through pleasant atmosphere by integrating humour with content of course through planned and spontaneous deviation from traditional style of lecture (Kher, Molstad & Donahue, 1999). Shiyab (2008) discusses roles of humour in teaching which involve learning quality increasing, firming of teacher – students connection, enhancement of teacher status among students, helping in interaction student – teacher, allegation of students on recognizing the quality of teaching, acting like compensation or reward to students for their attention and improvement of general social interaction without educational connotation. In two separate studies, Weaver & Cotrell (2001) have observed importance of using humour in teaching from teachers and students point of view and determined existence of significant disproportion between students and teachers opinions. Similar study was conducted by Torok & McMorris (2004) and established that students support often using humour by teachers as well as both students and teachers prefer positive types of humour even though sarcasm as a negative type of humour was on the fifth place of frequency of using and recommended type of humour. Another similar study (White, 2001) emphasizes that using humour in teaching reduces stress, gains attention and creates a healthy learning environment but recommends that humour need not be used to embarrass students, scare or retail them. Greatest variations are detected in attitudes of students and teachers about using humour for solving unpleasant situation where students have significantly highlighted benefit of using humour. Kozbelt & Nishioka (2010) extract positive correlation between humour comprehension and humour production.

In terms of mathematics (Paulos, 1982) humour and mathematics are not connected just on the level of humour specificity, logic of humour is illustrated through mathematical terms, theorems and theories. Humour is expressed as a cognitive process and encouragement of cognitive activity while attributes characteristic for mathematic as a science with humour – logic, forms, rules, humour structure are connected. The most trivial is illustration of elegance and economics of mathematical proof with the technique of telling jokes – to say enough, with right attitude, voice tonality and face expression but not too much to reveal the point before time. In intersection of mathematical and humoristic ideas there are paradox, brain teasers, puzzles and tricks. Demand for understanding certain type of humour involves recognition of logical thinking which is validly used in techniques of mathematical proof like *reductio ad absurdum*, assumption and wrong conclusion, equivalence, iteration and other. It is stated that without understanding what is exact, equal, congruent or natural we cannot realize wrong, unusual and unnatural (Paulos, 1982). More studies were conducted about the effect of using humour in language teaching. Using humour in teaching reduces tension, improves classroom atmosphere, increases relation student - teacher, facilitates learning and has a positive impact on language learning settings in a sense of increasing motivation as well as success (Aboudan, 2009). Teachers' primary goal to encourage and motivate students as well as to help them communicate in the target language is accomplished every time students are amused and entertained during an activity (Munoz-Basols, 2005). Ketabi & Simin (2009) also confirm detected humour efficacy as a very useful strategies in teaching and learning foreign languages. Among many benefits of implementing humour in the target language classroom several of them may be emphasized: creating a cooperative atmosphere to help students to better relate to one another, facilitating the acquisition of vocabulary, helping to distinguish figurative from literal memory, helping to develop visual memory and improving the capacity to solve linguistic problems (Munoz-Basols, 2005).

Minchew & Hopper (2008) gave review of logic reasons for using humour in teaching among which we emphasize encouraging language development, teaching grammar as well as methods for illustrating importance of word choice and clarity in composition. The reasons of using humour in the classroom can be associated with physiological and psychological benefits but the primary reason is to enhance students learning although when using humour as an instructional strategy it is suggested to perform it in a way to maximize its benefits as a teaching tool and simultaneously minimize its risks to the self-esteemed professional reputation of the professor (Deiter, 2000). Field (2009) considered that when teaching statistics to psychologists humour does not have such efficiency as a tool which helps and enhances learning so its action can be doubtful (Field, 2009). Neumann, Hood & Neumann (2009) have noticed that students have negative attitudes towards using humour in teaching statistics if they are already motivated for the subject, so they recommend selective humour using in teaching statistics even though most results of conducted research have shown that humour helped in providing amusement, sharing contents, retaining attention, increasing mood, motivation and reducing monotony as well as providing mental break. Summerfelt, Lippman & Hyman (2010) have investigated impact of humour in form of a pun on the memory by giving examiners various funny puns and detected that constraints and incongruity contribute to humour impact on memory, that is, memory is enhanced when pun items were infrequent. Students are more likely to remember the material if it is presented with humour but the methods of teaching are important in order to achieve the creative development and expression of humour in the classroom as well as the teachers who effectively prepare and use humour in the classroom will find that teaching is more fun and enjoyable (Deiter, 2000).

In this paper we have tried to find answers to the questions of using humour as an effective tool for teaching and communicating on mathematics and computer science courses, amount of using humour in teaching and forms of humour used in teaching. We also wanted to see whether using humour in teaching mathematics and computer science courses has encouraging effect on learning environment by creating pleasant classroom atmosphere without stress.

Research methodology

Research was conducted at the Faculty of Teacher Education in Osijek, Croatia, during the academic year 2010/2011 and again in academic year 2012/2013. It was repeated in academic year 2012/2013 on students in order to test the differences in participants' attitudes concerning of using humour in mathematics and computer science courses teaching. The total number of participants in academic year 2010/11 was 284 students and 7 teachers, and 126 students participated in research in 2012/13. Participants were familiar with the purpose of the research and research was conducted anonymously so the privacy of users was guaranteed. Involved students were between the ages of 18 to 23, from first to fifth year of study and in majority of female gender, considering that most students of the faculty where research was conducted are women. During the research two questionnaires were applied. First one for the students and the second questionnaire with corresponding questions for the teachers of mathematics and computer science courses.

Created questionnaires were anonymous and online available so the choice of approaching and filling the questionnaire was completely arbitrary for students and teachers. They could fill out the questionnaires either at Faculty or at home at any time that was convenient for them. Questionnaire was consisted of 28 questions.

Questions in the questionnaires can be distributed in several categories:

- general data about examiners;
- assessment of frequency of using humour in mathematics and computer science courses teaching;

- recognizing types of humour which are used in mathematics and computer science courses teaching;
- assessment and importance of impact of using humour in teaching (classroom atmosphere, learning abilities, students motivation).

For statistical data processing programs Statistica10 and MS Excell 2010 were used. The collected data were downloaded from online survey, imported in Statistica 10 and MS Excell 2010 and then sorted by given criteria for further processing.

Results and discussion

By observing frequencies of using humour in teaching at mathematics and computer science courses majority of students (62,32% of examinees in academic year 2010/11 and 73% of examinees in 2012/13) have assessed that teachers use humour one to three times on average during one lecture in mathematics as well as for the computer science courses (50% of examinees in academic year 2010/11 and 59% of examiners in 2012/13). Students have also assessed that the optimal amount of humour usage (frequency of teachers humour usage) during lectures contribute to better and more pleasant atmosphere in the classroom. Neither one student has assessed that humour should not be used at all, while minority of students have stated that humour should be used 12 or more times during one lecture, detailed results can be seen in Table 1.

Table 1. Comparative illustration of students' assessments of frequencies of using humour and optimal using humour in teaching mathematics and computer science courses.

Question	How often (on average) your mathematics teacher uses humour during lecture?		How often (on average) your computer teacher uses humour during lecture?		In your opinion, what is the optimal amount of using humour during one lecture for creating pleasant learning environment?	
Frequency of humour usage	**2010/11**	**2012/13**	**2010/11**	**2012/13**	**2010/11**	**2012/13**
Do not use humour	18,31%	15%	34,87%	25%	0%	0%
1-3 times during lecture	62,32%	73%	50%	59%	8,10%	14%
4-7 times during lecture	16,55%	12%	12,61%	13%	71,48%	69%
8-11 times during lecture	2,46%	0%	2,10%	3%	17,96%	13%
12 times or more during lecture	0,35%	0%	0,42%	0%	2,46%	3%

Generally looking at students' assessments we can say that teachers of mathematics and computer science courses should increase current amount of humour used in their lectures hence students' expectation would be fulfilled. This fact has its basis on students and teacher assessments which states that teachers mostly use humour one to three times on lectures while students mostly consider that optimal amount of using humour in lectures is four to seven times during one lecture (see Table 1).

Analyzing the questionnaire filled by students it was shown that students consider that teachers of mathematics and computer science courses on Faculty of Teacher Education:

- mainly use humour that is sometimes (30,63% of examinees in academic year 2010/11 and 34,92% in academic year) or mostly (27,82% % of examinees in academic year 2010/11 and 38,89% in academic year 2012/13) connected with the subject of the lecture;
- mostly use humour in main part of the lecture (34,15% of examinees in academic year 2010/11 and 34,92% in 2012/13) and the least during exams/colloquium (2,82% of examinees in academic year 2010/11 and 1,59% in 2012/13);
- present humour mostly in verbal form (62,68% of examinees in academic year 2010/11 and 57,94% in 2012/13);
- do not use inappropriate humour during lectures (89,79% of examinees in academic year 2010/11 and 94% in 2012/13).

Students have assessed how often teachers of mathematics and computer science courses use specific appropriate type of humour during their lectures. They were assessing frequency of using funny pictures (illustrations), funny stories (anecdotes), funny examples, funny questions and problem tasks of humourus content.

Even though there are no significant differences in frequencies of using specific type of humour in teaching, it can be seen (Picture 1) that the most frequently used in the academic year 2010/2011 are funny stories and in the academic year 2012/2013 funny examples. From the data given in Table 1 and Picture 1 it can be concluded that teachers of mathematics and computer science courses on the Faculty of Teacher Education should continue using thematically appropriate humour, humorous stories, examples and tasks as well as involving humorous contents in all parts of the lectures more frequently. We have checked how students feel about effect of humour on classroom atmosphere during lectures at mathematics and computer science courses. Students gave positive opinion about the effect of humour in teaching mathematics on their relaxation during lectures (54,58% of examinees in academic year 2010/11 and 46,83% in 2012/2013 considered that they are significantly relaxed when humour is used in teaching) while on analogous question concerning computer science courses slightly lower percentage of students gave the same positive response (46,83% of examinees in academic year 2010/11 and

46,03% in 2012/2013). 41,90% of examined students in academic year 2010/11 and 37,30% in 2012/2013 completely agree with the statement that mathematics teachers are more approachable if they use humour during lectures while 41,90% of examined students in academic year 2010/11 and 35,71% in 2012/2013 have the same opinion for computer science teachers (see Table 2).

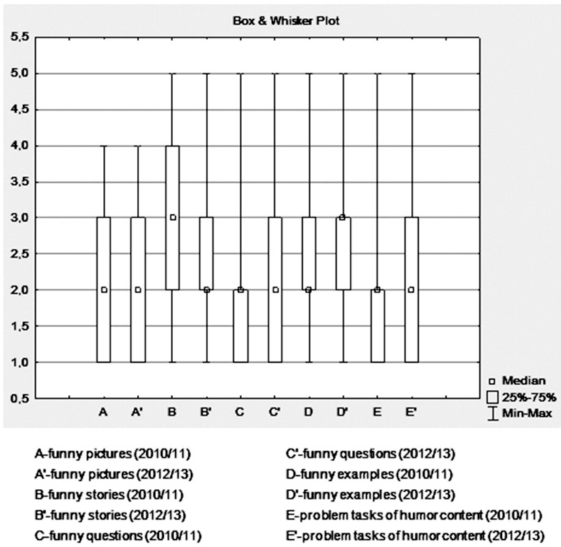

Figure 1. Using specific types of humour.

Table 2. Teacher approachability.

Statement	Mathematics teacher is more approachable when using humour in teaching		Computer Science teacher is more approachable when using humour in teaching	
Level of agreement	**2010/11**	**2012/13**	**2010/11**	**2012/13**
5-totally agree	41,90%	37,30%	41,90%	35,71%
4-mostly agree	28,87%	34,13%	26,06%	34,92%
3-neither agree nor disagree	23,94%	26,19%	24,65%	25,40%
2-partially disagree	3,52%	1,59%	4,23%	3,17%
1-totally disagree	1,76%	0,79%	3,17%	0,79%

Students have also assessed their confirmation with the statement that using humour in teaching mathematics and computer science courses affects on their learning ability in lectures by creating more pleasant lecture atmosphere, where 41,50% of examined students in academic year 2010/11 and 23,81% in 2012/2013 fully agreed with the statement within mathematics context, that is, 40,85% of examined students in academic year 2010/11 and 23,81% in 2012/2013 considered the same within computer science courses context. It was shown that humour has impact on increasing students' interest to specific courses. Only a small percentage of students answered that using humour in teaching reduces their interest on subject (see Table 3).

Table 3. Impact of using humour in lectures on students' interest for mathematics and computer science courses – students' opinion.

Course	Mathematics courses		Computer Science courses	
Academic year	**2010/11**	**2012/13**	**2010/11**	**2012/13**
Significantly increases	16,20%	14%	12,68%	13%
Moderately increases	33,45%	15%	28,17%	21%
Little increases	24,30%	32%	29,23%	32%
No, reduces	2,46%	0%	1,76%	0%
Does not impact on interest	23,59%	39%	28,17%	34%

Questionnaire with analogous questions for teachers was also filled by mathematics and computer science teachers. Primary data analysis showed that teachers' answers overlap student ones in terms of frequency of humour usage (71,43% of examined teachers of mathematics and computer science courses declared that they use humour one to three times on average during one lecture), teacher accessibility (66,67% teachers who participated in this research agree with the statement that humour usage makes them more approachable to students), humour impact on students interest in course (42,86% of examined teachers consider that humour usage in lectures increases students interest for that subject) and importance of using humour (28,57% of examined teachers consider that using humour in teaching is extremely important for creating a pleasant classroom atmosphere). These results have confirmed that students' estimates are realistic. 86% of teachers who participated in this research have completely supported positive effect of humour on students' relaxation during lectures. Considering a small number of the mathematics and computer science teachers (n=7) who participated in this research further data analysis of their answers was not conducted.

Taking into account the variables *importance of using humour in mathematics* ($p = 0,278$) *and computer science* ($p = 0,088$) *teaching* as well as *using humour increases ability of learning mathematics* ($p = 0,195$) *and computer science courses* ($p = 0,123$) there were no significant differences in participants attitudes of both groups (2010/11 and 2012/13) on the level of significance $p = 0,05$. Both groups of students mostly agree that humour has a significant role in mathematics and computer science teaching. Nevertheless, taking into account variable *humour impact on students relaxation during lecture* on the level of significance $p = 0,05$ there is significant difference between observed groups of students considering mathematics courses ($p = 0,009$) as well as computer science courses ($p = 0,011$). Examined students who enrolled 1st, 2nd and 3rd year in academic year 2012/13 assess in a higher percentage (8,00%, 22,22% and 20,25% respectively) that humour does not impact on their relaxation during mathematics classes in accordance to participants of earlier year (6,25%, 11,54% and 7,41%).

It is interesting to notice that student who enrolled 1st year in academic years 2010/2012 and 2012/2013 have assessed in a smaller percentage than students enrolled in higher years that humour usage during mathematics lecture doesn't contribute to their sense of relaxation (6,25% of examinees enrolled in 1st year in academic year 2010/11 and 8,00% in 2012/2013 consider that humour usage in mathematics lectures does not affect on their feeling of relaxation during lectures).

Obtained data could find its explanation in a fact that students of the first year are still adjusting on a change of the learning environment.

Conclusion

Even though there are many results and much research about the impact of using humour in teaching on all levels of education, there was relatively little research conducted in the field of mathematics and computer science courses as well as most of them are out of date. Our research results have shown that teachers are familiar with the possibilities of using humour in a sense of teaching quality improvement as well as its positive action on working atmosphere and learning environment. Students and teachers assessment of humour presence in teaching equals to one to three times on average during each lecture. Research showed that students clearly support using humour which affects on achieving pleasant working atmosphere, improving learning during lectures and increasing sense of teacher availability.

Humour is also considered as the effective factor in mathematical reflection because most types of humour with techniques that involve entertaining tasks and comical contents of the problem, can be used at math lessons and thus have influence on a positive classroom climate so as on positive interaction between the teachers and students. Mathematics and computer science courses are fruitful area for involving humorous content in the lecture flow from many reasons. Content oriented humour can be applied as a recommended way of breaking monotony and as additional motivation for active participating in lectures. Limitations of this research involve limited sample of students from one discipline, unbalanced ratio of male and female respondents and small sample of teachers. Hence some pointers for future research can include sample of students from various disciplines, better gender distribution as well as incorporating wider area of courses into research.

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Humor u nastavi matematike i informatike – da ili ne?

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Sažetak. Cilj ovoga rada je ispitati ulogu humora u poučavanju matematike i informatike na Učiteljskom fakultetu u Osijeku. Rad istražuje koristi li se i u kojem intenzitetu humor kao učinkovito sredstvo poučavanja i komunikacije na nastavi matematike i informatike te identificira koji oblici prihvatljivog humora se koriste. Također, u radu se proučava ima li upotreba humora na nastavi matematike i informatike poticajni utjecaj na studente i okolinu za učenje stvarajući ugodnu "razrednu" klimu bez stresa. Tijekom istraživanja kreirala su se i upotrijebila dva upitnika. Jedan od kreiranih upitnika koristio se za ispitivanje stavova studenata o upotrebi humora u nastavi, odnosno željelo se vidjeti jesu li ili nisu studenti sudjelovali u nastavnom procesu koji je sadržavao humor u nekom od prihvatljivih oblika, je li taj humor djelovao poticajno na njihovo učenje i ukupni dojam o kolegiju na kojem je humor bio upotrijebljen te kakav je utjecaj upotreba humora imala na komunikaciju s nastavnikom koji ga je upotrebljavao. Drugi upitnik služio je za prikupljanje podataka od samih nastavnika, a koncipiran je analogno prvom upitniku. Odgovori dobiveni upotrijebljenim upitnicima analizirani su, a rezultati su pokazali da premda nastavnici matematike i informatike prepoznaju dobre strane upotrebe humora na nastavi, u praksi ga koriste rijetko ili ponekad.

Ključne riječi: humor u nastavi, unaprjeđenje okoline za učenje, stav studenata, utjecaj humora na komunikaciju

Readiness of primary teachers to apply contemporary math teaching

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Abstract. Observing pupils results in mathematics in Bosnia and Herzegovina and Croatia we are confronted with bad grades, much of negative attitude of society towards math and poor results that pupils obtained in the various external evaluation. Poor results require reconsideration of all the parameters that have an impact on the results of mathematical education, and one of the most important is a teacher. With modes, enthusiasm, educational goals, methodological competence and knowledge a teacher affects on competence arise in their students, and because of that students results in mathematics also. Contemporary math education sets goals and models opposite than traditional teaching. Our research shows preference of classroom teachers to use contemporary or traditional concept of teaching mathematics. The results showed the most teachers have modern thinking about the mathematic teaching and a high level of awareness about the benefits and necessity of modernizing math teaching. Teacher's high level of awarnes toward contemporary approach we considered as good indicator of the progress of our teaching practice.

Keywords: math teaching, traditional concept of teaching, contemporary concept of teaching, primary teacher, preferences

Introduction

Mathematics as a subject has a special place and significance in the educational system, practically all developed countries. This specialty is reflected in the fact that this is a subject that is being taught from the very beginning of compulsory education, followed by a continuation of the overall general education, drainage, a large number of weekly working hours and which is generally compulsory subject in state graduation exam. All of the above shows that mathematics is recognized as one of the bases of the education system as a necessary element of every culture and as a subject that is the basis for almost all specific learning content in the various branches of human activity. Despite the importance and meaning of mathematics, student scores can not be fully satisfied. The results of students in PISA survey in 2006 and 2009 was conducted in Croatia indicated the underdeveloped mathematical knowledge, competence and value in use of mathematical knowledge. Similar results were obtained by students in Bosnia and Herzegovina in an international study TIMSS in 2007. The fact is that in most developed countries are not fully satisfied with the results of their students in math. Therefore, educators, counselors, psychologists, mathematicians, teachers, professors, parents and politicians have performed extensive research to find the cause and determine ways to combat and eliminate failure in mathematics. Their long reflection, research and experimentation led to the change in the concept of teaching mathematics in schools from the concept that dominated the traditional teaching of mathematics, which was characteristic of the second half of the 20th century, the concept of modern mathematics teaching characteristic of the end of 20th and the beginning of the 21st century.

The traditional concept of teaching mathematics involves the school in which he emphasized the role of the teacher as an unquestionable authority, highly hierarchical, where the assignment of teachers to transmit knowledge to students and to guide them trodden paths of knowledge to understand new mathematical fact. That concept in teaching mathematics can be displayed briefly as a model of teaching and learning that relies primarily based on the work of teachers, which is dominated by individual approach to teaching and assessing of mathematical knowledge, and nourishes and emphasizes the competitive spirit among students (DeShawn Kemp, 2007).

In the traditional teaching of mathematics to encourage and develop procedural knowledge and skills, and the teaching is focused on the adoption of certain amounts of content with the prescribed curriculum and applied mathematical theoretical knowledge in solving a number of standardized, typical mathematical problems. Educators have traditionally placed a heavy emphasis on the development of declarative and procedural knowledge (Miller and Hudson, 2007), while the conceptual mathematical knowledge is neglected. Fosters the learning formulas and algorithms memorized, and special attention, especially in the initial teaching of mathematics attaches developing and automation of computational skills in students. In order to achieve the desired level of automation as soon discarded vivid aids for computation, so that the "traditional approach to teaching numeracy vehemently opposed the use of fingers in addition and subtraction" (Vlahovic-Štetić, Vizek Vidovic, 1998, 3). In the traditional approach, believes that students are relying on your fingers in calculating the gain permanently available aid for which will not even try to reach the desired level of automation computing. Through traditional teaching mathematics students learn mathematical content through a separate lecture topics, within which the application of the learned practicing maths textbooks, and finally checked content acquired through written tests with similar missions.

This approach to mathematical content did not lead to satisfactory results, because in this way the students have learned the knowledge and skills they do not know and fail to apply in real, personal or professional contexts. "Limitations and disadvantages of traditional lecturing classes are becoming more visible and more dramatic with increased production of information and transfer of knowledge and memorizing content become useless" (Jurdana-Sepic and Milotic, 2005, 20). Therefore, in the past twenty years, increasingly implemented various reforms in mathematics education, who are trying to raise the level of understanding of mathematical concepts, concepts and content, and the level of mathematical thinking, and that instead of promoting the adoption of greater amounts of knowledge and the development of numeracy skills, are increasingly turning to the development of student competencies and preparing students to apply mathematics in realistic contexts. Reformed teaching of mathematics, which is trying to raise the quality of mathematics education and improve student achievement and results in terms of actual conceptual conceptual understanding of mathematics and the application of skills and knowledge of its content, we describe the expression contemporary mathematics classes.

The teaching of mathematics is much less attention paid to the amount of acquired content, and a lot more understanding of mathematical terms and concepts, discovering relations and developing competencies required for the application of mathematics in everyday life, raising the mathematical culture of pupils and students introduced in the abstract sphere (Prodanović, 1981).

So that students really need to understand mathematics is to discover and develop (build), which requires a lot of their own students' engagement, discovery, exploration, and the classroom environment rich communication and collaboration. Modern teaching mathematics such preferred learning style and views of learning, teaching and assessment of mathematics that shift the focus of the curriculum with memorization, mechanical learning, adoption and application of facts and processes on a conceptual understanding and logical, creative thinking. Therefore, the main goal of modern mathematics education is no longer the final transfer of mathematical knowledge, but, according to Bruner principle that learning is an act of discovery (Bruner, 1961), and that learning path discovery increases internal motivation and intellectual power of students, rather than training students for independent learning (Dakić, Elezovic, 2003), and for detecting and establishing relations, for reasoning and logical reasoning. The focus of learning mathematics is no longer in memory of concepts and procedures, but on the understanding of mathematical concepts and the student's own discovery of mathematical relationships and the application of what students know and learn in a variety of realistic, problem situations. It starts from the fact that "learning procedures without conceptual understanding is pointless and ultimately useless. Anyway, machines can do procedures far better than humans" (Addington et al, 2000, 1074). Conceptual understanding of mathematics implies the knowledge and understanding of the concepts, structure and processes, application of skills in realistic contexts, critical and self-critical thinking and reasoning, connecting with everyday life, create your own strategies to solve problems and displaying mathematics different mathematical symbols and forms. Modern teaching mathematics is based on a clear and unambiguous set of standards in mathematics education, in both the selection of content, as well as in the selection of competencies that the students must develop.

Contemporary education mathematics has set clearly defined competencies which it should strive in the teaching of mathematics at all levels of education. "Ambitious standards are required to achieve a society that has the capability to think and reason mathematically and a useful base of mathematical knowledge and skills" (NCTM, 2000, p. 29), and such a society is necessary in countries that want to be competitive on the world market. For this reason, it is important to investigate what the teaching of mathematics in this country and realize that mathematical competence encourage and develop in our students, to our current situation indicate the direction of change we need to make in order to raise the level of mathematical competence of our students and their competitiveness in the global market.

Teaching of mathematics in Bosnia and Croatia

Observing the situation in Bosnia and Herzegovina and Croatia, we are now, more than ever, we are faced with poor grades of students in mathematics, much of the negative attitude of society towards her output and poor student scores obtained in the various projects of external evaluation. Students can not cope in problem situations and can apply the acquired mathematical knowledge and skills in specific situations. In Bosnia and Herzegovina conducted international study TIMSS 2007, which was attended by students of the eighth grader. The results of students ranked Bosnia and Herzegovina at the 27th place (among 49 participants), with an average number of 456 points per student, which she placed in the country's significantly below the TIMSS of measuring the average of 500 points. The Republic of Croatia has participated in an international project PISA 2006 and there is ranked 36th place, with an average of 467 points per student, which places the country significantly below the average of OECD countries. It turned out that the Croatian students are able to conclude only directly, use only one source of data to solve problems and use only the basic algorithms, formulas and procedures, and to interpret the results directly. Croatian students are not able to conceptualize, generalize, connect various sources of information, display problems in different ways, thinking in advanced mathematical way, to develop new strategies to solve problems and generally managing the unknown, problem situations. Disorientation Croatian and Bosnian students in the application of the acquired mathematical knowledge and skills in the project confirmed the external evaluation of the final year, conducted in April 2008. Average scores of students was 52,9%, indicating a bad connection and usability learned mathematics during classroom teaching.

Poor results lead us to review all factors that have an impact on the results of mathematical education, and one of the most important is just a teacher (teacher, teacher) who teaches math. Our area of interest is primarily the initial teaching of mathematics which is being implemented in classrooms under the guidance of classroom teachers. "The teacher's role is indispensable in teaching mathematics, especially in elementary instruction, and therefore is irreplaceable and its impact and yield success or failure of teaching" (Pejic, 2003, p. 9). The teacher plans and manages the educational process, the selection of problems on which the students work, directing communications and leads students toward conceptual understanding of mathematics and leads them into logical abstract thinking. Teacher influences on students in many explicit and implicit ways. “Teachers’ beliefs, knowledge, judgments, thoughts, and decisions have a profound effect on the way they teach as well as on students’ learning in their classrooms. In addition, teachers’ beliefs, thoughts, judgments, knowledge, and decisions affect how teachers perceive and think about teaching a new curriculum that they receive and to what extent they implementt he training or curriculum as intended by the developers” (Peterson et al., 1989, p. 2).

A quality teacher in the teaching of mathematics must be open to the unexpected child explanations, interpretations and strategies applied, must improvise, discuss, argue and show enthusiasm and excitement of mathematical discovery. All this requires changing the teaching practices of knowledge transfer finished turning towards unpredictable Pupil detection and review of mathematical knowledge. “Implicit in the call for greater mathematical proficiency from students is a call for teachers to change their instructional practices. Teachers are expected to learn how to organize their instruction around student inquiry, problem solving, and mathematical discussion and to make their mathematics teaching more interactive, more investigative, and more focused on reasoning and understanding.” (Goldsmith, 2001, p. 57).

All this requires a change and new ways of thinking, perceiving and realizing teaching teachers, which is time consuming and difficult process that requires constant monitoring and learning new methodological insights. “The vast majority of today’s American mathematics teachers learned the traditional mathematics curriculum in the traditional way. Hence they have neither models nor experience teaching in the ways that would best facilitate their students’ development of mathematical understanding.” (Schoenfeld, 2002, p. 20). That the teachers really know the modern mathematical standards and realize their implementation in their teaching practice, it is important that during their study and life-long learning experience such an approach to mathematics and its teaching. This means that in its methodological and mathematical courses involved solving problems, learning new math concepts and new ways of looking at them. Teachers are responsible for the education and training of teachers are obliged to find ways to bring them a contemporary approach to mathematics, and this can be achieved using the current approach in their own teaching, connecting them with teachers who use such an approach, encouraging debate and providing them with the relevant literature (Taylor, 2002).

Research

This research was conducted within the framework of a broader research competencies that class teachers encourage the teaching of mathematics[1], and for this

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[1] The wider research was conducted within the framework of a doctoral dissertation, PhD. Irene Mišurac Zorica titled “Standards of mathematical competence in the primary teaching of

study was isolated part of the survey that is used here further analyzes.

Research goal

We examined the propensity of primary teachers applying the guidelines of traditional or contemporary teaching mathematics. Preference given model points out the knowledge of the concept, and thus creates the basis for its application in the classroom. For this purpose, we selected 12 characteristic statements by which we believe we will best distinguish these two models work, and where we want to measure modernity and traditionalism in the approach to teaching mathematics subjects. We set 12 incomplete statements, and respondents had to choose between one of two offered continuing claims. One of the attachments was a traditional approach to teaching mathematics, and the other contemporary.

Underlying presumptions (hypotheses)

In this paper, we started from the assumption, based on our several years of working with primary school teachers, the teachers in the areas of Bosnia and Herzegovina and the Croatian generally not sufficiently familiar with contemporary standards of mathematics education. This assumption was based on the fact that the teachers who are working with children, mostly educated themselves in the traditional school system, and guidelines that are adopted during their own education carried over into his teaching practices. In addition, the low standard of living, the general dissatisfaction of teachers in these areas, the unavailability of modern literature, the insufficiency and frequent inability to attend seminars and workshops, migration as a consequence of the recent war and other problems present in these areas, further hampering of follow modern methodical mathematical knowledge. It certainly should add a small number of books on teaching mathematics translated into our language, which seeks to obtain and read the literature, and a large number of teachers aggravated knowledge and regular monitoring of modern knowledge.

However, the positive changes in education in recent years, the reform of compulsory education in Croatia and some parts of the country, the new curricula, the general democratization, modernization of schools and open them to new knowledge, entering many modern and advanced programs (e.g., Step by Step, RWCT – Reading and Writing for Critical Thinking, and others), have also sensitizes teachers and receptive to some modern methods and forms of work (specially team work), use of new educational technologies (especially computers, "smart boards"), set up as a student learning of the subject at the center of the teaching process, self-evaluation, self-assessment and the like. All this certainly affects the changing attitudes toward his and other people's teaching, changing goals of mathematics education, the changing expectations of the students, both in content and in the application of these contents, changing the mode of teaching them (though not always consciously) approaching modern standards of mathematical education.

One of the hypotheses in our study was that class teachers put more emphasis on traditional guidelines of mathematical education than contemporary. We also

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mathematics" under the mentorship of Professor. Dr. M. Pejic, defended at the University of Sarajevo, the 2010th.

assume that there are no differences in the contemporary approach to teaching mathematics among teachers from Bosnia and Herzegovina and the Croatian teachers and teachers from the eight-year and nine-year schools in Bosnia. Another hypothesis from which we started was that teachers with higher education more emphasis on contemporary approaches in the teaching of mathematics teachers with higher (or secondary) education. Last hypothesis that we set was that younger teachers, beginners with job tenure up to 5 years, spend Modern teaching mathematics than their counterparts teachers with longer tenure are as were educated at a time when modern mathematics teaching guidelines already started (or at least should start) penetrate the highly educated institution

The research sample

To investigate the situation in the initial teaching of mathematics in these areas, we conducted an extensive survey of 400 primary teachers from the Federation of Bosnia and Herzegovina and the Croatian Republic. The study included 211 teachers from the area of the Croatian and 189 teachers from Bosnia and Herzegovina. The sample was stratified according to three criteria: the state in which respondents were employed, except that we are in Bosnia and Herzegovina, particularly schools that are observed at the time of the research, the eight-or nine-year primary education, qualifications and years of service of respondents. Table 1 presents data on the variables mentioned above.

Table 1. Respondents by country, type of school, level of education, years of service and gender.

		N			total N	total %
		M	**F**	**0**		
country and type of school	Bosnia and Hercegovina (8-year school)	8	80		**88**	**22**
	Bosnia and Hercegovina (9-year school)	13	87	1	**101**	**25**
	Croatia	12	199		**211**	**53**
	Total	**33**	**366**	**1**	**400**	**100**
level of education	secondary education	3	13		**16**	**4**
	higher education	18	200		**218**	**55**
	university degrees	12	148	1	**161**	**40**
	indefinitely		5		**5**	**1**
	Total	**33**	**366**	**1**	**400**	**100**
years of service	until 5	4	75		**79**	**20**
	6–10	4	62		**66**	**16**
	11–20	9	131	1	**141**	**35**
	20–30	8	51		**59**	**15**
	more than 30	8	47		**55**	**14**
	Total	**33**	**366**	**1**	**400**	**100**

In selecting the entire sample, as well as its specific categories, we considered meeting the main criteria of modern pedagogical research methodology, both in terms of size, representativeness, homogeneity, reliability and cost effectiveness. Therefore, we can assume that the results obtained in the sample relating to the entire population of primary teachers in Bosnia and Herzegovina and Croatia.

From Table 1 we see that the number of male respondents (33 or 8%) used an incompatible number of female respondents (366 or 92%), so we gave up examination of possible differences in susceptibility guidelines of modern mathematics teaching subjects by gender. We also notice that the subgroup of respondents with secondary education (16 or 4%) disproportionately small compared to the subgroup of subjects with higher (218 or 55%) or a university degree (161 or 40%). Group of respondents with secondary education are also not taken into account when analyzing the results of the qualification. Other subgroups were considered compliant and we compared them to the planned variables.

Results

Respondents are each assigned a value of 1 claim if they decide to answer a traditional model work, or 2 if they opted for a response that indicates the contemporary model in teaching mathematics. After entering the data for each of the respondents we calculate the arithmetic mean of all values obtained in individual claims, which may range from 1,0 (which would indicate a completely traditional approach) to 2,0 (which would indicate a completely modern approach). We call the resulting measure present value of each of the respondents. The results for each question, as well as the overall measure of modernity, are shown in Table 2.

As we can see from the results obtained, the arithmetic mean of the total sample contemporary measures is relatively high ($M = 1,81$), indicating a common choice answers that show a contemporary approach to teaching mathematics. Looking at the individual claims, we also see that all of them have the arithmetic mean greater than 1,5, indicating that in all these assertions respondents inclined to choose a more contemporary approach to the more traditional. From the obtained results we can assume that the teachers surveyed, at least verbally, show a relatively good knowledge of and affinity modern approach to teaching mathematics.

Tallest result of respondents had the statement "If a student came to the exact solution, it should be encouraged to: (a) read the results, (b) clarify their thinking" ($M = 1,97$), and lowest in the statement "To be good learn math should: (a) the full thinking, (b) to resolve as many tasks" ($M = 1,58$). This result shows us that teachers recognize thinking as one of the key elements of teaching mathematics, but they are still quite oriented to the amount of resolved tasks.

Thus, high scores in all the arguments, and the high overall measure of modernity can not be taken as evidence of a modern approach to teaching mathematics in our patients, but we interpret it as a knowledge and appreciation of the principles

Table 2. Preferences of contemporary methods in teaching.

DECLARATION	N Valid	N Missing	Mean	Std. Deviation	Percentiles 25	Percentiles 50	Percentiles 75
Complexlife problems in which it appears mathematics in teaching: 2) should often be used: 1) should be used only in Additional Mathematics Class	397	3	1,75	,435	1	2	2
Mathematicsshould be understood as: 1) a group of concepts and operations that should be overcome, 2) a unique and cohesive whole	395	5	1,85	,357	2	2	2
Itis better that a student solve a math problem: 2) in a way that he devise: 1) in the manner which is demonstrated in class	398	2	1,69	,465	1	2	2
Ifa student solve the problem correct, he should be encouraged to: 1) read the result, 2) clarify his thinking	399	1	1,97	,164	2	2	2
Inmath teaching it is better to solved: 2) one task in several ways: 1) more tasks on the same way	399	1	1,79	,408	2	2	2
Whenstudents learn math it is most important that children know: 1) quickly and accurately solve the related tasks 2) to explain what they are doing and know where to use it	397	3	1,93	,252	2	2	2
Whenstudents solve math problems it is better to apply: 2) diverse strategies 1) the same strategy	397	3	1,91	,284	2	2	2
Inthe problem is the most important 1) result, 2) process	395	5	1,69	,462	1	2	2
Inmathematical problems mainly exist: 2) many ways of arriving at the correct solution: 1) only one correct way of arriving at the correct solution	399	1	1,82	,385	2	2	2
Tobe well-learned math it should 2) a lot to think: 1) to solve as many tasks	393	7	1,58	,494	1	2	2
Childrenneed to learn as many math contents, 2) as much allow them their development opportunities; 1) as provides curriculum and textbook	397	3	1,92	,276	2	2	2
Concretematerials to assist in calculating the pupils need use: 2) as long as they need; 1) only in the first grade	400	0	1,94	,247	2	2	2
total quantity of contemporary	400	0	1,8073	,17322	1,7500	1,8333	1,9167

of modern mathematics teaching. These results also tell us that teachers' guidance modern teaching math are considered desirable in view of the results of their teaching.

In order to determine whether there are differences in the degree contemporary among respondents from the general set up categories (ie state education system, education, years of service), we set aside the results of each of the subcategories and display them in Table 3. Before conducting t-test and analysis of variance homogeneity of variances was tested by a field test of homogeneity of variances (Leven test) which showed that the variances are not significantly different ($p > 0,05$).

Table 3. Total quantity of contemporary respondents in relation to general variables.

	Type of school			**Level of education**		
	Bosnia		**Croatia**	**secondary**	**higher education**	**university degrees**
	8-year school	**9-year school**				
	Mean	**Mean**	**Mean**	**Mean**	**Mean**	**Mean**
Total quantity of contemporary	1,74	1,79	1,85	1,69	1,80	1,84
	1,76					

	Years of service				
	<5	**5–10**	**11–20**	**21–30**	**>30**
	Mean	**Mean**	**Mean**	**Mean**	**Mean**
Total quantity of contemporary	1,82	1,79	1,81	1,84	1,76

We notice that the respondents in Croatia likely to select answers that show a contemporary approach to mathematics ($M = 1,85$) than those in Bosnia and Herzegovina ($M = 1,76$). Although the difference in results between respondents from Bosnia and Herzegovina and the Croatian is not great, we conducted t-tests to determine its significance. The results ($t = -4,903$; $df = 398$; $p < 0,01$) indicates that the difference is statistically significant, and that respondents in Croatia reported a more contemporary approach to the initial teaching of mathematics than participants from Bosnia and Herzegovina.

Since from Table 3 we see that there are some minor differences in the degree of modernity and among respondents from the eight-year school and nine-year in Bosnia, we were interested in whether the observed differences are statistically significant. The results of these two subgroups of patients from Bosnia, we wanted to compare with the results of a separate Croatian respondents. Therefore, we conducted an additional analysis of variance between groups of respondents from the three types of educational systems in relation to their present extent. It has been shown that the observed differences in the results from three subjects observed the educational system to the extent present significant ($F = 14,4$; $df = 2$; $p < 0,01$), and in order to accurately determine between which pairs of subgroups these differences were statistically significant, and we spent post-hoc test. The results showed that the differences between the present extent of the Croatian respondents $(M = 1,85)$ and subjects from elementary schools in Bosnia $(M = 1,74)$ statistically significant $(p < 0,01)$, as well as differences between subjects Croatian and nine-year respondents from schools in Bosnia $(M = 1,79)$ $(p < 0,05)$. Differences in the degree of contemporaneity between teacher and eight-year and nine year schools in Bosnia and Herzegovina were not statistically significant.

In order to determine which of three statements respondents observed educational system most agree or disagree, the results of respondents in individual claims, we presented in Graph 1.

Legend on the right shows the abbreviations of certain statements in the survey in the same order in which they are listed in Table 2, and the higher (lower) column in each statement indicates the modern (traditional) approach of the respondents from the education system.

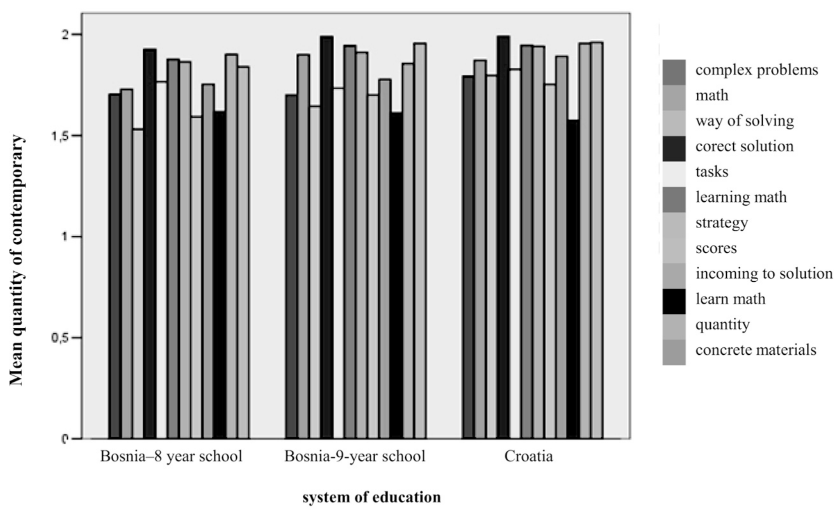

Figure 1. Respondents results in relation to the education system

We note that the only claim in which subjects were shown the Croatian traditional approach from the residents of Bosnia and Herzegovina, the statement that “In order to learn mathematics should: (a) a lot of thinking, (b) to solve as many tasks” (assertion shows dark green in Figure 1). The fact that many teachers are inclined Croatian traditional claims about the amount of math problems, a possible consequence of the orientation of Croatian traditional mathematical curriculum for which we have already found that it is quite focused on the amount of content and not enough on the process. The results of most of the respondents differ in the claims, “It is better for the student mathematical problem solved: a) in the manner of his own design, and b) in the manner indicated on the hour” (beige color) and “Mathematics should be understood as follows: a) group concepts and operations that need to be overcome, b) a unique and cohesive whole” (light green color), in which the most traditional approach showed teachers from elementary schools in Bosnia. They are also in nine of the twelve statements showed the most traditional approach to the initial teaching of mathematics. Teachers from schools in Bosnia nine-year the most traditional approach expressed in the statement, "mathematics is better to solve: a) a task in multiple ways, and b) the same way multitasking" (yellow) and "Children need to master math as much as them, a) allow their developmental potential, b) prescribe the curriculum and the textbook" (orange color).

Comparing the present measure of patients according to their length of service (Table 3), we find that teachers with tenure 21–30 years are most apt expression of the modern approach to teaching mathematics ($M = 1,84$). However, among all categories of respondents according to seniority could not perceive a clear link or form that would explain the small differences in the degree of modernity among them. To determine the significance of differences between subjects with different subcategories service in much of modernity, we again conducted an analysis of variance, and it is shown that among these categories there are no statistically significant differences in the expression of modernity measures ($F = 1,543$; $df = 4$; $p > 0,05$). We conclude that the length of service is not a parameter that affects the measure of present classroom teachers.

Analyzing the results of the respondents according to their qualifications, we noticed that the difference in mean size between the modern categories of respondents with a higher education are very low (respondents with college degrees, 1,8, and respondents with a high 1,84). However, the *t*-test significance of differences between respondents with university and college degrees has shown ($t = -2,403$; $df = 377$; $p < 0,05$) that it is statistically significant. In other words, respondents with higher education reported slightly more modern approach to teaching mathematics than those with higher education.

This result partly confirms our hypothesis that teachers with university degrees spend some modern teaching of mathematics teachers with college degrees. Although these results do not take as proof of actual promotion of contemporary competencies in students, it is possible that teachers with higher education during his four-year study better and become more aware of the importance of the implementation of modern teaching of mathematics teachers with college degrees.

To get the best possible conclusions from the results obtained, we examined separately the category of respondents with a college degree, you are again divided into two groups of subjects from Bosnia (nine-year primary school and college) and a subgroup of patients from the Croatian. We observed that among these subgroups of respondents with a college degree there are certain differences in the degree of modernity, including the Croatian teachers express a great deal of modernity ($M = 1,83$), teachers from schools in Bosnia nine-year slightly lower ($M = 1,79$), and teachers from elementary schools in Bosnia lowest ($M = 1,76$). To see whether the observed differences in the extent of modernity between subgroups of respondents with a college degree from the three educational systems statistically significant, we conducted an additional analysis of variance, and the results showed a statistically significant difference among them ($F = 4,568$; $df = 2$; $p < 0,05$). Post hoc tests showed that respondents with a college degree from the Croatian statistically significantly different from subjects from elementary schools in Bosnia, while no significant differences in relation to teachers with higher qualification from nine-year school does. We can conclude that teachers with higher qualification Croatian and nine-year schools in Bosnia reported contemporary attitude of teachers with college degrees in elementary schools in Bosnia.

Respondents with higher education are divided into two categories, and the results show that highly educated respondents indicate Croatian contemporary approach ($M = 1,87$) than the highly educated respondents in Bosnia ($M = 1,78$). The significance of these differences, we have examined the t-test showed that this difference was statistically significant ($t = -3,389$; $df = 158$; $p < 0,01$). We can conclude that Croatian teachers reported more contemporary approach in the teaching of mathematics teachers from Bosnia and Herzegovina (especially teachers from elementary schools) regardless of their qualifications.

Though declared the measure of modernity expressed by the respondents in the survey can not be taken as proof of their holistic approach to modern and contemporary real encouraging competence, certainly points to a better awareness of the need to modernize the teaching of mathematics teachers in Croatian. As it applies to teachers with college and university education, we can suggest that it is not causing a formal university education, but is the result of better education of the lifelong progression of classroom teachers. In recent years, Croatia has been working on modernizing the system of promotion of teachers, reinforced by the Agency for Education in organizing professional training, introduced the system of county assets, approved and funded numerous educational projects, organized a number of re-education teacher with two years of study, etc. Teachers are recognized as the "key to all success in education. Teacher shapes the educational process, the main instigator and contributor to students in developing their knowledge and skills" (Guide to the CNES, 2005, p. 6). The Plan and the development of the education system since 2005 by 2010 (2005), as one of the priorities set the continuous professional training of teachers and the teaching staff. All this certainly raised the level of awareness of Croatian teachers about what is modern mathematics classes.

We must not forget that teachers from Bosnia and Herzegovina have shown a high measure of modernity, especially teachers from nine-year schools. Since Bosnia reformed education system through the introduction of nine-year school and new, customized curriculum, this fact probably influenced the teachers' awareness of the importance of modern teaching mathematics.

Conclusion

Readiness of primary teachers contemporary or traditional concept of teaching mathematics in the paper, we investigate the selection of 12 questions, including one offered an answer suggested by modern, the other traditional approach to teaching mathematics. It was clear that this measure should not be taken as proof we actually apply the guidelines of modern mathematics teaching in their own classroom practice, but we were interested to know whether teachers guidance of modern mathematics teaching and whether they are more desirable than selecting concept of traditional teaching.

The results showed that teachers are more often preferred the modern answers to all questions, so we concluded that the majority of teachers indicates contemporaneity of considerations on the teaching of mathematics and a high level of awareness about the benefits and necessity of modernizing teaching basic mathematics. High degree of modernity indicates knowledge of guidelines and characteristics of contemporary mathematics teaching from teachers and their assessment is advisable to opt for a contemporary model of teaching (rather than traditional), which is considered a good indicator of improving our teaching practices. Such results are extremely pleased because they represent a good basis for the changes in recent years, we try to introduce the system of compulsory education students.

All of the above justifies the extension of the duration of studies at the Faculty of Education, the reforms being implemented in the education and professional and continuing education of teachers. Knowing the guidelines of modern mathematics teaching and new, contemporary ways of thinking and attitudes of teachers are the first steps in a positive change of the results of student achievement in mathematics. However, we should take some time to complete the implementation of the guidelines and the achievements of modern education in our teaching practice, and thus a lot of time that their application results in better student scores in math.

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Spremnost učitelja razredne nastave za primjenu suvremene nastave matematike

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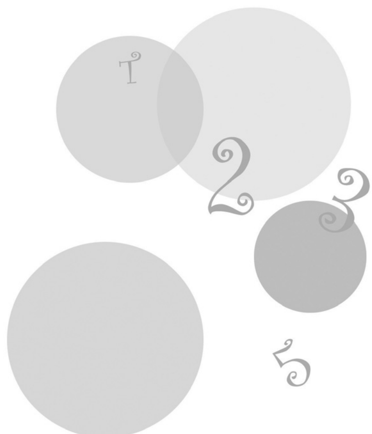
Sažetak. Promatrajući rezultate učenika u matematici na području Bosne i Hercegovine i Republike Hrvatske, suočeni smo s lošim ocjenama učenika u matematici, negativnim stavom velikog dijela društva prema njoj te lošim izlaznim rezultatima učenika dobivenima u raznim projektima vanjskih vrednovanja. Loši rezultati navode nas na preispitivanje svih parametara koji imaju utjecaj na rezultate matematičkog obrazovanja, a jedan od najznačajnijih je upravo učitelj (nastavnik, profesor) koji poučava matematiku. Načinima rada, entuzijazmom, ciljevima koje postavlja pred učenike, metodičkim kompetencijama i znanjima učitelj utječe na kompetencije koje razvija kod svojih učenika, a time i na rezultate učenika u matematici. Suvremena nastava matematike postavlja izmijenjene ciljeve i načine rada u odnosu na tradicionalnu nastavu. U radu smo ispitali sklonost učitelja razredne nastave primjeni suvremenog ili tradicionalnog koncepta nastave matematike. Rezultati su pokazali da većina učitelja pokazuje suvremenost u razmišljanjima o nastavi matematike te visoku razinu osviještenosti o prednostima i potrebi osuvremenjivanja početne nastave matematike. Visoka mjera suvremenosti ukazuje na poznavanje smjernica i karakteristika suvremene nastave matematike od strane učitelja te njihovu procjenu da je poželjno opredijeliti se za suvremeni model nastave (a ne tradicionalni), što smatramo dobrim pokazateljem napredovanja naše nastavne prakse.

Ključne riječi: nastava matematike, tradicionalna nastava, suvremena nastava, učitelj razredne nastave, sklonosti

5

Examples of good teaching practice



Teaching practice is the source of scientific research. The task of scientists in education, as well as related sciences, is to, by means of research and with the aid of their respective governments, improve the quality of teaching practice. The link between science and practice is created through efforts of benevolent scientists and critical practitioners. Only through costant interaction of scientific activities by noted mathematicians, psychologists, computer scientists, educationists, educational specialists and the critical work of teachers in the field can we, in the socially acceptable atmosphere, count on the achieved mathematical potential in pupils and students. The famous Stanford mathematics professor and popularizer of natural sciences Keith Devlin (Devlin, 2000) claims in one of his books that evolution endowed humans with a brain which can, besides the reactions to outward sensory stimuli, think *offline* about objects or items that resemble them. Yet, *offline* thinking is difficult because the brain accepts the new abstraction by trying to make it more real through the process of familiarization which is the result of direct contact with abstract concepts, making this a demanding process. Teachers are not evaluated according to well thought-out shortcuts which they use to lead pupils towards a passing grade, but according to their resolve to use the teaching process to lead pupils towards endeavors by means of which they will move from that which is concrete to that which is abstract.

Data processing and presentation in primary school at elementary level – Dealing with teaching experiences

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Abstract. Themes which, in the subject of mathematics, deal with information processing. Have been introduced into the syllabus in Slovenia during the transition to the nine-year schooling, in 1998. These themes have also remained a part of the syllabus after 2011, when the syllabi have been updated. The decision to introduce these themes was made with the purpose of increasing mathematical literacy and improve the quality of effective education regarding the processing, presentation, reading and interpretation of collected data, all in connection with an introduction to statistics. Pupils tackle their first cases in their pre-school period, later they upgrade their knowledge in school every year. They come from simple demonstrations introduced by professional staff in schools, to concrete, through graphic to a symbolic level. Here they consider the strategy of solving mathematical problems and the associated solving phases. In the 1^{st} and 2^{nd} grades they start with simple presentations and data reading with demonstrations using columns and lines. The figure presentation offers them a transition from the concrete to the graphic level. In 3^{rd} grade they pass over to independent collecting, presentation, reading and interpretation of data. They also become used to employing various ways of data collection. In the 4^{th} and 5^{th} grades they can already use the gathered knowledge and independently plan research. In this paper you can find descriptions of cases which indicate the didactic manner of introducing any presentation. Difficulty gradation from one class to the other has also been indicated. These are themes which make classes more interesting and offer many sensible possibilities for connections with other subjects. The activities have been planned in such a way that they guide the pupil through the experience. This manner of operation offers to the pupils a sensible path to knowledge improvement, learning the skill of orientation in a presentation and gaining the skill of the reading of data from a presentation. The introduction of these themes can also be executed through ICT. We can manufacture the material ourselves or look for them in cases available on the world wide web.

Keywords: Mathematical literacy, information processing, presentations, problem solving strategies, didactic path

Introduction

Mathematics is a subject which can be organized during lectures in such a way that pupils experience it as something pleasant. At the same time, the notion that this is a subject without which everyday life is not possible, grow stronger and stronger.

What we need to achieve good results in the learning and teaching process is the control over various forms of communication. We have to get closer to the pupils in the sense of getting to know their way of thinking, accepting and organizing information.

Here we all heavily depend on all of our five senses. While gathering information from the environment, most people rely on one of three ways of gathering information from the environment; what we see, what we hear or what can be produced or felt. That is why technical literature distinguishes between visual, auditory and kinaesthetic modalities and learning styles connected to them (Deproter, 1996). An individual's learning style is a combination of accepting, organizing and processing information.

In this the way – modality, in which an individual perceives information, plays an important role. The teacher is the one who plans the learning process. He/She must do this in such a way that the best possibilities to best use their natural abilities are offered to the pupils, but that they develop other ways of gathering information from the environment at the same time.

The 'Information processing' section includes information gathering and information presentation. This is a field that develops the pupil's mathematical literacy. This is defined as the pupil's ability to effectively think, analyze and explain ideas, while they organize, solve and interpret the solutions to mathematical problems in various life situations (Pisa, 2006).

In this article I will mainly focus on the presentation of gathered and processed data with various presentation models, with emphasis on the presentation using lines and bars, as well as pie charts. These are the three ways that are introduced in the pre-school period as an introduction to statistics and work with them is continued in the firs and second triad. In the second triad pupils have so much knowledge and experience that they can use what they know and individually execute a simple research assignment.

Statistics offers many options for interdisciplinary connections with other subjects and connections between various mathematical themes. Thus it is beneficial to the expansion of the mathematical horizon, helps evolve mathematical thinking and encourages critical thinking about the world we live in. With all this it helps to form a comprehensive learning experience (Cotič, 2003). The results of the TIMSS 1999 study show that the introduction of these themes into the syllabus was legitimate, as it was found that:

— *'Slovenia has statistically made important progress in the use of graphic presentations in mathematics classes,*

— *the field of information presentation, analysis and probability have become, according to the results, the best field in mathematics in Slovenia'* (*Japelj, 2000*).

Information processing – general

Syllabus

In the syllabus mathematical themes are divided into different sections. Operative goals are stated first for each section. They are derived from general goals and themes and are upgraded, supplemented and refined. The distinction between mandatory and optional goals is also stated. The mandatory goals lead to knowledge necessary for general education at the completion of primary school education. They are intended for all the pupils, that is why a teacher must include them into his/her lessons. The optional operative goals are intended for adding and deepening knowledge and are chosen by the teacher and included into the syllabus according to the pupils' abilities and interests (Syllabus, 2011).

With the transition to nine year primary schooling in Slovenia in 1999, we have introduced the 'Information processing' themes into various syllabi. They have been introduced on all levels, from the first grade on. Previous generations met with these themes a lot later, in high school. This section contains themes from statistics, combinatorics and probability.

The table below shows the themes of the 'Information processing' section which are connected and upgraded from grade to grade.

Table 1. Themes of the 'Information processing' section in the syllabus (SOURCE: Syllabus 2011 and Kurikulum 1999).

<table>
<tr><th colspan="6">THEMES – INFORMATION PROCESSING:</th></tr>
<tr><th>PRESCHOOL PERIOD</th><th>1st GRADE</th><th>2nd GRADE</th><th>3rd GRADE</th><th>4th GRADE</th><th>5th GRADE</th></tr>
<tr><td colspan="2">Tables.</td><td>Tables.</td><td>Tables.</td><td colspan="2">Counting recording/ Counting techniques. Presentation of data in a table.</td></tr>
<tr><td colspan="4">Presentations: Pictograms, bar chart, line chart.</td><td colspan="2">Bar chart, line chart, pie chart.</td></tr>
<tr><td colspan="2"></td><td colspan="4">Tally marks.</td></tr>
<tr><td colspan="3">Simple statistical research.</td><td>Research.</td><td colspan="2">Investigation (use of information processing knowledge).</td></tr>
</table>

In the preschool period we cannot yet talk about real study and learning of information processing but about the gathering of first experience in data presentation. In the lower grades pupils gather experience with the collection and organizing of data, presentation and chart reading and commenting on the presented data. The gradually progress to independent interpretation and critical judgment of other interpretations.

Some important concepts, connected to information processing and their introduction

Tables and information gathering

In the preschool period pupils enter the world of statistics through a coherent experience and active participation. They gather their first experience in the use of symbols with which they record events, situations and data. They get to know their first graphic presentations, they design and interpret them.

In the first grade pupils present data with a provided table, during a controlled activity on the factual level.

Each activity, in the preschool period as well as in the first grade, is introduced on the factual level and only progress to the graphic and later symbolic levels if the children's abilities allow it.

In the second and third grades the pupils gather information with a table and present the gathered data as clearly as possible. In the fourth grade they get to know and use various counting techniques and record their counting into tables. In the fifth grade they can logically define classes of data arrangement and use reliable counting techniques. (Syllabus, 2011).

Presentation, reading and interpretation of gathered information

In the first grade pupils read data from tables and present them with a figure presentation using a line or bar chart. Up until the third grade they use the line or bar chart for presenting data. With the presentation the pupils read the data or compare the various pieces of information between one another. The required and sought-after information indicated by the questions regarding the presentation is gathered from the presentation itself. (Syllabus, 2011).

In the fourth and fifth grades pupils can arrange data by themselves, present the data with line or bar charts. They also get to know another way of presenting gathered information, the pie chart, which is connected to fractions. In the pie chart pupils can read and compare shares of an individual piece of information. In the fifth grade they can present the shares (half, quarter) by themselves (Syllabus, 2011).

Research, Investigation

From the third grade onward the pupils more or less do all the research by themselves, they gather data, organise it, present it with one of the known presentation models and interpret it.

In the fourth and fifth grades they use the gathered knowledge on data processing in new situations and independently, in groups or individually conduct an investigation and present their findings. They are able to provide interpretations of a higher quality and critical assessment of the presented data (Syllabus, 2011).

On research and presentations

Research planning

Every research begins with planning. We decide what we are going to research and how we are going to approach the subject. We indicate our expectations and pose questions, for which the research is going to provide answers (Cotič, 1999). In the preschool period and in the first two grades the research is simple. The teacher prepares everything necessary and includes children or pupils into the continuation of the research. Older pupils are capable of searching for and proposing ideas for the cases worthy of research. Older pupils can carry out the research from the beginning to the end independently.

Information gathering

First, the information has to be gathered. The younger children are helped with that. We also pick a theme that does not require gathering of extensive data, so that the database does not become too large and difficult for the children to navigate. Information gathering is also associated with the place and time of gathering, the methods of gathering and recording information and the number of people included in the research. Because the amount of data is large, one of the most useful ways to record data is to use tallies. We can also use crosses, dots, geometric shapes or something similar.

The figures below present examples of tables and ways with which to achieve clear information gathering.

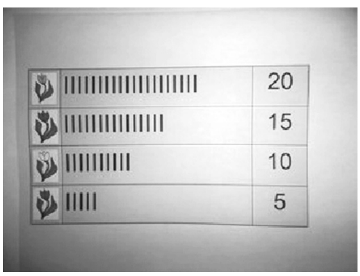


Figure 1. Tally mark chart (Pisk, 2013).

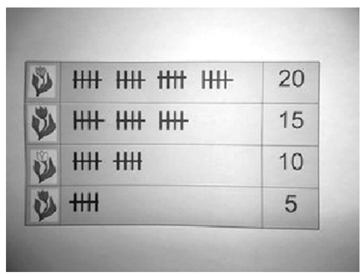
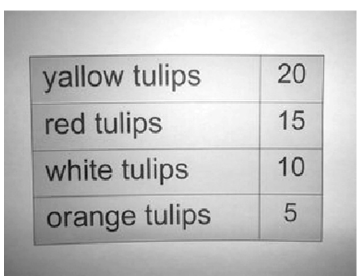


Figure 2. Tally mark chart 2 (Pisk, 2013).



yallow tulips	20
red tulips	15
white tulips	10
orange tulips	5

Figure 3. A table with the gathered data (Pisk, 2013).

For this step we plan and choose a way that will allow us to overview the collected data in the next step. The older pupils can define classes for individual data groups by themselves, but the younger ones have to be helped with this. The gathered information has to be organised and made clear. If necessary, the inappropriate data is excluded. We also choose a way in which we are going to most clearly record and present the data. In the preschool and school period we usually use tables. Later this helps us with the presentation design and the presentation of data in graphic form (Cotič, 1999).

We can design a line or bar chart, or a pie chart.

Examples of various presentations for the preschool and school period

For information processing we use the line or bar chart and from the fourth grade on the pie chart. All of the mentioned presentations can be made into pictograms, when individual pieces if information is presented with an image, photograph, a drawing of an object which is the element of the research we are doing or we can use a symbolic presentation, where a piece of information in a certain space is represented by a cross, circle, line,. . . or by coloured or hatched spaces.

At first we direct pupils towards space colouring. Later, for economical reasons, we progress to data marking in the spaced with a cross. The presentations in the first grades offer data in such a way, that each space represents the data for one element, later one space can represent a larger number of elements. This is indicated with a legend, which is an integral part of the presentation. A title is also a mandatory part of the presentation.

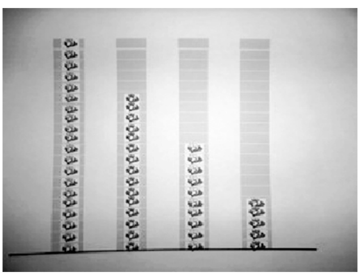


Figure 4. A pictogram – columns (Pisk, 2013).

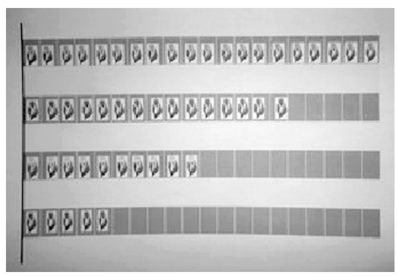


Figure 5. A pictogram – lines (Pisk, 2013).

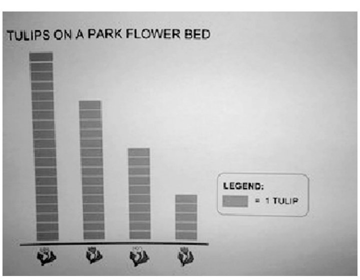


Figure 6. A symbolic presentation 1 – columns (Pisk, 2013).

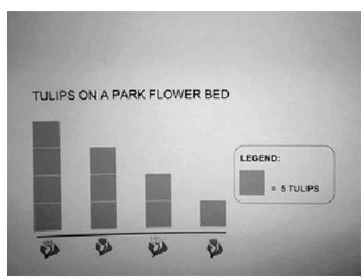


Figure 7. A symbolic presentation 2 – columns (Pisk, 2013).

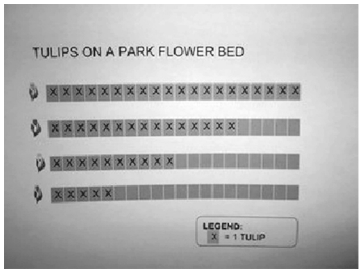


Figure 8. A symbolic presentation with lines with the use of crosses (Pisk, 2013).

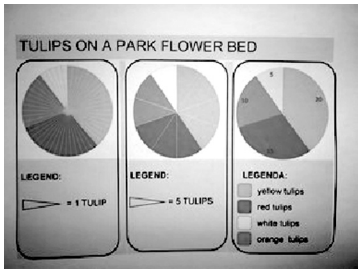


Figure 9. A pie chart – three examples in the order of difficulty (Pisk, 2013).

Data analysis

The presentation offers us an overview of the data and is of great help with the analysis and verbal interpretation of thus presented data, as it is non-verbal and in this way generally readable. It has to be equipped with a title and a legend. Data presented in this way can be critically assessed, we can find out which facts they point out and which questions we can answer on the basis of this data. At the same time we can find out if the research has answered the questions we have posed at the beginning (Cotič, 1999).

Verbal data explanation – interpretation

The data offered to us by the collected data and the presentation can be verbally explained. The older the pupils are, the more we progress into interpretation. The read data has to be given meaning and proper reflexion. This kind of interpretation allows the pupils to get used to critical judgement of the presented data, to deduction and analysis. Gradually it should lead the pupils to the discovery that one of the tasks of statistics is the anticipation and prediction of events on the basis of gathered information (Cotič, 1999). Each good interpretation opens a way to ideas for new statistical research.

The didactics of presentation introduction – from the factual to the symbolic

Displays allow us to present processed data. With the help of prudent didactical steps we will introduce pupils to presenting and reading of the collected data. We have to offer them the knowledge of various ways of presenting data through active participation and through a concrete experience. From the concrete experience we lead them to the graphic level and lastly to the symbolic level. This is achieved by planning the introduction and executing it over four didactic steps: A live display, an in-space display, pictogram and symbolic presentation. I will explain all four didactic steps on an example.

The pupils are offered coloured A4 sheets of paper from which each pupil folds his/her own paper boat. The situation we get from this is appropriate for information processing as we inadvertently wish to know which colours have been picked and how many boats of a certain colour have been made. That is why we gather data. The pupils form groups on the basis of the colour of their boat. We discover that we can see the pupils standing together have boats of the same colour, but if we wanted further information on the gathered data we encounter certain difficulties. We announce that we are going to present the collected data more clearly, so that we can read and comment it quickly.

The live display

Classrooms with floors made of materials which by themselves offer spaces – squares and rectangles – are very appropriate for the formation of the ’live display’. If a classroom does not offer that it makes sense to ’draw’ a grid on the floor of the classroom. We can do this very quickly with a coloured or white sellotape.

The pupils that have boats of the same colour come and gather and form their line with their boats in their hands. As many lines as there are colours are formed. Here we have to be careful that all lines start on the same line. We direct the pupils to observe the line lengths and find out which one is the longest, the shortest, longer than, shorter than,. . .

We have thus formed a live display in which the pupils are standing and in their hands they are holding the object we used for the basis of our research. This kind of display offers the pupil a possibility to be actively involved in the formation of the display and at the same time triggers the interest for the analysis of the gathered information and search for the answers to the questions which appear along the way.

The In-space display

In the second didactic step we are still staying on the factual level.

For the progression to the new form of display – the in-space display – we take advantage of the situation. The pupils standing in the display do not have a good overview of all the data. With the next move we will prepare a display that will offer an overview of the gathered and presented data to everyone. We encourage the pupils to suggest what could be done in order to have a better overview of the data. We lead them to the suggestion that each of them leaves the display, but puts down his/her boat where they were standing. The pupils are standing around the display. They have a view of all the data and objects subject to the process are still a part of the display. The overview of the data allows them to find out how many of individual boats there are, of which there is more/less, are there more red or blue ones,. . .

Figure presentations/pictograms

In the third didactic step we prepare an opportunity for the pupils to progress from the factual level to the graphic. Object used for the information processing is given in graphic form. In our case these will be the pictures of boats in colours used by the pupils, when they were making their boats. We prepare the pictures in the vicinity of the display on the floor, they can also be photos of the boats. Pupils find the picture/photograph of their boat and then they replace the boat in the display with its picture. For younger children we prepare pictures of the same size as the spaces on the floor. When pupils pick up their boats, they replace it with a picture that fills the entire space in the display. With older pupils we can perform this step with pictures that are smaller than the spaces in which the pictures are placed. A display prepared like this is very clear. At any given moment we can see from the display how many individual boats there are and for which colour be search for/report the data. All this can be interpreted from the lengths of the bars and colours of the boats.

Symbolic presentations

In the last step we execute the transition from the graphic to the symbolic level. Up to this moment the data was very evident in itself, they gave us all the information we wanted to get form the display/presentation. In this step we will present the data with symbols. For this purpose we can use crosses, circles or colour all the spaces in the grid. This can also be done in the display constructed on the floor, on which we have performed all three previous didactic steps. One of the possible ways is to simply turn over the sheet of paper with the picture of the boat on it. The image disappears, the only clue to the data is the blank piece of paper. The picture of the boat can also be replaced by a sheet of paper which can be white or any other colour, it this case it makes sense for the piece of paper to be the same size as the space in the display in which the picture of the boat was situated before. We can use white paper, to lower our expenses, but there is nothing wrong with using any other colour. It does, however, make sense for all the sheets of paper to be of the same colour.

From here on the display itself does not offer all the data we need to read or interpret the display, that is why the presentation has to be equipped with a legend and data below the line where the bars or lines start. At each starting point we add a boat of the same colour the bar or line represents. With the legend we provide information for each individual space coloured in the grid, how many elements it represents. Each of these didactic steps allows us to read data from the presentation and discuss it. All this is planned and executed in a way which observes the characteristics and abilities of the pupils included in the activity. In the preschool period we perform the presentation on the factual level, this means that we use the first two didactic steps, the third one if the pupils are up to it. In the first and second grades the pupils are already able to progress to the symbolic level so that is when we usually add the third step to the activity. Later on we mostly present data on a prepared grid on a larger sheet of paper or a blackboard. Together with the pupils we can perform data presentation on the graphic level and then progress to the symbolic. At first we colour the spaces in the grid, but later the pupils find out for themselves that it more economical to use hatching or crosses to mark the representative spaces.

Information processing and ICT

Modern communication technology (ICT) provides us with the materials for data processing and presentation. The interactive blackboard also offers very practical possibilities. We can also use PowerPoint and Excel. Many freeware programs available on the internet also come in useful when working on assignments like these. HotPotatoes is suitable for assignments including questions and answers, supplement assignments or matching assignments. We can also prepare our own e-materials in PowerPoint, we design it so that individual slides offer a presentation with some incorrect answers and the correct one. The answers can be animated. Pupils progress to the next assignment with the choice of the right answer. These kind of assignments come readymade and are available on the internet.

Final thoughts

Themes connected to data processing offer the possibility for interdisciplinary connectivity, the also offer connections inside the scope of mathematical themes and sections. Situations which offer cases from everyday life are not difficult to come by. And this is the power and attractiveness of these materials. Activities which are planned and executed in this way give guidelines to experiential learning. There are enough possibilities to organise and execute the activity on various difficulty levels observing the various abilities of the pupils, we differentiate the activity. The pupils perceive mathematics as a pleasant experience and confirm that mathematics is closely connected to everyday life.

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Obdelava podatkov in prikazi v osnovni šoli na razredni stopnji – Ukvarjanje z učnimi izkušnjami

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Povzetek. Vsebine, ki pri predmetu matematika pokrivajo temo obdelava podatkov, so bile v Sloveniji v učne načrte vpeljane ob prehodu na devetletno osnovno šolanje, leta 1998. V njem so se ohranile tudi po letu 2011, ko so bili učni načrti posodobljeni. Odločitev za vnos teh vsebin v učne načrte je bila sprejeta z namenom, da bi povečevali matematično pismenost in izboljšali kakovost učinkovitega izobraževanja na temo obdelave, predstavitve, branja in interpretacije zbranih podatkov, vse tudi v povezavi s prvimi koraki v statistiko. Učenci se s prvimi primeri srečajo že v predšolskem obdobju, nato pa v šoli vsako leto svoje znanje nadgradijo. Prehajajo od preprostih prikazov, ki jih strokovni delavci v šoli vpeljejo preko konkretne, skozi slikovno do simbolne ravni. Pri tem upoštevajo strategijo reševanja matematičnih problemov in v povezavi s tem faze reševanja. V 1. in 2. razredu začnejo s preprostimi predstavitvami in branjem podatkov s prikazi s stolpci in vrsticami. Figurni prikaz jim ponudi prehod iz konkretnega na slikovno raven. Nato pa v 3. razredu preidejo na samostojno zbiranje, prikazovanje, branje in interpretiranje podatkov. Navajajo se tudi na uporabo različnih načinov zbiranja podatkov. V 4. in 5. razredu so že zmožni uporabiti pridobljeno znanje in samostojno načrtovati preiskavo. V prispevku so opisani primeri, ki nakazujejo didaktično pot pri vpeljavi kateregakoli prikaza. Nakazano je tudi stopnjevanje zahtevnosti iz razreda v razred. To so vsebine, ki popestrijo pouk in ponujajo mnogo zelo smiselnih možnosti za povezave z drugimi predmeti. Dejavnosti so načrtovane tako, da učenca vodijo skozi izkušnjo. Tak način dela učencem ponuja smiselno pot k izgradnji znanja, urjenju spretnosti orientiranja v prikazu in pridobivanju veščine branja podatkov iz njega. Obravnavo teh vsebin lahko izpeljemo tudi z uporabo IKT. Gradivo si lahko izdelamo sami ali pa ga poiščemo med primeri, ki so dostopni preko spleta.

Ključne besede: matematična pismenost, obdelava podatkov, prikazi, strategije reševanja, didaktična pot

Open-ended mathematical tasks for primary school youngsters

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Abstract. The school curriculum emphasizes that maths should be a practical and useful subject which students can understand and apply in everyday life to solve various problems. Particular goals of the national curriculum are aimed at developing students' competencies. Mathematical competence will enable a student to develop and apply mathematical thinking in solving simple problems. Students should realize the importance of maths during their education because it develops their awareness and understanding of the environments in which they live. It should enable pupils to manage risk and cope with change and adversity. PISA, being an important aspect of mathematical literacy, emphasizes that Maths should promote pupils' social and personal lives. It improves their schooling and teach them how to spend their free time in a more useful way. We all know that Croatian fifteen-year-old students did badly in mathematics last year. It was due to a traditional way of teaching Maths in our schools, the use of boring teaching methods and strategies where a set of exercises are done in a non-creative way (while according to PISA assessment items include a combination of multiple choice and open-ended tasks).

The teaching of mathematics is defined by the Curriculum which has strictly separated teaching themes, key terms and achievements, where more attention is paid to the realisation of the curriculum and not to the application of knowledge. Students usually learn everything by heart and cannot apply these competencies to real-world contexts. Being teachers practitioners we have noticed that teaching in a creative way is much more efficient (instead of using traditional methods). We apply various new methods while explaining a new subject matter.We have to use different types of exercises as means of stimulating and encouraging the best possible progress for all pupils. Maths should contribute to the development of pupils' sense of identity through knowledge of the social heritage of diverse society and of the local, national and global dimensions of their lives.

Examples of open-ended and problem-solving tasks will be shown in this paper. These exercises require an integration of data while solving problems (which we try to incorporate into teaching). Primary-school teachers, living in the town of Sisak, should take into consideration this kind of teaching.

Keywords: mathematical competence, mathematical education, mathematical functionality, mathematical tasks, role of a teacher

Maths instruction in the initial grades of elementary school

Maths teaching in lower grades should be interesting and diverse, challenging and intellectually stimulating. In the initial elementary school grades pupils like maths and gladly accept all given exercises. However, as they move to higher grades, they find maths more and more difficult, they are less ready to face mathematical challenges and less successful in solving the given problems. Students' attitudes towards maths are positively linked to motivational processes and achievements in maths (Kadijevich, 2008). Considering the fact that lower grades lead to more negative attitude towards maths, it's evident that students will "love" maths less and put weaker efforts into it. The reasons for failing could be various: overburdened programmes which aren't followed by sufficient number of revision and practice lessons, long-term practice of the same type of exercises (often too easy for larger number of students in the class), insisting on mathematical operations practice (rather than solving problems), contents inappropriate for students' abilities, monotonous and traditional (and exclusively frontal) teaching which is implemented by the teacher, not relating maths contents to reality... School teaching must consider children's informal knowledge, and the research have shown that children can successfully solve very complicated mathematical problems if the situation in them is connected to their life experience (Saxe, 1988; Baranes et al., 1989, acc. to Vlahović-Štetić, 2009). The stated authors suggest the importance of teaching children to think strategically, and doing so on familiar situations, because then they will be capable to apply that knowledge in new situations. So, it is necessary to confront students with more complex life situations in which the accent won't be on the classical calculations (speed, precision and only one right solution!), but on the problem solving which is related to their realities. The expected goals are achieving basic mathematical literacy, integration of mathematical competencies in the student's overall intellectual and life development – gaining useful knowledge for a successful life. Students (and their teachers and parents), in the first four grades, expect maths teaching to be fun, useful, and for themselves to be as successful as they can be in acquiring and assessing maths knowledge (Mesić, Basta, 2012).

The assigned curriculum accentuates the implementation of maths, which should be viewed as a practical and useful school subject. Students have to understand it and know how to apply it on various problems and in their surroundings because, if they acquire mathematical competence in such a way, they will be prepared for life-long learning. Along with the assumption that the existing syllabus in everyday schooling is so implemented, numerous external factors should have influence on maths teaching: the application of ICT technology in education, external evaluation of students' academic achievements, international research and students' knowledge and skills assessments (for example, PISA, PIRLS; TIMSS).

Maths teaching is defined by the curriculum, it has clearly separated teaching themes, key concepts and achievements. In that instruction more attention is given to content realisation (within the set time-table) and educational goals than to connecting the acquired knowledge which is most commonly 'served' to students in a structured way and disconnectedly with other subjects or real-life situations. So, the greatest difficulty in the external evaluation of educational achievements of the fourth graders (implemented in 2006) came from integrated exercises which required particular knowledge of science in order to solve the mathematical exercise correctly.

According to the methodological analysis of PISA exercises, it was noticed that the exercises of assessing mathematical literacy in PISA testing were more demanding than the exercises implemented and being solved in maths teaching (and textbooks) in the Republic of Croatia – according to the contents (areas of statistics and probability!), the degree of questions' complexity and formulation of the same. PISA exercises accentuate the interpretation (especially graphic illustrations), argumentation, assessment and deeper understanding of mathematical content, which are stimulated by textual open ended exercises. It is probable that exactly the difference in demands between PISA exercises and the ones from Croatian mathematical instructional practice is the cause of poor results of Croatian fifteen year-olds on PISA testing of mathematical literacy. The Report of PISA's work group for maths in Croatia was published in 2008 (Braš, Roth et al., 2008, p. 163), and although it suggested it "would be wise to take those PISA demands which can enrich our maths teaching", that application is still not visible in maths teaching in lower grades of Croatian primary schools. The research done in 2008 has shown that the greatest accent in maths teaching is on solving mathematical exercises and the development of logical way of thinking, and less on the use of maths in everyday life and on the development of critical reflection on the concepts and procedures. So, the transformation of the Croatian educational system should include strengthening students' mathematical competencies as well.

Open ended tasks

Open ended tasks should have a more important place in contemporary maths teaching, 'more modern' instruction which will require students to solve mathematical problems in different ways, and in doing so create and interpret assumptions and present the chosen way of work. There are two types of open ended tasks: tasks with one solution with more different approaches to it, and tasks with more different solutions. The advantages of open ended tasks are numerous:

- For students – they are more active because they can examine their ideas and assumptions more often, and in doing so they stimulate various forms of thinking, concluding and verbal expression.
- For teachers – they gain feedback: about the way students think ('see' the process and not only the final result), about verbal expression in explaining and argumentation, but also about the possible drawbacks of students' knowledge or understanding of the concepts which student apply in the course of work.

Table 1 shows open ended and integrated tasks in maths teaching, designed by the authors of this paper.

Table 1. Examples of open ended and integrated exercises in maths teaching for students of the fourth grade.

<table>
<tr><td>
The table contains information about how much fish the salesmen sold on Christmas Eve in some Croatian fish markets.
<table>
<tr><th>Market in Split:</th><th>Market in Osijek:</th><th>Market in Gospić:</th></tr>
<tr><td>1234 kg of cod
495 kg of carp
874 kg of hake
508 kg of pilchard</td><td>645 kg of carp
766 kg of hake
827 kg of octopus
398 kg of trout
278 kg of catfish</td><td>649 kg of trout
249 kg of hake
391 kg of pilchard
189 kg of carp</td></tr>
</table>
1. How many kilograms of fish were sold in the market in the flat area of the country?
2. How many kilograms of fish were sold in the market in the hilly area of the country?
3. How many kilograms of fish were sold in the market in the seaside area of the country?
</td></tr>
<tr><td>A sportsman has been preparing for an athletics competition. During January, February and April he ran 45 kilometres a day; and during May, July and August 39 kilometres a day. The other months he ran 40 km a day. How many miles did he run during spring?</td></tr>
<tr><td>Caravan is allowed to carry 965 kilograms of cargo. A farmer has loaded it with picked vegetables: potatoes, onions, cabbage and beans. How many kilograms of each vegetable could he have loaded in the caravan?</td></tr>
<tr><td>Martina has been preparing to spend the winter in Austria. She needed to change kunas into euros and she found out that she can get 100 euros for 750 kunas. She wanted to buy 950 euros. How many kunas did she need?</td></tr>
<tr><td>Brown bear has the body mass of 500 kilos. How many children have the same body mass as the brown bear?</td></tr>
</table>

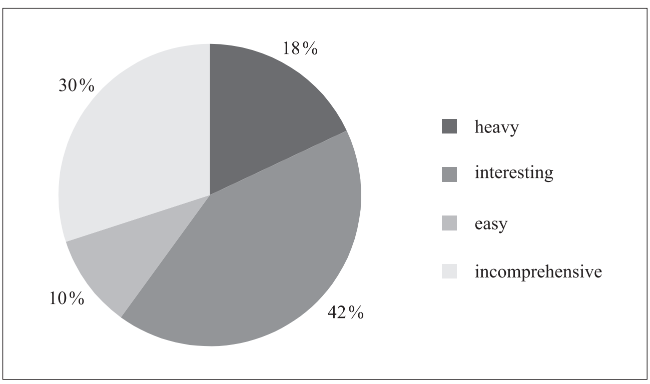

Figure 1. Fourth grade students' opinions about the open ended exercises.

Fourth grade students (Primary School Viktorovac Sisak and Primary School 22nd of June, Sisak), which solved the presented exercises, encountered problems in dealing with them because they needed to use the information from science content in order to calculate the mathematical task, and most problems were caused by the open ended exercise with multiple answers, "because they didn't know which one will be correct. " When they were encouraged to state the impressions about the exercises, they declared: “interesting” and “incomprehensive” (Figure 1 – Fourth grade students’ opinions about the open ended tasks; Table 2 – Students’ impressions about the tasks).

Table 2. Students’ impressions about the exercises.

The second exercise was the most interesting. The third exercise was a little harder, but I think it is very well designed because you have to think a lot.
I didn’t solve the second nor the third exercise because they are very difficult for much logic, and I can’t understand that. I should have known how to do the calculations and learned a bit more.
I found nothing hard, but I don’t like reading the exercises much. I prefer to think, and very well.
It takes a lot of logic and knowledge to solve these exercises. All in all, I would like to deal with this type of exercises more often.
First exercise was confusing, the second was interesting and incomprehensive, and I didn’t know the third.
I had to remember the third grade subject matter from science in doing all the exercises.
I found the third exercise the most interesting because it has many solutions.

Students’ answers show that, in great percentage, the exercises were interesting because they can relate to them (sports, nature), but, at the same time, they are incomprehensive because reading comprehension is still a big problem at that age, and they are also not used to apply data from other areas in dealing with mathematical problems.

Teacher and maths instruction

The vision of life-long learning and constant professional growth demands a teacher who will know how to think critically, who is capacitated for reflection and evaluation, who knows how to find or secure the prerequisites for the development of every individual student, knows how to encourage and support students in the learning process (Vujičić, 2007). Teacher can and should influence the creation of motivating and challenging educational environment by setting broader goals in maths teaching, by choosing more efficient strategies, methods and work procedures because, according to the results of national assessment at the end of the 8th grade (implemented in 2008), students whose teachers were counsellors or mentors have shown greater success (experimental external assessment of educational achievements in primary schools of the Republic of Croatia, 2007).

In preparing and planning the maths instruction, teacher should design the appropriate instructional methods, forms of work and teaching tools which stimulate creative and cooperative work. The results of implemented research (Arambašić et al., 2005) have shown that teachers should be given the support in order to develop their positive attitudes and believes about maths, and they should be directed towards the choice of teaching methods most appropriate for their students. The application of various teaching strategies requires the teacher’s creativity and necessary time for designing complex mathematical exercises (Mesić, Basta, 2012).

Teachers' opinions about the application of open ended tasks

107 class teachers participated in the research implemented in November 2012, at the professional meeting of the County Expert Council of the city of Sisak. Most of the interviewed teachers (70%) have a university degree, 4 teachers (3,7%) have a postgraduate degree, and the other teachers have a college degree. Teachers were reminded of different types of tasks which can be applied in initial maths teaching, with the accent on open ended and integrated tasks.

According to the questionnaire's results, all teachers use open ended and integrated tasks in regular and additional lessons. It's been noticed that, in doing so, they use integrated exercises more often (Figure 2) than open ended tasks (Figure 3).

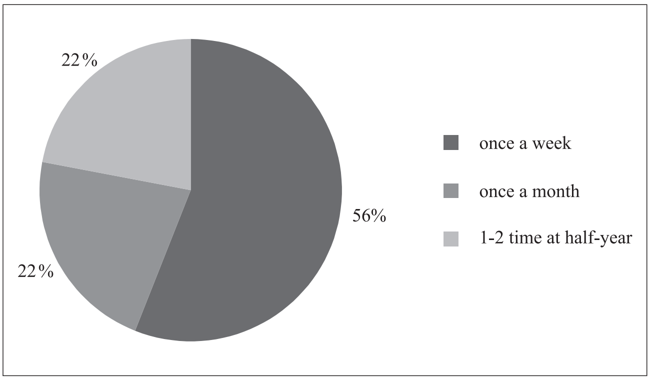

Figure 2. Integrated tasks application frequency.

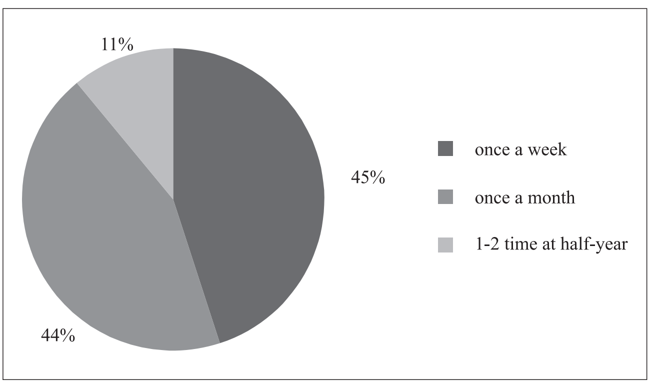

Figure 3. Open tasks application Frequency.

According to teachers' opinion, textbook sets used in maths lessons in the first four grades don't contain enough integrated tasks; moreover, open ended tasks with more possible answers are altogether absent. So, in designing and making open ended exercises (Figure 4), large number of teachers relies on their knowledge/abilities and literature, while they mostly design integrated exercises themselves, using literature, but with no help from experts (Figure 5).

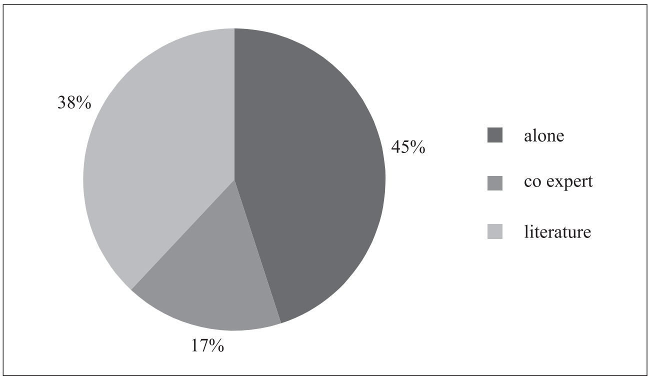

Figure 4. Help with designing and making open ended taks.

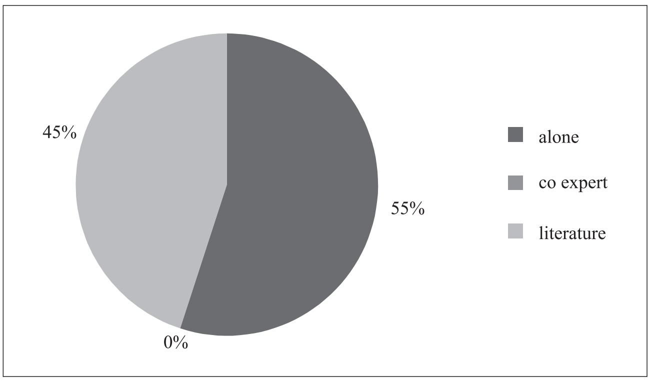

Figure 5. Help with designing and making integrated tasks.

Some of the answers to the open question, 'Which competencies (abilities, skills) do children develop with open ended exercises', are shown in Table 3, while Table 4 presents some of the teachers' answers to the question, 'Which competencies (abilities, skills) do children develop with integrated exercises?' Table 3 – Teachers' answers to the question: Which competencies (abilities, skills) do children develop with open ended exercises?

Table 3. Teachers' answers to the question: Which competencies (abilities, skills) do children develop with open ended exercises?

'Which competencies (abilities, skills) do children develop with open ended exercises?'
The ability to concentrate, think logically, make connections and conclusions independently. Promoting their own opinion and higher thinking abilities.
Developing thinking and logical inference abilities, capacitating students for written and graphic expression, development of knowledge application in everyday life, developing systematicness and independence in working, and interest in the subject.
Mathematical competencies: application of mathematical thinking in solving everyday life problems, assessing.

Innovativeness, problem solving, critical thinking development, stimulating research and new ideas creation, cooperation development.
Mathematical thinking and reasoning ability (mastering mathematical ways of thinking; abstraction and generalisation, where relevant for the question, and mathematical modelling (i.e. analysing and creating models) with the use of existing models for the posited questions.
Comprehension of exercises, ideas presentation and interpretation of possible solutions.
Creative and critical thinking (reexamination), spotting important relationships and connections, verbal expression.

Table 4. Teachers' answers to the question: Which competencies (abilities, skills) do students develop with integrated exercises?

'Which competencies (abilities, skills) do children develop with integrated exercises?'
Logical concluding, analysis and synthesis, connecting teaching contents, application of knowledge they possess.
Capacitating students for logical thinking. Developing abilities of knowledge application on various examples. . .
Ability to connect learned contents from several subjects, logical thinking and inference, ability to convey conclusions and reasons which led to them.
Developing assessment skills, systematisation in working.
The ability to connect the acquired knowledge from all subjects, application in everyday life.
Proper knowledge application; seeing, determining, explaining, proving, assessing.

In the course of designing, applying and assessing open ended and integrated exercises teachers encounter many problems. Some of them are stated in tables 5 and 6.

Table 5. Teachers' answers to the question: Which problem have you encountered in the course of designing, applying or evaluating open ended exercises?

'Which problem have you encountered in the course of designing, applying or evaluating open ended exercises?'
Lack of time for the use of pictures or stories appropriate for children's' better understanding of the textual exercise.
There are problems during the exercises presentation, since they demand solving in several steps. They are often incomprehensive for children (particularly in the initial grades), so they should be additionally explained by drawings, and then there is a problem of time – e.g., two exercises can be solved during one lesson. They are useful for students' latter mathematical thinking.
Some students sometimes don't understand what they are required to do, more often in exercises with more than one correct answer. Sometimes I don't know how to score answers which include mistakes in the calculation, but give the correct answer.
The lack of professional literature, considering that students' set of textbooks doesn't contain sufficient number of such exercises.
More time is needed for designing the exercises (although they are more challenging and interesting), little number of exercises is solvable in one lesson, less number of students understands them and they carry more points.
The necessary time for designing the exercises is a problem because there isn't enough literature and ready-made worksheets.
There are no problems, apart from the fact they require students' preparation for solving this type of exercises.
I need more time to correct students' exercises and to think over the way of evaluating solutions, and the given grade is lower than in the usual mathematical evaluations.

Table 6. Teachers' answers to the question: Which problem have you encountered in the course of designing, applying and assessing integrated exercises?

'Which problem have you encountered in the course of designing, applying and assessing integrated exercises?'
Little time for solving integrated exercises in regular lessons.
Problem is designing exercises with actual and interesting topics, and sometimes also scoring the results.
Some students haven't learned all the contents from other subjects, which makes the implementation of integrated exercises hard. In evaluation, I noticed that some students don't have a habit of drawing a sketch of the exercises in which they would see the important connections, and which would make solving and evaluating the exercises easier.
Where to write the grade in the school directory if a student 'lost' points on spelling errors, or because of not knowing the relief features (and alike from science lessons) – and he/she set the exercise mathematically well and came to the right answer?
Students often forget to write the wanted answer in words, they often read the exercise superficially, and then mix mathematical operations.
Students don't read the exercise carefully (and more than once), scoring is the problem, and there is a deficit of literature which would help us.
Problem is the time needed to do the exercises (considering their lack in the textbook sets). Maybe if there was more cooperation between teachers in exchanging work materials.

Conclusion

It is necessary to enrich maths teaching with various (less traditional) ways of explaining the subject matter, with the choice of exercise types, teaching methods and strategies, to pass the positive attitudes towards maths onto the students, to strive towards the implementation of maths within various contexts, to derive mathematical problems from everyday meaningful situations. Considering that the research amongst class teachers has shown that the textbook sets don't contain sufficient number of described exercises, the teacher's role is important, and he/she needs to design interesting and life's textual exercises (open type, integrated) in which the students will use higher thinking abilities. Students find open ended exercises interesting, their interest in maths is more expressed because they are motivated – and that again depends on the teacher, on his professional and methodological creativity. As problems they encountered, teachers stated: insufficient maths timetable (considering the programme and demands of modern instruction), lack of their own time for designing exercises, because the textbook sets are lacking, but also superficial reading of the exercises on behalf of the students.

Creative teacher recognises him/herself in various ways of explaining the subject matter, in choosing the types of exercises, in setting challenging problems for students to solve, in attempting to make maths as interesting as possible for students and to integrate it in real life – "we use maths in everyday life, science, shops, trade and industry, because it is a powerful, complex and unambiguous mean of communication, explanation and assessment. " So, teacher's competencies, except necessary professional knowledge and skills for teaching (alongside methodological and didactical knowledge, team and cooperative spirit), should contain positive attitude towards mathematical contents, desire for creation of active and stimulating surroundings and work materials, but also an active contribution to introducing the necessary changes in the Croatian educational system.

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Zadatci otvorenog tipa za učenike mlađe školske dobi

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Sažetak. Propisani nastavni program naglašava primjenu matematike na koju treba gledati i kao na praktični, korisni predmet čije sadržaje učenici moraju razumjeti i mogu znati primijeniti pri rješavanju raznih problema u svome okružju. Nacionalni kurikulum usmjeren je na učeničke kompetencije, pri čemu se matematička kompetencija odnosi na osposobljenost učenika za razvijanje i primjenu matematičkoga mišljenja u rješavanju problema u nizu različitih svakodnevnih situacija. Stoga bi, tijekom matematičkog obrazovanja učenici trebali uvidjeti važnost matematike u svojim životima, povezati je sa svakodnevnim stvarnim i smislenim situacijama i primjenjivati je u raznolikim kontekstima. I PISA, kao važan aspekt ispitivanja matematičke pismenosti, ističe rješavanje i korištenje matematike u raznolikim situacijama uključujući osobni život, školski život, rad i slobodno vrijeme, lokalnu zajednicu te društvo. Znamo kako su u ispitivanju matematičke pismenosti prošli hrvatski petnaestogodišnjaci, a neki od važnih razloga iskazanog (ne)uspjeha sigurno su tradicionalni način ostvarivanja nastave matematike u našim školama, uporaba klasičnih nastavnih metoda i strategija, te naglasak na rješavanju zadataka zatvorenog tipa s uvježbanom tehnikom rješavanja određenog tipa zadataka (dok PISA matematičku pismenost procjenjuje kombinacijom oblika pitanja višestrukog izbora i pitanja otvorenog tipa).

Nastava matematike definirana je Nastavnim planom i programom, ima jasno odijeljene nastavne teme, ključne pojmove i postignuća, pri čemu se više pažnje posvećuje realizaciji NPiP i obrazovnih zadataka nego povezivanju usvojenih znanja koja se učenicima najčešće 'serviraju' strukturirano i nepovezano s ostalim nastavnim predmetima ili stvarnim životnim situacijama. Kao učiteljice praktičarke uočile smo da je u nastavu matematike potrebno unositi više kreativnosti u obliku raznolikih načina objašnjavanja gradiva, u izboru tipova zadataka, nastavnih metoda i strategija, u pokušaju da se matematika svakom učeniku učini što zanimljivijom i integriranom u stvarni život

U radu će biti prikazani primjeri zadataka otvorenog i problemskog tipa koji zahtijevaju integriranje podataka prilikom rješavanja problema (koje pokušavamo ugraditi u nastavnu praksu), te promišljanja učitelja razredne nastave grada Siska o primjeni takvih zadataka.

Ključne riječi: matematička kompetencija, matematičko obrazovanje, funkcionalnost matematike, matematički zadatci, zadatci otvorenog tipa, uloga učitelja

Educational effects of the interactive method in studying the Pythagorean theorem in elementary school

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Abstract. The purpose of this paper was to show that traditional methods of teaching mathematics in elementary school do not produce satisfactory effects. The implementation of contemporary teaching methods, such as the interactive method, aims at maximum involvement of students, by means of which the passive listener turns into an active participant of teacher-student, teacher-group and student-group communication. The effects of interactive education methods are much stronger than those of the teacher-fronted instruction, and that is the goal of modern teaching.

Keywords: educational effects, interactive method, the Pythagorean Theorem

Introduction

The title of this paper refers to the name of a great mathematician. Before delving into mathematics, it is important to make a reference to Pythagoras. Pythagoras of Samos is by many considered to be the first "real" mathematician. He is a very important personality who contributed to the development of mathematics, even though in essence very little is known about his work in mathematics. Pythagoras was born on a Greek island Samos circa 569 B.C. (today, this island belongs to Turkey), a son of a wealthy and prosperous tradesman with whom he travelled a lot. During these journeys, young Pythagoras encountered various teachers and thinkers of his time, who taught him philosophy and science (especially Pherekydes, Thales and Anaximander). In the year 518 B.C. Pythagoras founded a religious school of thought, which had many followers – the Pythagoreans. The exact date and circumstances of Pythagoras' death are unknown.

Around the year 508 B.C. Pythagoras' school was attacked by Philolaus, a nobleman from Croton, who had previously been rejected by Pythagoras owing to his unsuitable character. Pythagoras ran away from the attack to Metapontumand and it is believed that he died there circa 475 B.C. Unfortunately, none of Pythagoras' works have been preserved. The secrecy and collaborative efforts characteristic of his school make it difficult to discern Pythagoras' results from the work of his followers.

The major areas of Pythagoras' interest were the concept of number, triangle and the abstract idea of proof. Naturally, Pythagoras is today remembered for the well-known Pythagorean theorem: "In any right triangle, the area of the square whose side is the hypotenuse is equal to the sum of the areas of the squares whose sides are the two legs (the two sides that meet at a right angle)." Even though this theorem is named after Pythagoras, it was known to ancient Babylonians a thousand years before Pythagoras was born. Namely, Pythagoras did not discover the Pythagorean theorem (as many are quick to presume), but he was the first one who proved this theorem and hence it was named after him. According to some sources, when Pythagoras proved the theorem, he sacrificed an ox in honor of the gods who enlightened him.

Educational tasks of mathematics teaching

The tasks of mathematics teaching are educational, pedagogical and functional.

Educational tasks of mathematics teaching in elementary school, and consequently the teaching of the Pythagorean theorem, are to acquire knowledge. This corresponds to the following formulations: introduce, instruct, perceive, understand, acquire, explain, describe, define, prove, apply, deepen, etc. The educational aims when teaching the Pythagorean theorem are:

- to understand the geometrical interpretation of the Pythagorean theorem by means of a model with rectangle surfaces above the sides of a right triangle;
- to learn how to write, read and interpret the equation $a^2 + b^2 = c^2$
- to learn to solve standard problems by applying the Pythagorean theorem to a right triangle;
- to learn how to say and apply the inverse Pythagorean theorem.

Work methods

Educational aims in traditional teaching used to be realized by means of employing of various work methods. These teaching methods represent teachers' and students' work procedures within instruction, during which students acquire knowledge and develop abilities. These teaching methods are divided into:

- Verbal methods
- Visual methods

When applying verbal methods, the teacher's verbal expression is dominant. The verbal methods are:

- Method of oral presentation: narrating, explaining, describing, elaborating, proving,. . .
- Method of conversation or dialogue: conversation, discussion, dialogue, controversy, interview.

Visual methods connote the arrival to knowledge based on the written word, graphic documentation, conducting of experiments, etc. Visual methods are divided as follows: documentary and demonstrational methods. Documentary methods are: work method using course books, as well as the method of written and illustrative work. Demonstrational methods are: the method of demonstration and the method of experimenting and observing. The teacher's quality of work equals the success of students, so the overall achievement and educational results depend on the proper application of work methods. It is very important during the teaching process to apply various work methods and combine them. Therefore, a question may be posed whether all the methods are equally valuable. The choice of work methods depends on: – the content of the subject matter; – the type of class unit; – the developmental stage of students; – the teaching technology, etc. The aim in choosing work methods is to allow the student to fulfill the educational aim of the given teaching unit as easily as possible. Numerous research has shown that traditional teaching and work methods are not adequate in the present age. The organization of contemporary teaching and its methods requires much more efficiency. One of the most significant methods in contemporary teaching is the interactive method.

Interactive methods

Firstly, it is necessary to distinguish between the terms interactive teaching, interactive learning, interactive methods, interactive procedures, active classes, workshops, etc. Interactive teaching is a process the results of which are relatively permanent changes in thinking, emotions and behavior, which come to exist based on experience, tradition and practice embedded in social interaction. Interaction can be defined as an activity that a person A directs towards person B, who then replies with an activity Y (Robert A. Hinde, 1997). As can be seen from this definition, an either positive or negative valence can be recognized within social interactions. Interaction in school-related conditions is treated in literature within a frame of cooperative learning, pair work, group work, project-methods and other models applied in teaching. Numerous authors emphasize that interaction in school-related conditions has so far been neglected. Precisely because of that, it is not advisable to separate interactive learning from the social context of its realization. More specifically, if the social dimension were to be subtracted from interaction, there would be a danger of every action, even the individual one, being treated as interaction. Consequently, every learning might be treated as interactive, and that means that co-education would be neglected as it has been thus far.

Active learning is a conceptual syntagm that is often considered equivalent to interactive learning. This kind of equating, however, is not justified for at least two reasons: 1) active learning may be individual, 2) active learning may be single-direction – from subject to object, medium or other subject, but not the other way around. If interactive and active learning were to be equalled, it would occur that a situation is treated as interaction in which one side is passive, which can correspond to the term of active learning, but not interactive. What is active learning? Active learning can be defined in relation to its opposite – passivity. "Active learning aims at being proactive and enabling. A person can be confronted with a choice, such as a dilemma whether to get involved in the ongoings of the world and realize that in reality he/she can be active or passive. If that person should choose consciously, based on their cognition and experience, to be active and act in accordance with the nature of that experience, they can make an impact on their reality. If they should choose to be passive or submissive, then they react based on constructions taken from others. Active learning entices people to assume responsibility for the manner they relate towards reality" (Morgan and Ramirez, 1984, p. 9). Based on the above-mentioned determinants, active learning can be defined as a learning process which an individual pursues consciously with the intention to understand the experience and act on their environment and society, as well as in order to change him/herself. Active learning may be one-directional, from X toward Y, but also two-directional, where X and Y can be persons, media or objects. Accordingly, active learning, as opposed to interactive learning, may but does not have to be realized within social interaction (Suzić, 2003).

Methods of interactive learning are those where learning through social interaction is dominant. Such methods are team method, mosaic-method, learning together, group project-method, structural approach; cooperative map conceptualization, collaborative learning, tribe method, cooperative scripts, as well as other methods which support coeducation and social interaction within the learning process. If during instruction interactive methods are predominant, it can be said that this instruction is based on interactive learning, but if interactive learning methods are applied sporadically, this instruction can be labelled traditional or other, depending on the type of dominant activity or application of methods.

Interactive teaching is the kind of teaching where methods of interactive learning are predominantly applied. If during the majority of mathematics instruction interactive methods are not applied, we may not consider this interactive mathematics teaching, or the interactive teaching of any other school subject.

Interactive teaching procedure implies one or more activities which constitute two-way actions between two or more persons or groups, and are performed within the framework of interactive and other teaching methods. An example of this is a teacher performing a teacher-fronted presentation, and at some point asking the auditorium to spread the fingers of their left and right hands, and then put the palms together so that the fingers are intertwined; after that, the teacher asks the auditorium to identify which thumb of which hand is on top. This activity, as is obvious, implies interaction, but it is not a teaching method; it is a procedure which should serve as an introduction into the presentation about the relation of the left and the right brain hemisphere. This example relates to the application of interactive teaching procedure in traditional teaching. A sequence of such activities is applied as a part of every interactive teaching method. The mere act of showing emotions will produce mutual trust among children, as well as towards the teacher, and children will feel a stronger sense of belonging to the group, as well as a certain degree of emotional safety.

The aim of this elaboration is the development of emotional competency in children. Such competencies are:

1) emotional consciousness, recognizing one's own and other people's emotions;
2) self-confidence, clear sense of one's abilities and limitations;
3) self-control, control of distracting emotions and impulses;
4) empathy and altruism;
5) truthfulness, building standards of honor and integrity;
6) adaptability, flexibility in accepting changes;
7) innovation, openness to new ideas, approaches and information. (Suzić, 2002).

The aim aspect of interactive learning refers primarily to accomplishing the desired outcomes of interaction. What is accomplished as the goal of interaction should be desired by all the participants in interactive learning. The teacher who creates interaction during instruction should start with the presupposition that the set aims are not desired by all the students, therefore it is necessary to dedicate time in order to arrive at a concensus regarding the desired aims and, in accord with such a decision, apply suitable methods and techniques. However, it would be unrealistic to expect that all students will always accept all the aims set forth by the group. In that case, such individuals should learn to cooperate with regards to group aims aware that they have the right to withhold the autonomy of their commitment, and in the meantime attempt to work towards the acceptance and/or fulfillment of their own aims within group interaction.

The working aspect of interactive learning refers to accomplishing the set aims while working on the subject matter or program, in other words those aims that are to be accomplished through interaction. The direction of communication in interaction depends on the aims that are being fulfilled through interactive learning. The content of work dictates the direction of the communication. Sometimes it is possible that there are many optimal directions in interactive communication. In addition to the content, the direction of communication also depends on the number of participants in interaction, as well as on the organizational setup or the work model. Communication in a group may flow through all channels, from every student to every other student or to the group as a whole. If within this kind of communication the leader blends in with the others, they do not stand out, and there is the so-called laissez-faire, unchanneled, or free communication. Aims constitute memorizing, knowledge, separating the important from irrelevant, the affective encompasess experience, emotional consciousness, recognizing emotions, group work implies completion, the understanding of other individuals and groups, and aim orientation signifies intentionality, knowledge of the trade or profession.

The main idea/guideline in the realization of these aims should be:

- What I hear – I forget
- What I see – I remember
- What I do – I understand.

The tasks needed to fulfill the aim should be completed by means of creating concepts of content and activity realization. The student should be placed in the role of the creator and executor, brought into an immediate connection with the content (student creator, coworker and implementer). The starting points of interactive teaching, and consequently the employing of interactive methods are: – children learn in different ways, – children learn best through their own activity, – learning is an active construct of knowledge through personal activity, experience and exchange.

Interactive teaching is done with the aim of applying and further developing the knowledge through practice. It is based on the concept of cooperative learning and stems from the fact that learning is a process of developing knowledge. The teacher's role is to lead the entire process, in the educational, as well as in a technical and organizational sense. It is evident that teachers spend a great portion of their time on activities related to preparation for the realization of the subject matter. They work much more on preparing the activities, student answer recording protocols and preparing students for completing those activities, and less on realization of the subject matter content, while the dynamics of their activities in traditional teaching is completely different.

In interactive teaching, students are regularly and carefully prepared, which cannot be said of traditional teaching. It is also evident that students are more and differently engaged in other stages as well, especially in the realization of the subject matter. The modernization of education assumes the permanent development of methods, forms and resources in teaching. Active methods of teaching mostly encompass:

1. *Brainstorming*
2. *Simulation*
3. *Method I KNOW – I WANT TO KNOW – I HAVE LEARNED*
4. *KWL-Table*
5. *Roleplaying*
6. *Insert-method*
7. *Case study*
8. *Cubing*

9. Project work

10. Workshop

11. Snowball

12. Demonstration method

Mathematics teaching is changing due to the demands placed on schools, and mathematics learning, by the contemporary society. The subject of mathematicsis adapted to the new technological and technical possibilities available to students (and also generally), so in this sense the modern school can not be envisaged without modern didactic aids, such as computers. Ideally, mathematics learning in school is aimed at solving problems with active participation of students themselves as much as possible. Here we are talking primarily about the so-called open-type problems, problems that can be solved via more than one path and the solution to which is uncertain.

Such problems demand an active research approach and (often) cooperation between several students. In its performance, contemporary teaching is dynamic, rich in various methods and activities. Interactive materials used in interactive teaching may be:

- PowerPoint presentations: PowerPoint presentations are mostly used in introduction to a certain unit, for historical overviews, etc. In certain cases, along with the presentation, a guidebook is provided, in which the content of each slide is described in detail.
- Mathematical projects: constitute a modern and widely spread form of work in mathematics teaching– solving simple, interesting and (often) applied problems. Other materials include quizzes, Sieve of Eratosthenes; interactive calculator; GeoGebra – dynamic applets.

The Pythagorean theorem and interactive methods

During the analysis of the Pythagorean theorem, i.e. proof and application, interactive methods of demonstration and insert-method were employed. *Insert-method* was used to test knowledge about triangles – each student was given a text. . .

— Students are familiarized with the procedures and are required mark the margins of the text.

Procedure: Put a tick mark in the margin if something that you read *confirms* what you already know. Put a plus sign "+" if the information is *new* to you. Put a minus sign "−" if the information you read is *contradictory* or *false* to what you know. Put a question mark "?" if you wish to *know more* about it, in this instance with the special emphasis placed on the triangle.

Method of demonstration: visual method based on the principle of what is obvious in teaching, and on the use of obvious resources (models, schemata, phenomena, characteristics, sketches,. . .)

— It has a wide application in teaching since perception and observation are important sources of knowledge;

— Combines well with almost all other methods.

Afterwards, with the help of PPt presentation of the Pythagorean theorem, pair work and possible correlation with other subjects (art classes and technical education), children create a a poster/billboard. Classes during which the application of the Pythagorean theorem was elaborated also included the method of demonstration, insert-method with PPt presentations and interactive material in terms of geometrical shapes (for example, when considering the height of an equilateral triangle, the height of the triangle should be drawn in the picture and after the demonstration of proof, formulas should be written down and then glued in the notebook).

Research results and conclusion

A research examining the effects of the application of interactive methods in teaching the Pythagorean theorem and the attitude of students towards instruction was conducted,.and a specific test was used to examine the outcomes of this type of teaching.

The research was conducted in elementary school “Mehurići” in Travnik by the author of this paper and the colleagues who teach mathematics.

This research should provide answers regarding the possibilities of implementing interactive content for the purpose of improving the efficiency of the teaching process, as well as about possible positive effects on students’ achievements in learning mathematics.

The research sample was comprised of two groups of examinees:

- students – 105 students of elementary school “Mehurići” in Travnik (7^{th} grade students, five classes total),
- teachers – mathematics teachers (from two schools in Travnik, 7 elementary school teachers)

In two classes, VII-a,c the teaching was realized in a traditional manner, while in classes VII-b,d,e it was conducted using the above-mentioned methods. The same teacher was in charge of instruction in all five classes.

The class results pertaining to success in mathematics are presented as follows:

Table 1. Review of previous knowledge in mathematics across classes.

Class	Grades					Total number of students
	5	4	3	2	1	
VII-a	6	4	7	2	3	21
VII-b	5	6	6	3	1	21
VII-c	4	7	6	2	2	21
VII-d	5	5	6	3	2	21
VII-e	5	6	5	5	1	21

The research was conducted in two segments: firstly a survey regarding work motivation and the level of interest in instruction, and secondly a control knowledge test, which was the same for all students.

- Motivation as an important condition for successful instruction and its outcomes was the first question posed to students.

It is interesting that as many as 46% of students replied that they studied mathematics only because they wanted a better grade, and 26% because they were really interested in the subject matter.

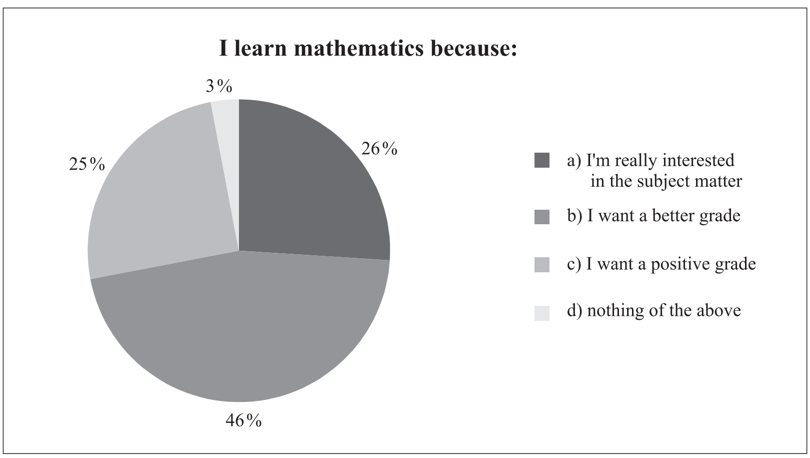

Figure 1. Results of students' survey about motives for learning.

Regarding the question on how interesting the classes were, in instruction where interactive methods were used, 98% of the students gave positive feedback, while in the instruction without the use of interactive methods (VII-a, c) the students replied that the classes were monotonous, boring, etc.

Table 2. Results of the test in percentages for each task of the test.

Classes	Test tasks					
	1st task	2nd task	3rd task	4th task	5th task	6th task
VII-b,d,e	87%	92%	82%	78%	66%	79%
VII-a,c	86%	91%	76%	59%	57%	69%

- The control test contained six tasks. The first and the second task were theoretical questions, the third task required the calculating of the volume and the surface of a right triangle if one side and the hypotenuse are known, the fourth task was to construct a square equal to the sum of two given squares. The fifth and the sixth task were the applications of the Pythagorean theorem (squared and equilateral triangle). The following are the test results.

The above table shows that the test results in three classes where the interactive methods of teaching were used are 46% better than the results in the two classes where the Pythagorean theorem was taught by means of traditional teaching. (Note: the control test and results were completed by a colleague, so as to retain objectivity). The results confirm that interactive methods in teaching produce better results, which was the aim of the research. These results yielded a further survey among teachers. Its aim was to examine teachers' interest in contemporary teaching.

It is interesting to point out that within this research 38% of examinees never even thought about using interactive methods, over 50% declared that they do not even know how to use a computer in mathematics teaching, while over 2/3 of examinees are not familiar with the use of so-called java virtual classrooms, and 14% do not know what interactive content and interactive teaching are.

Conclusion

Mathematics teaching through discovery with the help of a computer and the dynamic geometry program GeoGebra, as well as by means of educationally conceptualized and didactically shaped interactive digital educational materials, ensures the participation of all students in the classroom, improves motivation for learning mathematics, entices individual drawing of conclusions, as well as cooperation between students.

It is also important that students alone or in pairs learn at their own computers, and test and discover new concepts independently. Knowledge obtained in this way will be stored long-term, and the students will learn to think, solve each new problem much more easily and master the art of discovery. However, whether this is viable in real-life instruction depends largely on conditions in schools.

It has been shown that students are willing to cooperate in introducing changes in the teaching process, but if the mathematics teacher is not able to use computerized classrooms for the purpose of instruction, he/she will not be able to implement interactive methods.

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Obrazovni efekti interaktivne metode u izučavanju Pitagorine teoreme u osnovnoj školi

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Sažetak. Svrha ovog rada bila je da se pokaže da tradicionalne metode u nastavi matematike u osnovnoj školi ne daju odgovarajući efekat. Primjenom savremenih nastavnih metoda, kao što je interaktivna metoda ima za cilj maksimalnu angažovanost učenika, gdje se iz faze pasivnog slušaoca prelazi u fazu aktivnog učesnika u međusobnoj komunikaciji: nastavnik-učenik, nastavnik-grupa i učenik-grupa. Obrazovni efekti interaktivne metode su puno bolji od frontalnih oblika rada što jeste cilj osavremenjavanja nastave.

Ključne riječi: obrazovni efekti, interaktivna metoda, Pitagorina teorema

Touch Math

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Abstract. Current legal documents of a large number of European countries give advantage to educating children who belong to the same generation but have different levels of knowledge skills and abilities. At the same time we are witnessing a large number of exclusion of students from schools and the low achievements of a significant number of students in Europe (EC, 2001, according to Bartolo et al., 2007). Mathematics learning disorders are noticed in achievements that decline and are lower than expected considering age, education and intelligence level. In regular classes there is a strong requirement for adjustment and preparation for teaching mathematics, not only for the students with disabilities, but for all students. "Touch Math" is an innovative teaching method that enhances mathematical achievements in the field of summing, subtraction, multiplication and division. The "Touch Math" program ensures multisensory approach and gradual instructions that apply dot notation for each number (not the numerical line) and that can be used by students with difficulties remembering facts. Benefits of multisensory methods in teaching basic concepts in mathematics are confirmed by many studies (Scott, 1993, Thornton, Jones and Toohey 1983, according to Calik and Kargin, 2010).

Teaching in accordance with the method of the "Touch Math", based on the direct teaching approach, is effective and sustainable. It can be generalized and it has a social value in teaching the basic summing skills to students with disabilities.

Keywords: addition strategies, touch math, students with difficulties

Introduction

The teaching of mathematics in primary school is mostly concrete and inductive. Considering concrete objects and examples the teacher comes to abstract settings and generalizations (Kurnik, 2009). The main aim of teaching mathematics is to create an individual who is mathematically literate and will be able to apply the ways of mathematical thinking, not only in further education, but also in everyday life. The positive students' attitude towards mathematics is very important, because mathematics develops logical and rational way of thinking and reasoning. From its earliest age child is encouraged to do the activities like joining, sorting, sequencing and matching. All these procedures lead to the development of abstract thinking. Students should be gradually and appropriately taught to analyze, synthesize, generalize, specialize, abstract, concrete, induce, deduce, and spot analogies (Kurnik, 2009). By applying and combining appropriate activities students will gradually adopt the mathematical way of thinking.

Abilities for mathematics differ from individual to individual. There are many cases in which the student is not able to understand the basic mathematical concepts. Difficulties in learning mathematics, which are expressed in various ways, are present in children with different degrees of intellectual development Those difficulties can be caused by neurological dysfunction, insufficient development level of cognitive intelligence and higher mental functions, underdevelopment of the fundamental skills that are a prerequisite for the acquisition of mathematics, the existence of language difficulties and specific difficulties in reading and writing, irregularities in the teaching process or child's emotional condition (Mahesh C. Sharma, 2001). The prevalence of mathematics learning disorders is about 6.5% (Vlahovic – Štetić, 2009) depending on the criteria which determine it.

In the process of teaching there is a necessity for the appropriate adjustments according to the student's needs in order to ensure effective learning. These adjustments refer to the planning during education, differentiation of teaching methods, content preparation and preparation of evaluation on various levels (Spencer, 1998, Wood, 1992, according to Calik and Kargin, 2010). In formal education adjustment and preparation for teaching mathematics are required not only for the students with disabilities but for all students. Lock (1996; Calik and Kargin, 2010) states that the minor changes that teachers make in teaching mathematics content, do not only increase the number of correct answers but also help the students to understand the process more clearly. When the teachers clearly express objectives of the teaching, give instructions and make simple adjustments, the students' performance, success and their level of interest grow accordingly Furthermore, the objectives reflect the expectation of learning, which has an impact on students' success. According to Porter and Brophy (1988; according to Calik and Kargin, 2010) successful teachers always clearly express their expectations and objectives of education. While introducing the objectives to the students, successful teachers also explain in detail what a student needs to do to be successful, and what he / she will learn through classes (Christenson, Ysseldyke, and Thurlow, 1989; according Calik and Kargin, 2010).

Characteristics of the students with disabilities

Many students with disabilities have problems with mathematical calculations, solving mathematical problems and do not progress in math as quickly as their classmates without difficulties with math normally do (Cawley and Miller, 1989, Miller and Mercer, 1997, by Nora D. Green, 2009). The difficulties that students have may also be related to teaching styles and methods. Students with learning disabilities may have difficulties in memorizing, but also in the orientation, e.g. inability to use the numerical line (Miller & Mercer, 1997). Students who think they are not good at math are also ready to avoid it at all costs (Wadlington and Wadlington, 2008, according to Green, 2009). Multisensory approach to teaching mathematics should contribute to improving performance and success in math and to raising self-esteem in students with disabilities (Santoro, 2004: 28, according to Green, 2009).

Touch math

There is a lot of research on summation strategies, but only a small number of these studies is focused on children with disabilities (Groen and Parkman, 1972, Hughes 1986, according to Calik and Kargin, 2010). One of the most prominent research on different strategies that students use when solving summation tasks at different levels of learning is Carpenter's and Moser's research (1984; according to Calik and Kargin, 2010).

The first strategy is recounting. It consists of counting where students use fingers or other objects. For example, when solving the task $4 + 5$ a student begins by opening four fingers of one hand while counting to four and then he/she opens five fingers of the other hand and continues to count. In fact, the student counts all his/her opened fingers, in order to come to a solution, i.e. number 9 This strategy is limited, since the student can easily sum up only to 10, but majority of students use it in the initial stages of learning. When this strategy is mastered, the student usually needs to take a different strategy for summation tasks solving.

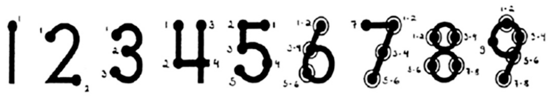
Another strategy is counting on fingers. This strategy is implemented by saying first number to add up and from this number is counted further on (Carpenter and Moser, 1984; Secada, Fuson and Hall, 1983; according to Calik and Kargin, 2010). For example, a student will solve a problem $4 + 5$ by saying the first number, in this case 4, and then he/she will start counting from 4.

The last stage of summation learning is storing and re-remembering facts on the summing from the long-term memory (Carpenter and Moser, 1984 according to Calik and Kargin, 2010). For example, during the time a student memorizes the summing task $4 + 5 = 9$. Not all of the students prefer to use strategies of recounting and counting on the fingers especially students with disabilities because counting on the fingers can be embarrassing for them because they can see how their class peers without disabilities quickly add up by using their memory, not helping themselves with fingers. That is why a lot of students with disabilities do not want to use the strategy of the summation on the fingers which could be seen by their class peers or by the teacher. One way to overcome this obstacle could be the use of tagging methods, by applying and using the dots, wherein the dots are associated with each number from 1 to 9, according to a certain pattern. By using this technique students count dots on the numbers rather than the fingers and over time they learn to count positions where the dots are and later the dots are removed from the numbers. *Method of tagging by using dots (mathematics through touch-touch math)* includes visual, auditory and tactile learning. Students mark the dots by touching them (dots on the numbers and dots in the circles) while looking at the number (visual) and while pronouncing the number (auditory) they touch it with a pencil (tactile). Students are taught to count dots on each figure in order to be assisted and helped in summing subtracting, multiplying and dividing. In summing students count forward and in subtracting they count backward (Bullock, Pierce and McClelland, 1989; according to Calik and Kargin, 2010). This is a method that helps students with disabilities to solve problems without using objects (Scott, 1993; according to Calik and Kargin, 2010). When using the technique of "touch math" students begin to learn the positions of dots on each number from 1 to 9, according to a certain pattern (sample). Once the task is mastered, the teaching begins with the most basic tasks of summing single-digit pairs. Students are taught to start with the first number, count all the dots on that number, and then continue to count the dots on the next number until all dots are counted. When the dots are removed from the paper, students can still touch them with the pencil using their memory. While reading mathematical tasks students are encouraged to read the task and the solution aloud in order to facilitate memorizing. The technique "touch math" occurs in learning adding up according to the same learning strategies that students naturally develop for addition problems solving. The system provides a method for teaching adding strategies involving recounting and counting, but does not require retrieval of stored information from memory, which is an area where many students with disabilities have problems. It is expected that the facts on adding up will be gradually stored in long-term memory of students. Benefits of multisensory method of teaching basic concepts in mathematics were confirmed by many studies (Scott, 1993, Thornton, Jones and Toohey, 1983, by Calik and Kargin, 2010).

Application in the teaching process

This school year we have used touch math in the first grade of the district school Sarvaš. There are thirteen students in this grade. The grade itself is extremely heterogeneous and requires an individual approach to every child, which includes being familiar with their prior knowledge, skills and experiences, as well as with their family circumstances. We were interested to apply the new teaching strategy of addition and subtraction recognizing that diverse groups are good and productive and providing each student with the opportunity to progress in line with their potential and to reach the expected results at their own pace. So we have decided to apply the touch math.

We have applied this teaching strategy to the entire class, regardless of whether students have difficulties or not. Students were first taught positions of the dots on every number from 1 to 9 according to the offered form (sample).



When they had mastered that, we started with adding up pairs of single-digit numbers. In the example, task $2 + 5$, the students first counted the dots of the first addend and then continued counting the dots on the second addend until they have reached the number 7. Then they verbally, aloud, repeated the entire task. The next step was to "remove" the dots from a larger number and to add up the smaller addend. When they had mastered this step, we removed all the dots, and taught them to continue counting to the largest number adding the dots that had been removed from all numbers. Students could continue, if necessary, to touch the dots with the pencil by using their memory. In calculating, the whole task was uttered aloud. We provided the students a method by which they could sum up in the way that they could recount and count. This method helped the students without disabilities to automatize adding up and to store it in the long-term memory. In the same time the students with disabilities were offered a method that does not require the retrieval of information stored in memory, but with time such information stores permanently too.

Teaching subtraction of single digit numbers was taught in a similar manner. In example 7 – 4, students started from minuend and they covered with their finger the dots number of subtrahend. They verbally followed the flow of computation. Then all the dots from tiles were removed, but students could still use the pencil in a way to touch the spots on a number. After applying this teaching method we evaluated students' progress. The results of written tests of all students, students who are educated by the regular program and students with disabilities were very high. The students without disabilities automatized addition and subtraction much faster, as expected. We also noted that the students in the calculation use their fingers less, or not at all. Next school year we plan to apply the methods of the *touch math* in order to teach multiplication of numbers up to 100.

Conclusion

An important social objective is to include all students in regular schools, what implies children with disabilities. In their everyday work teachers encounter children from different social, cultural and ethnic backgrounds, and therefore are the classes more and more heterogeneous (EC, 2003, by Bartolo et al., 2007).

In its legal documents the Republic of Croatia urges and supports integration. In the Republic of Croatia the system of primary education for the population of students with disabilities is regulated by the Law on Education in Primary and Secondary Schools (NN 87/08, 86/09, 92/10 and 105/10.), and by the regulations and directions of the Ministry of Science, Education and Sports. These documents (and other unlisted) give priority to education that respects and includes a variety of children, but at the same time we witness a large number of exclusion of students from schools and the failure of a significant number of students in Europe

(EC, 2001, according to Bartolo et al., 2007). Benefit of multisensory methods of teaching basic concepts in mathematics was confirmed by many studies (Scott, 1993, Thornton, Jones and Toohey 1983, according Calik and Kargin, 2010), but also by our own experience.

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Matematika dodirom

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Sažetak. Važeći dokumenti velikoga broja zemalja u Europi daju prednost obrazovanju djece različitih razina znanja, vještina i sposobnosti koji pripadaju istoj generaciji. Istodobno se određeni broj učenika isključuje iz škola zbog neuspjeha (EC, 2001, prema Bartolo i sur., 2007). Poremećaji učenja matematike uočavaju se u padu postignuća s obzirom na dob, obrazovanje i razinu inteligencije. U redovitim razrednim odjelima potrebna je prilagodba i priprema za podučavanje matematike, i to ne samo za učenike s teškoćama nego za sve učenike. "Matematika dodirom" inovativna je metoda poučavanja koja povećava matematička postignuća u procesima zbrajanja, oduzimanja, množenja i dijeljenja. Program "matematike dodirom" osigurava multisenzoran pristup i postupne instrukcije koje se koriste notacijom točaka za svaki broj (ne brojevnim pravcem) kojim se mogu koristiti učenici s poteškoćama pamćenja činjenica. Korist multisenzornih metoda u podučavanju osnovnih pojmova u matematici potvrdila su mnoga istraživanja (Scott, 1993; Thornton, Jones i Toohey, 1983; prema Calik i Kargin, 2010).

Poučavanje metodom "matematika dodirom", temeljeno je na neposrednom pristupu poučavanja, učinkovito je, održivo, može se generalizirati i socijalno je vrijedno pri poučavanju osnovnih vještina zbrajanja učenika s teškoćama.

Ključne riječi: strategije zbrajanja, matematika dodirom, učenici s teškoćama

Visualization of mathematical concepts – some examples from geometry teaching

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Abstract. The abstract nature of mathematical concepts requires more effective illustration. For beginners this is solved with difficulty because of good tools, e.g. Dienes set often does not concretize the concepts, but it also represents a higher level of abstraction. Children's use of everyday objects can help implement the transition between reality and abstraction. Examples of this process will be shown in this paper.

Keywords: concept teaching, teaching of disjunction, teaching of the concept of the plane

Introduction

When teachers say that spontaneous development occurs – maybe there is little scientific knowledge about the development of some mathematical concepts. It would be useful also for teacher-students to know more about this process. We are looking for concepts, which are not taught in an implicit way in the school. Development of these concepts comes from an action, hands-on activities and other experiences from every day life (Zimmermann-Cunningham, 1991).

Examples

First we will see two extreme examples. They will help us to understand the third example, the problems of understanding geometry among young low-achieving students.

Relationship between space and plane and three-dimensional shape representation in a plane

Why and how can we teach geometry to blind students? We have started a joint study with Emanuela Ughi from Perugia.

"We had to help a blind girl, 19 years old – a very brilliant student. We first interviewed her, and were surprised to discover that she had very confused ideas about some basic facts, like the use of the words horizontal, vertical, oblique (her miscomprehension seemed related to the work she made at school about cartesian axes). Moreover, she had no clear idea of a path that a stone describes when falling. And, also, when arriving to a corner of a building and changing direction, she had no idea that there was a size of the angle measuring her turning." — wrote E. Ughi.

In connection to this situation, a simple experiment will be showed to the audience of the conference about problems of "visualisation", when the picture cannot be seen.

The concept of infinity

Infinity is a hard concept also in the context of geometry (Jirotková, Littler, 2004). Poincaré's Disk Model for Hyperbolic Geometry makes it possible to look "closely" the geometrical infinity. We will show Lajos Szilassi's Bolyai.exe software and students' drawings of regular hexagon on hyperbolic plane ([4]).

Segments, angles, cube, polyhedron – low-achieving students' errors

Some students (age about 10) cannot copy a segment, or an angle. They have problems when they have to count edges, faces and vertices of a cube. They cannot understand pictures of cuboids in their mathematics textbook. We will present a short lesson in Perspective – there is rich literature on this topic, but often we forget that first we have to learn to "understand" the bird's-eye view. In European culture, this happens almost imperceptibly. But it does not always happen. In this case, teachers should help. Together with my colleagues we made a lion-polyhedron. It can show the gaps and can help the students to understand their task ([5]).

In connection to this problem, we propose experimental teaching for which we prepared slides which can serve for multiple purposes. The characters appearing in the slides, Daddy bear and Teddy bear, discover in each part of a story some mathematical relationships and learn new concepts. It serves to teachers as a model of the inquiry-based learning. In the frame of the story pupils' tasks got meaning and significance. The pictures show examples of hands-on activities. These are all simple, interesting activities, behind which deep mathematical relationships were found. The slides do not only show examples of hands-on activities, but also how children move to symbolic thinking. The children's experiences were described in words, images and symbols as it was done by characters from the story (Munkácsy, 2012).

Conclusions

Our studies have shown that many traditional visual tools and methods sometime are not useful for beginners in mathematics education, while some surprising situations will help us to find an effective solution. We think that the visualisation should get bigger role in the classroom. We think also that every appropriate methodological tool is a good tool for problem-solving in mathematics. The gifted students should be able to use the symbolic language of mathematics.

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A matematikai fogalmak szemléletes bemutatása – néhány példa a geometria tanításából

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Összefoglaló. A matematika foglamak absztrakt volta hatékony szemléltetést igényel. A kezdők számára ez nehezen megoldható feladat, mert sok jó eszköz, mint például a Dienes készlet számukra nem konkretizálja a fogalmat, hanem az absztrakciónak egy még magasabb szintjét jelenti. A gyerekek által is használt mindennapi tárgyak segítik a valóság és az absztrakt foglamak közötti kapcsolat megértését. Ebben az írásban ennek a folyamatnak néhány példáját fogom bemutatni.

Kulcsszavak: fogalmak tanítása, a diszjunkció fogalma, a sík fogalmának tanítása